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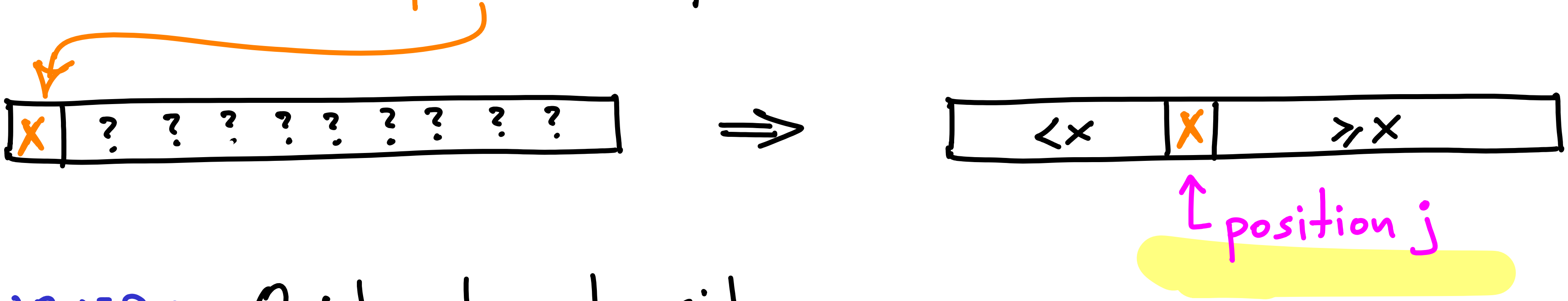

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$$T(n) = \quad ?$$

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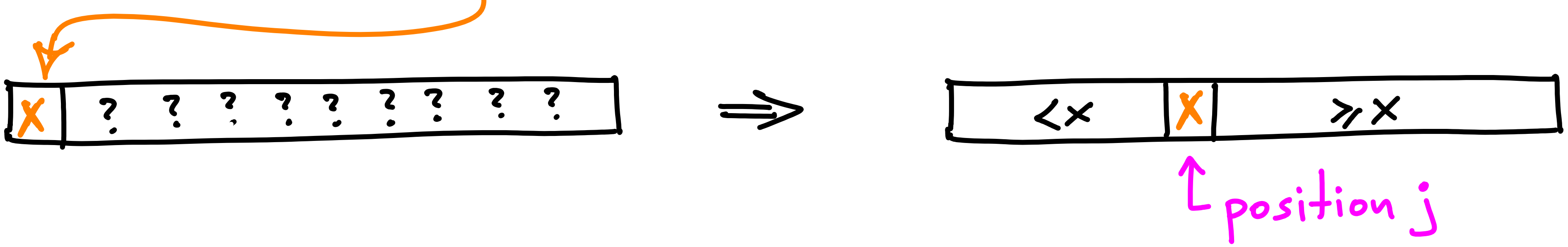

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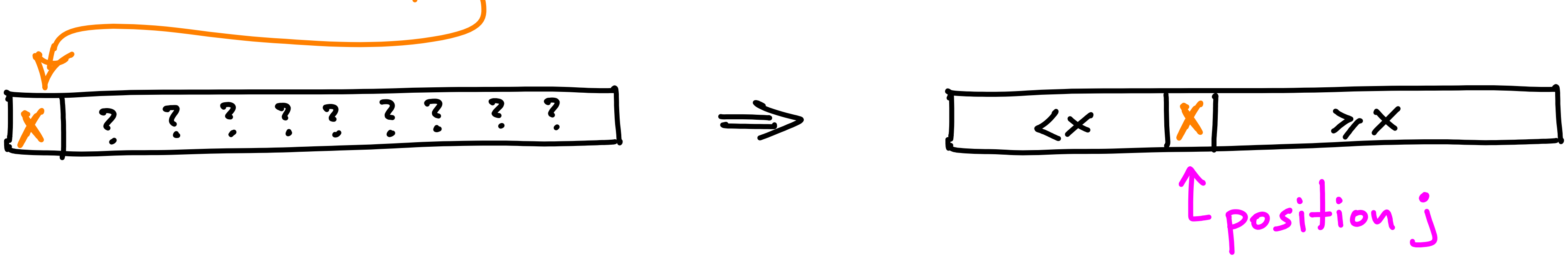

- **CONQUER:** Quicksort each side.

$$T(n) = \Theta(n) + T(j-1) + T(n-j)$$

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What is the worst-case time complexity, and why?

