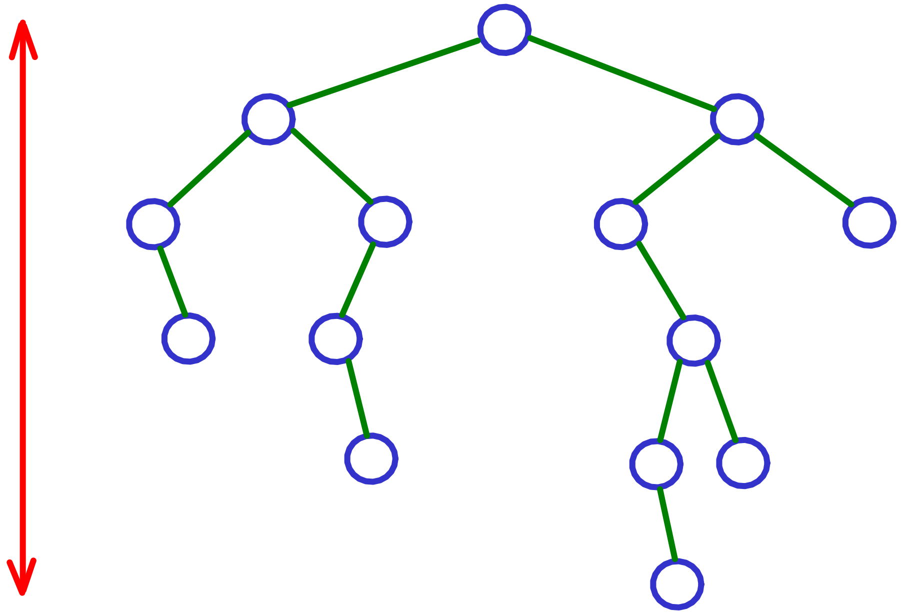
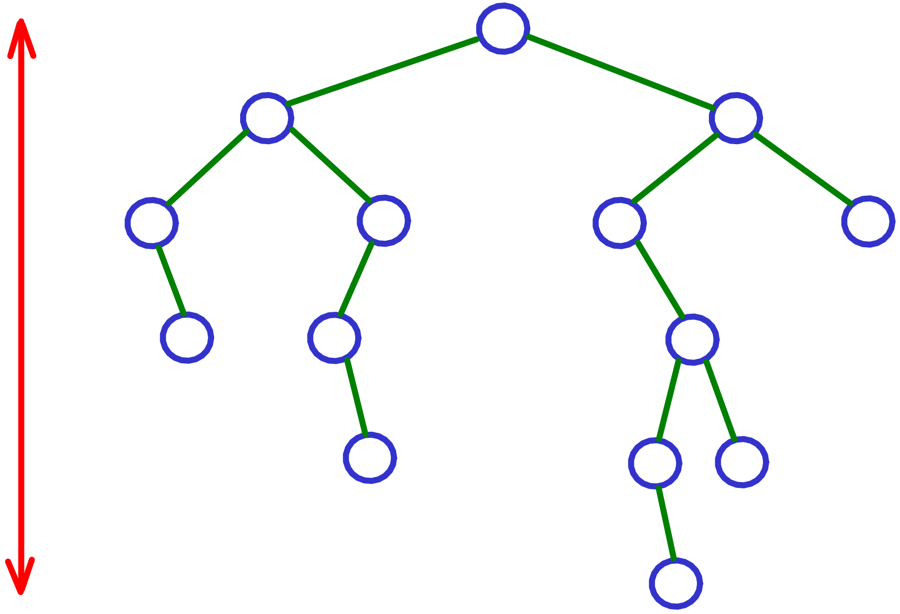
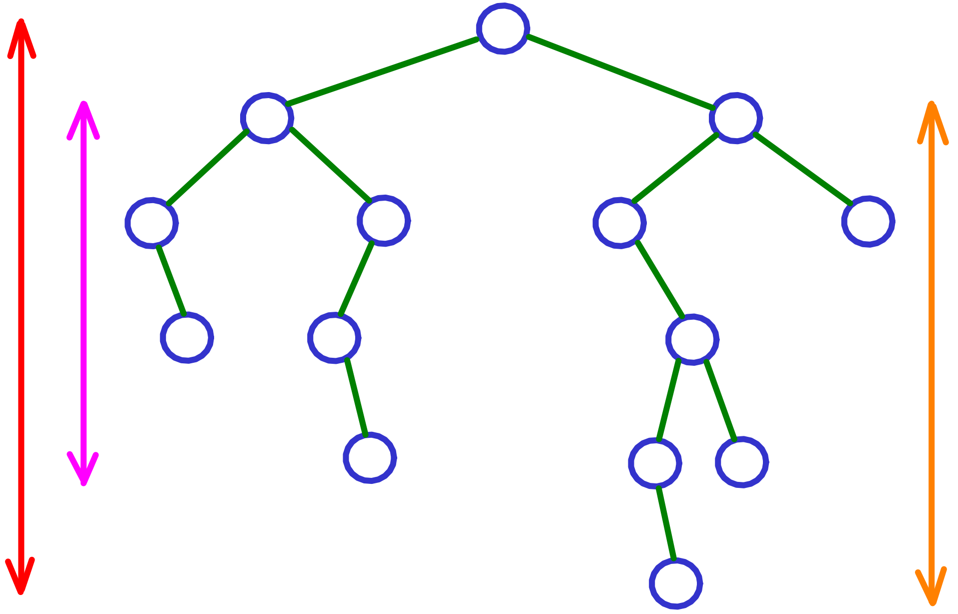


