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We are interested in $E[X]$

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linearity of expectation

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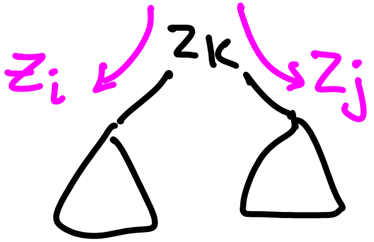

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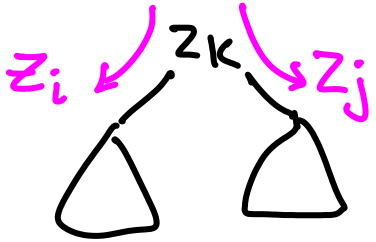

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$E[y] = \sum_t t \cdot P(y=t)$

For I.R.V.: $E[Y] = P[Y=1]$

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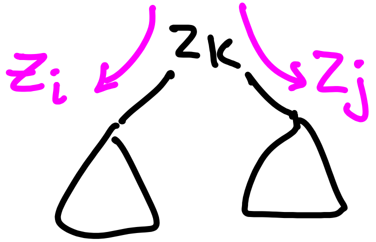

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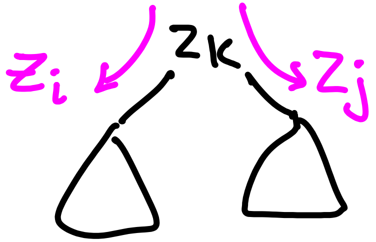

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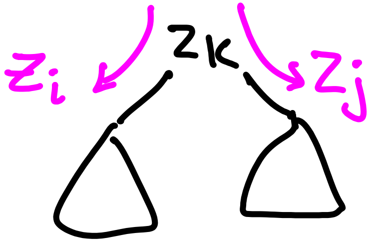

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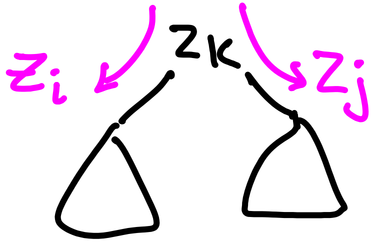

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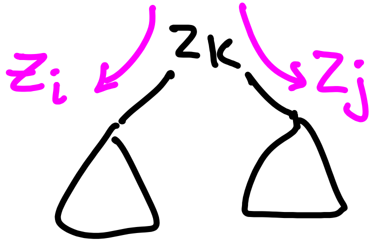

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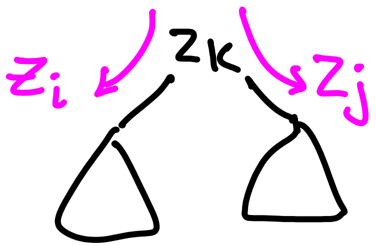

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