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$$T(n) = T(0) + T(n-1) + \Theta(n) \quad = \quad ?$$

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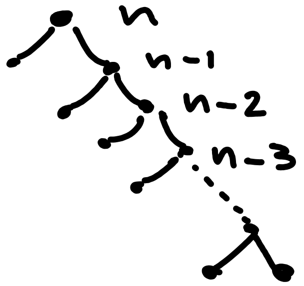

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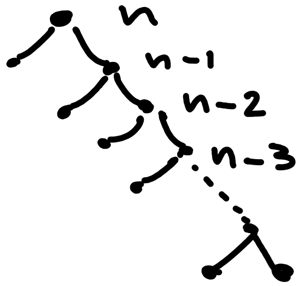

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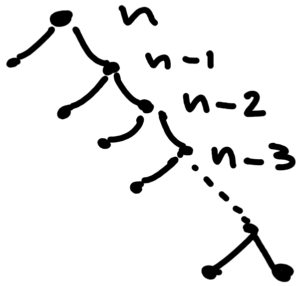

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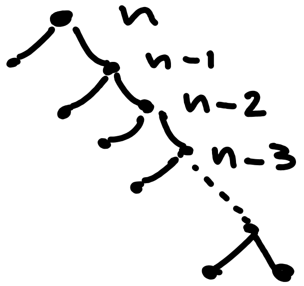

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$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + \Theta(n) \quad = ?$$

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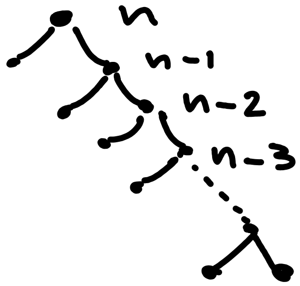

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e.g., $T(n) = T\left(\frac{n}{10}\right) + T\left(\frac{9n}{10}\right) + c \cdot n$

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$$
\begin{tikzcd}[column sep=small]
 & cn \arrow[dl, dash] \arrow[dr, dash] & \\
T\left(\frac{n}{10}\right) & & T\left(\frac{9n}{10}\right)
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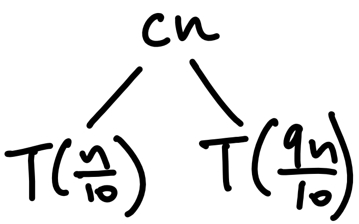