Expected height of randomly built BST

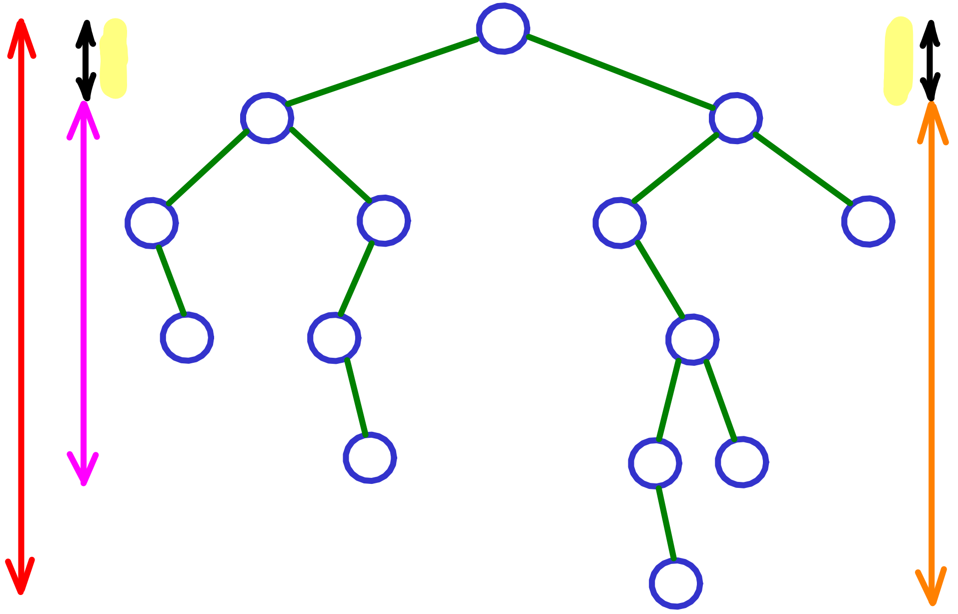


$\underbrace{H(n)}$ = ?

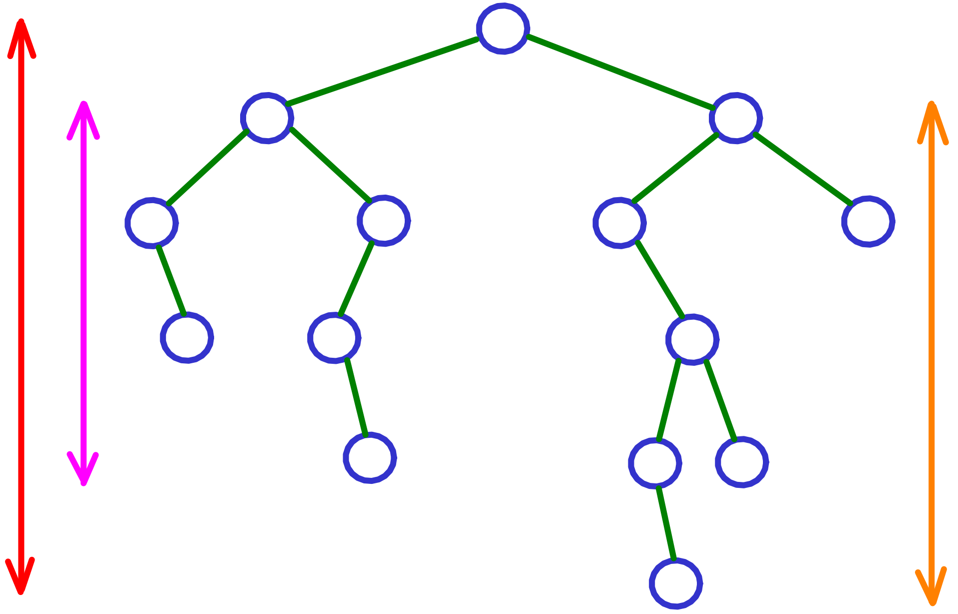
Expected height of a randomly constructed BST with n elements