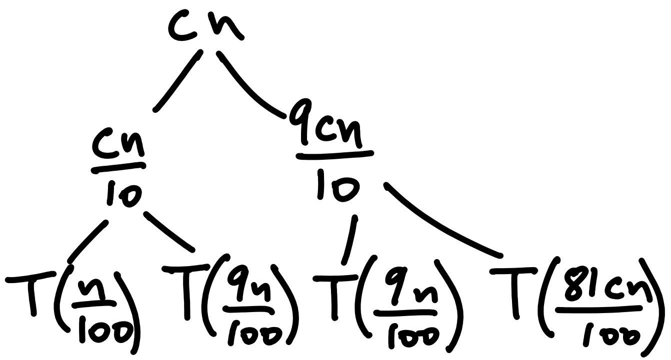

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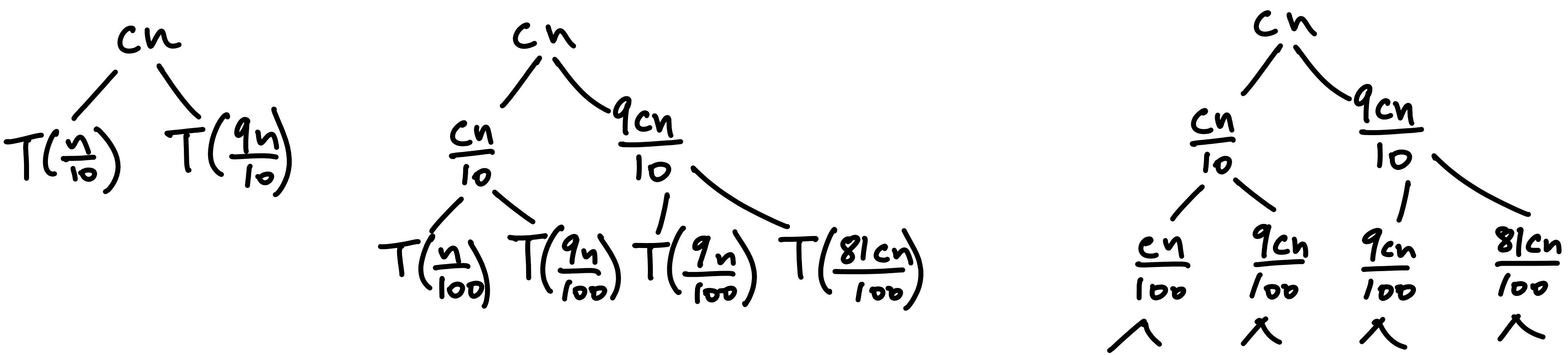

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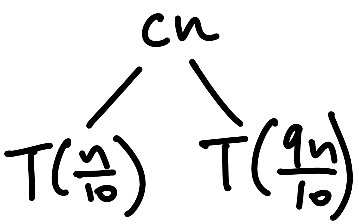

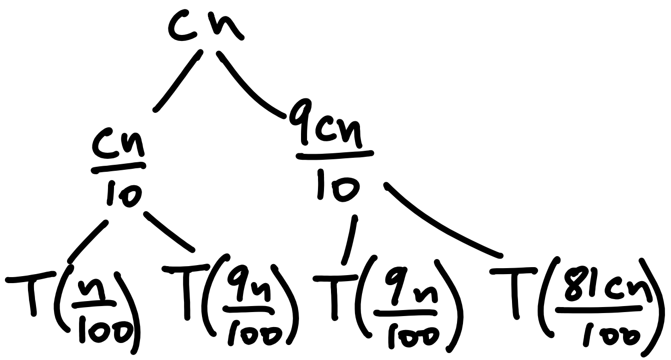

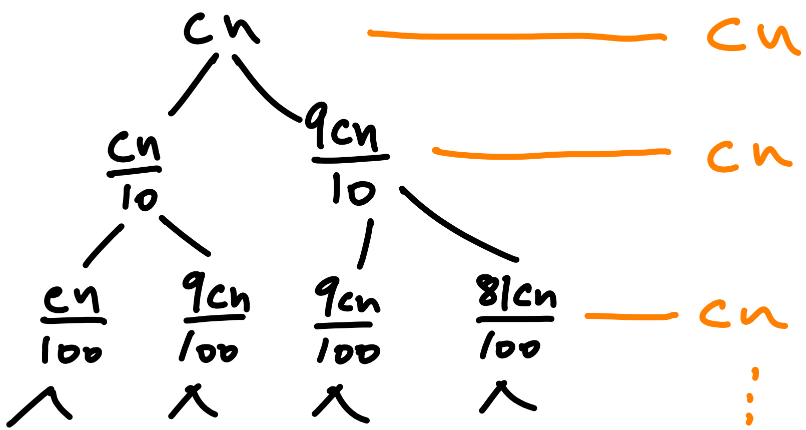

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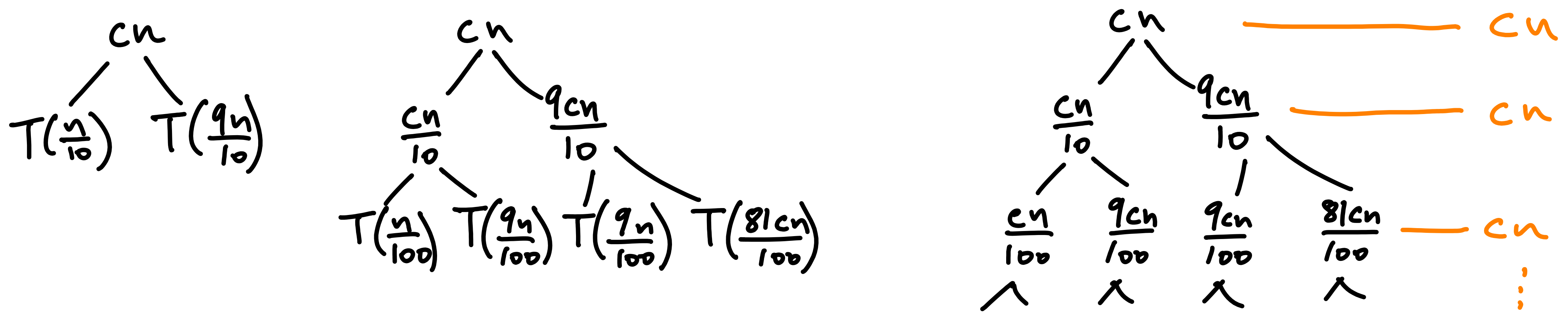

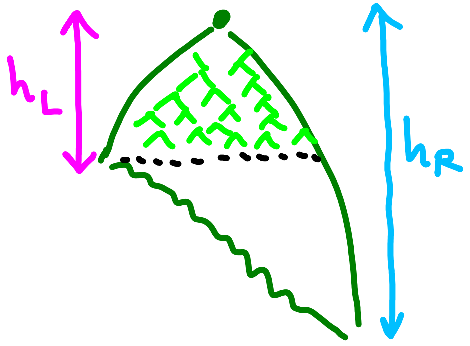

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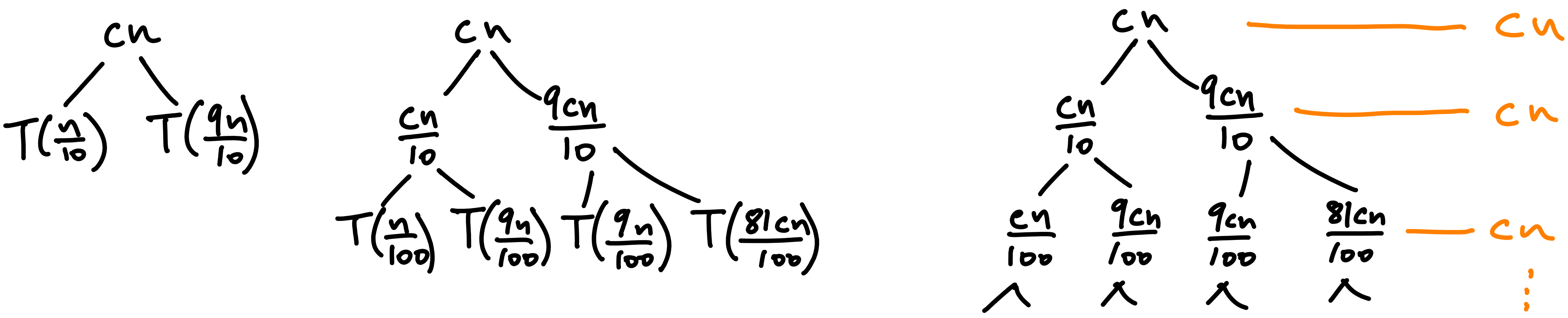