$H(n) = ?$

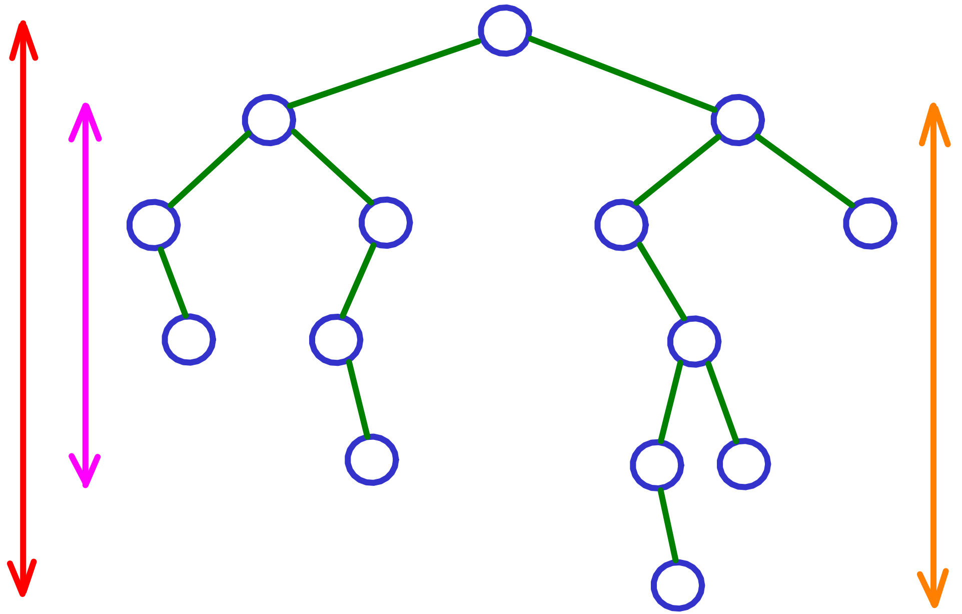


$$H(n) = 1 + \;?$$



$$H(n) = 1 + \max\{H(k), H(n-k-1)\}$$

for some random k $(0 \leq k \leq n-1)$

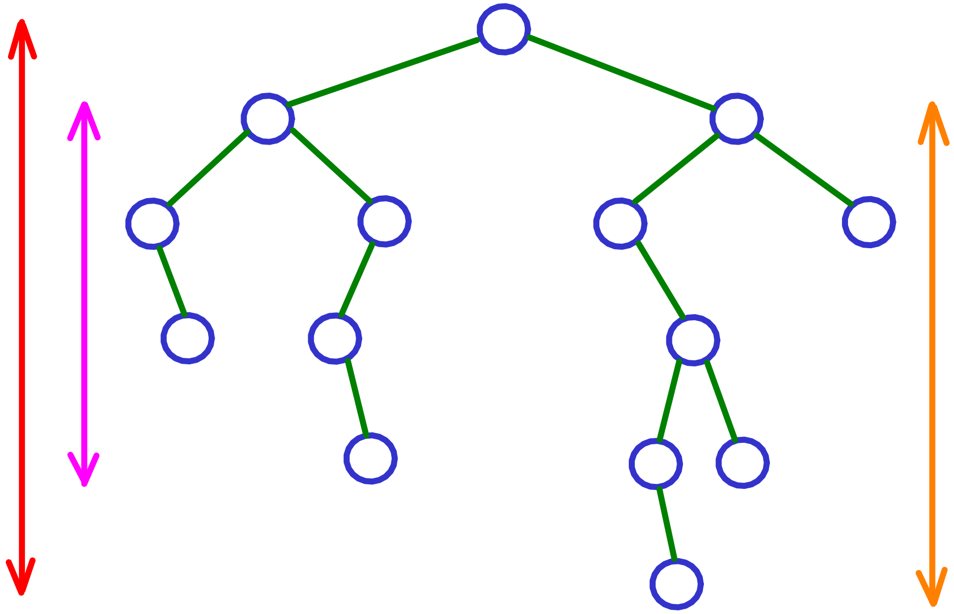


$$H(n) = 1 + \max\{H(k), H(n-k-1)\}$$

for some random k $(0 \leq k \leq n-1)$

If $\frac{n}{4} \leq k \leq \frac{3n}{4}$

$\hookrightarrow H(n) \leq$?



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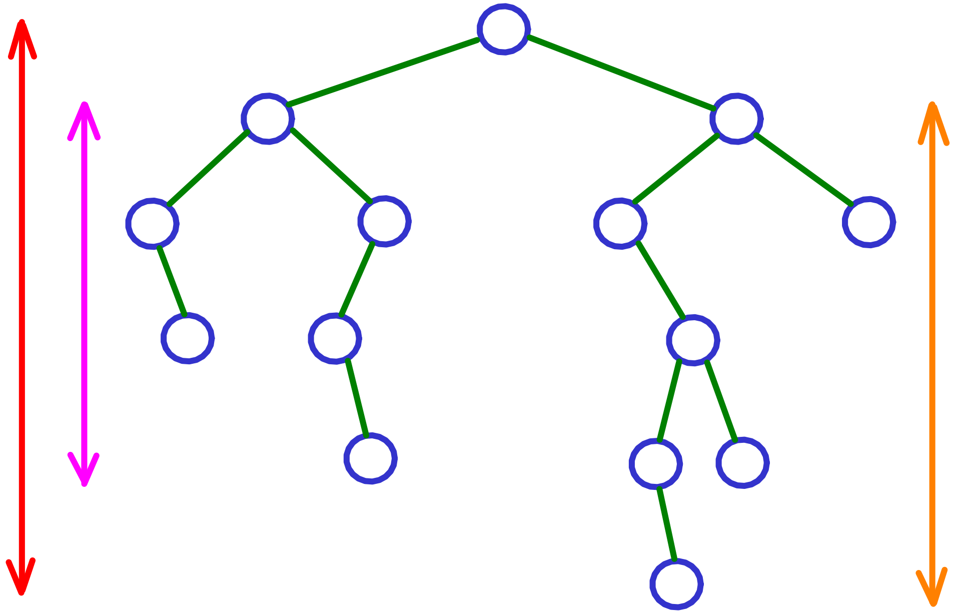
for some random k $(0 \leq k \leq n-1)$

If $\frac{n}{4} \leq k \leq \frac{3n}{4}$

$\hookrightarrow H(n) \leq 1 + H\left(\frac{3n}{4}\right)$



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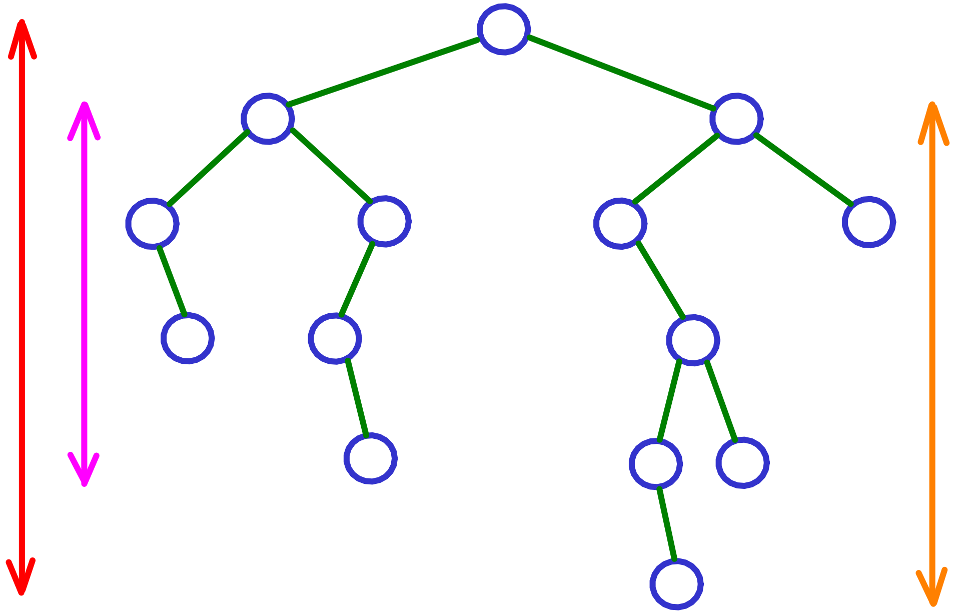