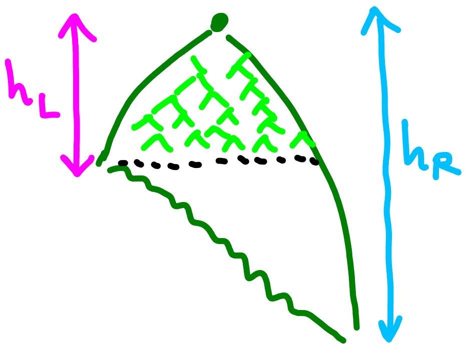

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$h_R \sim \log_{10/9} n$

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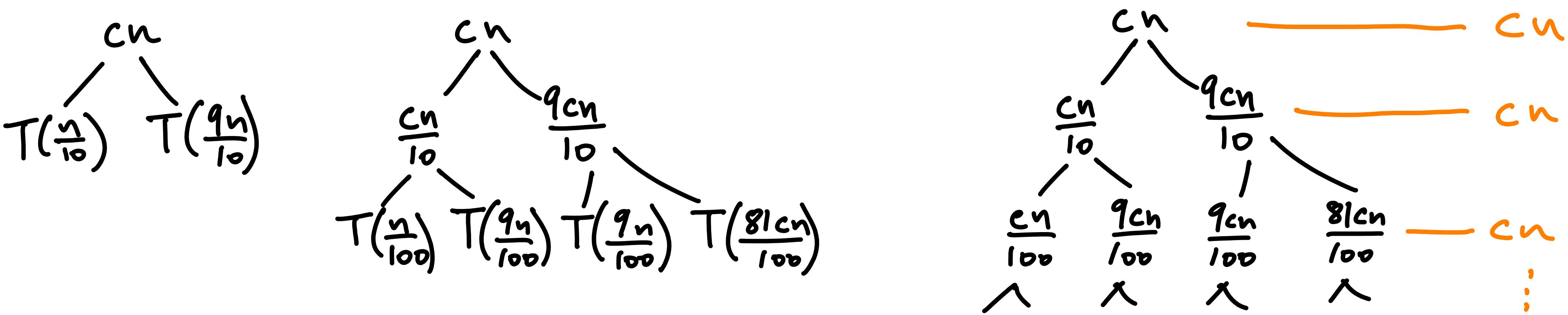

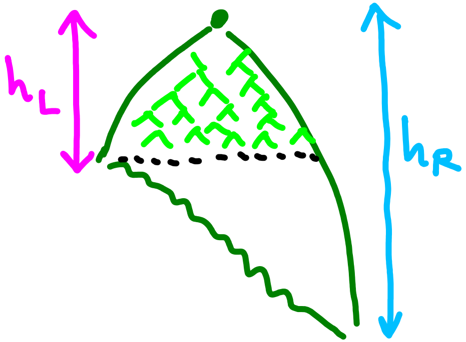

$h_L \sim \log_{10} n \rightarrow T(n) \geq cn \log_{10} n$

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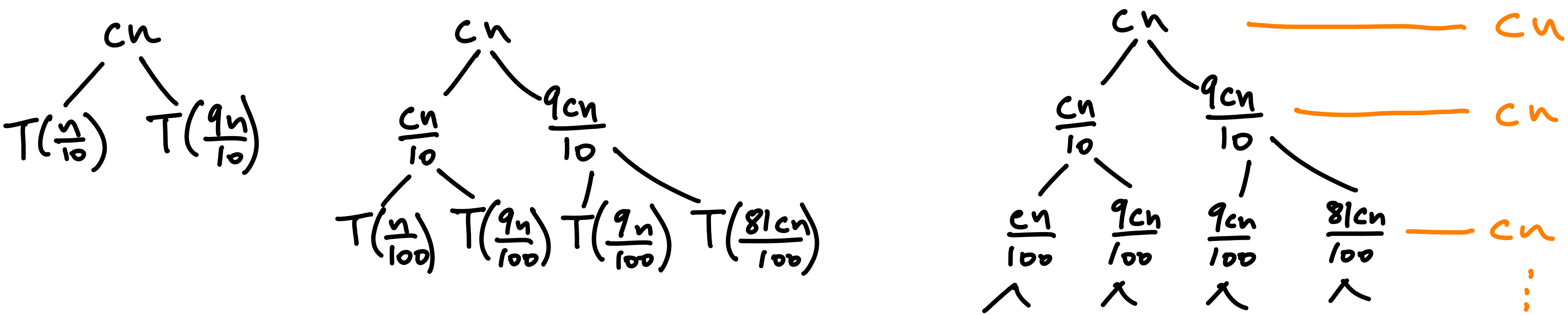