$$H(n) = 1 + \max\{H(k), H(n-k-1)\}$$

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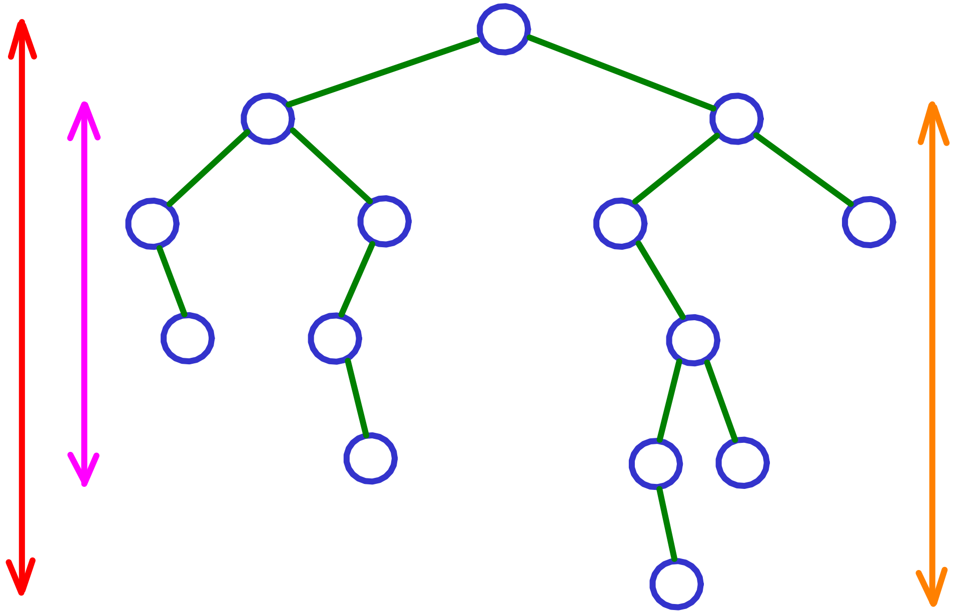
If $\frac{n}{4} \leq k \leq \frac{3n}{4}$

$$\hookrightarrow H(n) \leq 1 + H\left(\frac{3n}{4}\right)$$

else $H(n) \leq 1 + H(n-1)$

$$< 1 + H(n)$$



$$H(n) = 1 + \max\{H(k), H(n-k-1)\}$$

for some random k $(0 \le k \le n-1)$

If $\frac{n}{4} \le k \le \frac{3n}{4}$

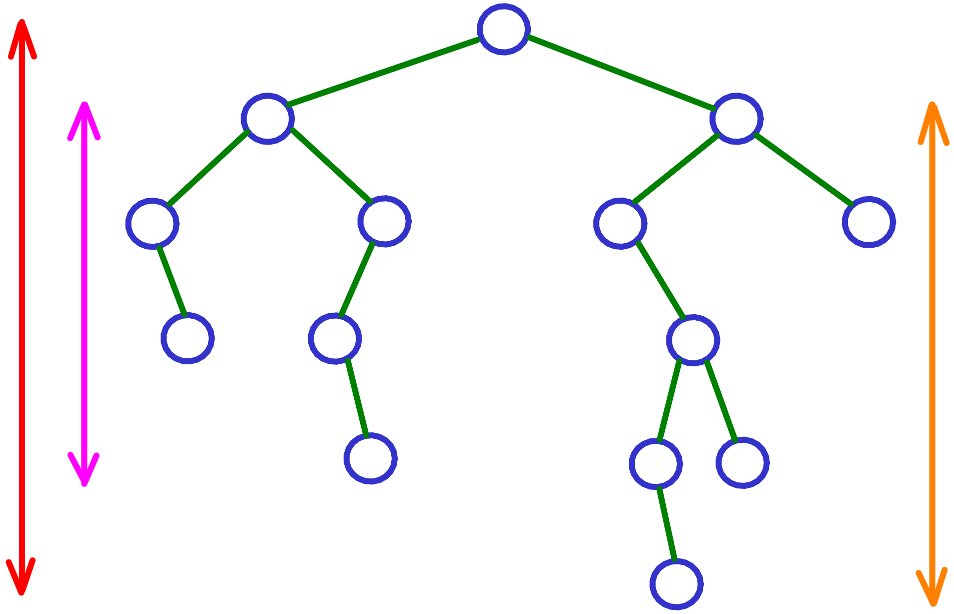
50%

$\hookrightarrow H(n) \le 1 + H\left(\frac{3n}{4}\right)$

else $H(n) \le 1 + H(n-1)$

50%

$< 1 + H(n)$



$$H(n) = 1 + \max\{H(k), H(n-k-1)\}$$

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50%

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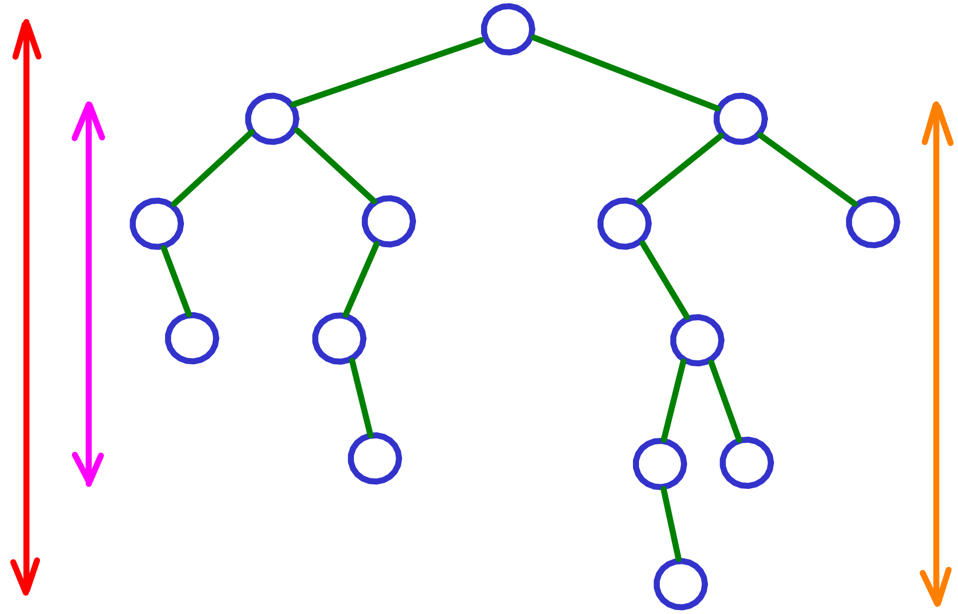
50%

$< 1 + H(n)$

Expected height will be:

$H(n) \leq$?



$$H(n) = 1 + \max\{H(k), H(n-k-1)\}$$

for some random k $(0 \leq k \leq n-1)$

If $\frac{n}{4} \leq k \leq \frac{3n}{4}$

50%

$\hookrightarrow H(n) \leq 1 + H\left(\frac{3n}{4}\right)$

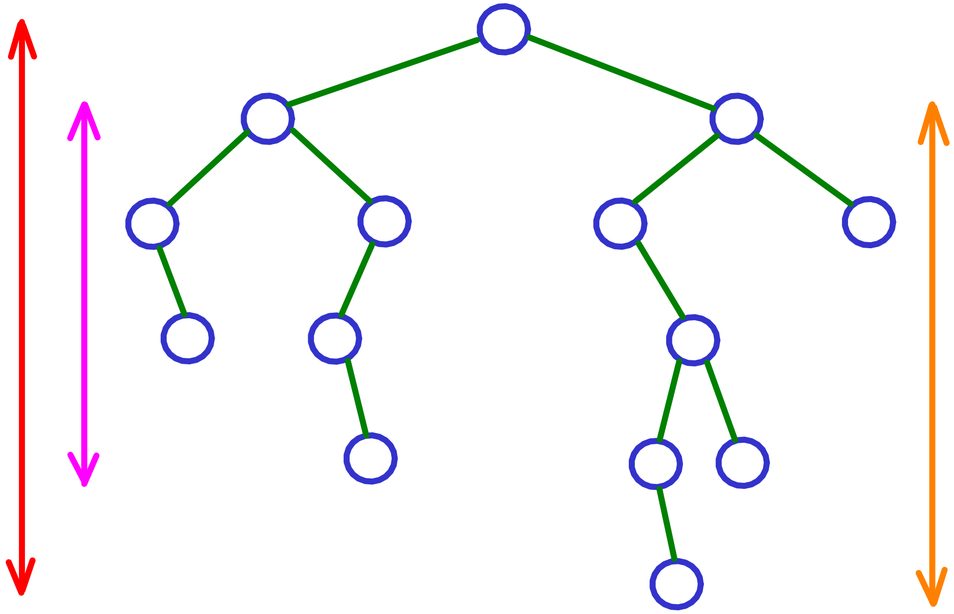
else $H(n) \leq 1 + H(n-1)$

50%

$< 1 + H(n)$

Expected height will be:

$$H(n) \leq 1 + \frac{1}{2} H\left(\frac{3n}{4}\right) + \frac{1}{2} H(n)$$



$$H(n) = 1 + \max\{H(k), H(n-k-1)\}$$

for some random k $(0 \leq k \leq n-1)$

If $\frac{n}{4} \leq k \leq \frac{3n}{4}$

50%

$\hookrightarrow H(n) \leq 1 + H\left(\frac{3n}{4}\right)$

else $H(n) \leq 1 + H(n-1)$

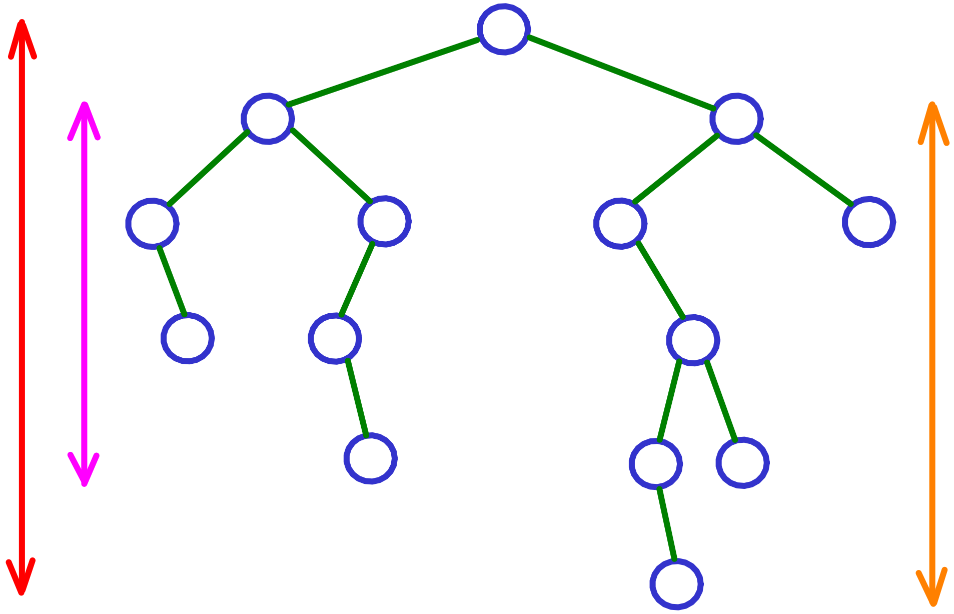
50%

$< 1 + H(n)$

Expected height will be:

$$H(n) \leq 1 + \frac{1}{2} H\left(\frac{3n}{4}\right) + \frac{1}{2} H(n)$$

$$\frac{1}{2} H(n) \leq 1 + \frac{1}{2} H\left(\frac{3n}{4}\right)$$



$$H(n) = 1 + \max\{H(k), H(n-k-1)\}$$

for some random k $(0 \leq k \leq n-1)$

If $\frac{n}{4} \leq k \leq \frac{3n}{4}$

50%

$\hookrightarrow H(n) \leq 1 + H\left(\frac{3n}{4}\right)$

else $H(n) \leq 1 + H(n-1)$

50%

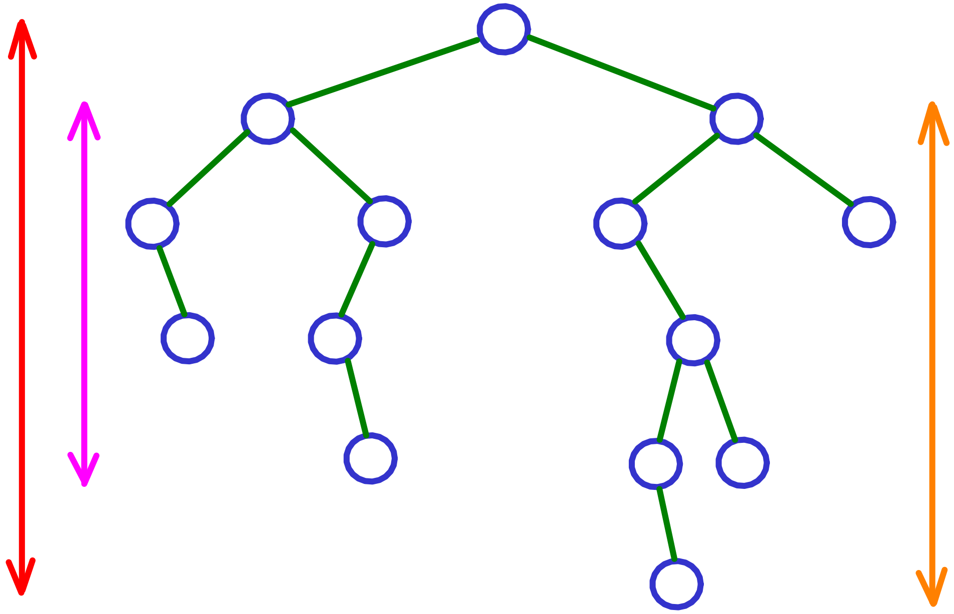
$< 1 + H(n)$

Expected height will be:

$$H(n) \leq 1 + \frac{1}{2}H\left(\frac{3n}{4}\right) + \frac{1}{2}H(n)$$

$$\frac{1}{2}H(n) \leq 1 + \frac{1}{2}H\left(\frac{3n}{4}\right)$$

$$H(n) \leq 2 + H\left(\frac{3n}{4}\right)$$



$$H(n) = 1 + \max\{H(k), H(n-k-1)\}$$

for some random k $(0 \le k \le n-1)$

If $\frac{n}{4} \le k \le \frac{3n}{4}$

50%

$\hookrightarrow H(n) \le 1 + H\left(\frac{3n}{4}\right)$

else $H(n) \le 1 + H(n-1)$

50%

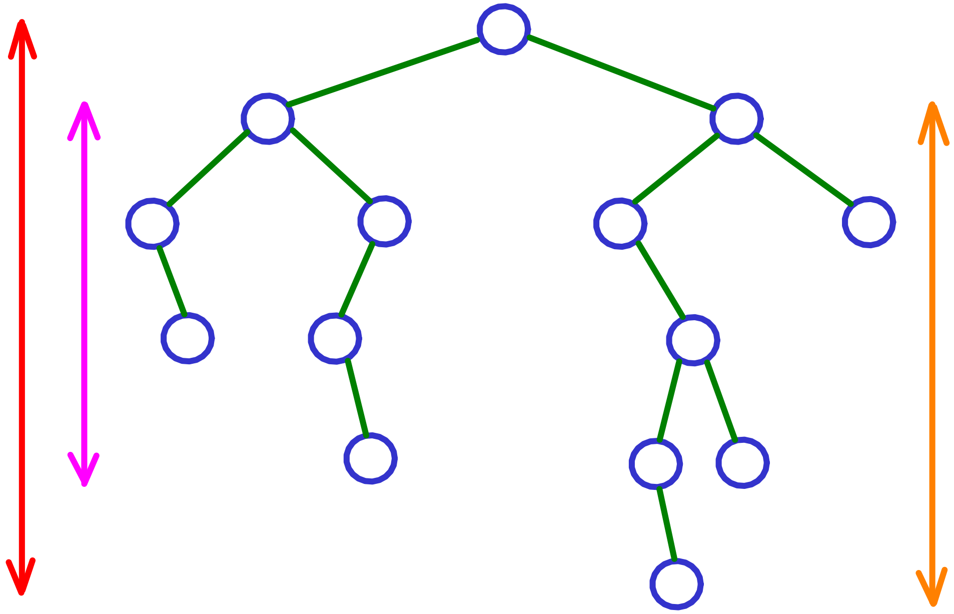
$< 1 + H(n)$

Expected height will be:

$$H(n) \le 1 + \frac{1}{2} H\left(\frac{3n}{4}\right) + \frac{1}{2} H(n)$$

$$\frac{1}{2} H(n) \le 1 + \frac{1}{2} H\left(\frac{3n}{4}\right)$$

$$H(n) \le 2 + H\left(\frac{3n}{4}\right) = O(\log n)$$



$$H(n) = 1 + \max\{H(k), H(n-k-1)\}$$

for some random k $(0 \le k \le n-1)$

with more rigorous analysis we can get $\sim 3\log n$

If $\frac{n}{4} \le k \le \frac{3n}{4}$

50%

$$\hookrightarrow H(n) \le 1 + H\left(\frac{3n}{4}\right)$$

else $H(n) \le 1 + H(n-1)$

50%

$$< 1 + H(n)$$

Expected height will be:

$$H(n) \le 1 + \frac{1}{2} H\left(\frac{3n}{4}\right) + \frac{1}{2} H(n)$$

$$\frac{1}{2} H(n) \le 1 + \frac{1}{2} H\left(\frac{3n}{4}\right)$$

$$H(n) \le 2 + H\left(\frac{3n}{4}\right) = O(\log n)$$