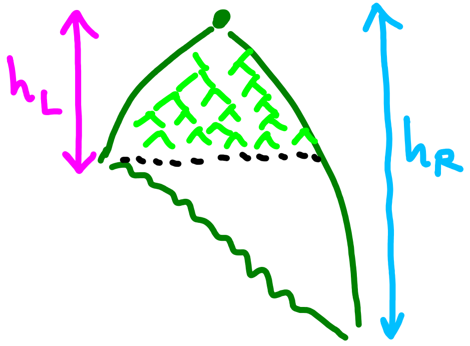

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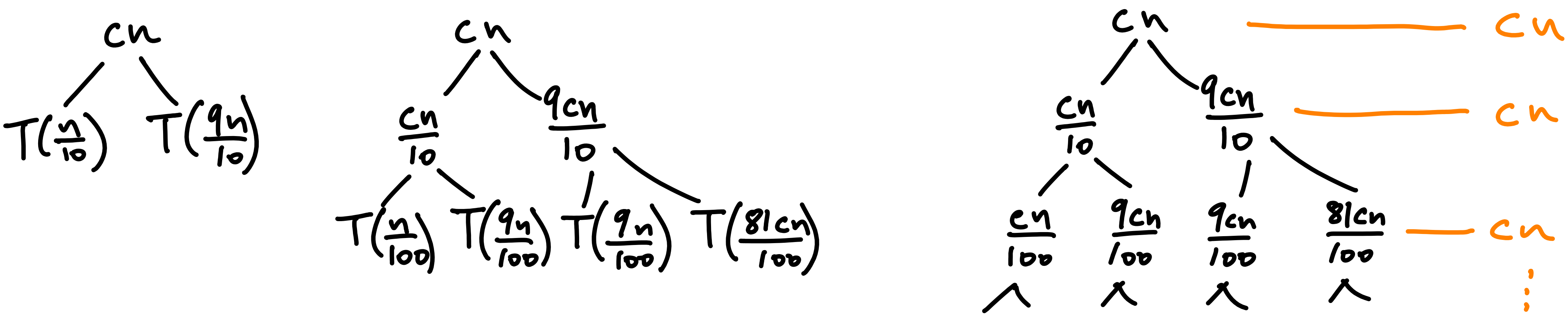

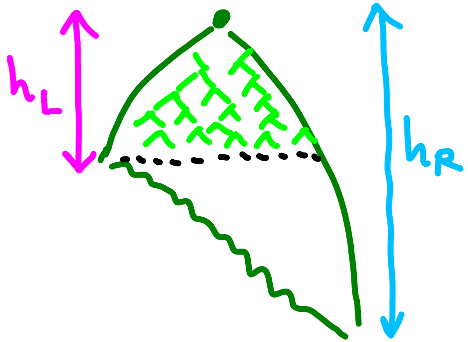

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Any constant-fraction-split will give $\Theta(n \log n)$

Expected time

Expected time: call a partition balanced if pivot ranks in $[\frac{n}{4} \dots \frac{3n}{4}]$

(redefining what balanced means)

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Worst case if balanced: $T(n) \leq T\left(\frac{3n}{4}\right) + T\left(\frac{n}{4}\right) + cn$

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Each partition has a 50% chance of being balanced

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$$T(n) \leq \frac{1}{2}\left(T(n)+cn\right) + \frac{1}{2}\left(T(\frac{3n}{4}) + T(\frac{n}{4}) + cn\right)$$

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$$T(n) \leq T(\frac{3n}{4}) + T(\frac{n}{4}) + 2cn$$

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$$T(n) \leq T(\tfrac{3n}{4}) + T(\tfrac{n}{4}) + 2cn \quad = O(n\log n)$$