

PARTITION

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Input: an array (or linked list)

7	8	3	12	7	15	20	30	7	10	29	14
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PARTITION

Input: an array (or linked list)
↔

PIVOT - An element in the array.

(either given as input, or random, depending on context)

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PARTITION

Input: an array (or linked list) \longleftrightarrow

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(either given as input, or random, depending on context)

wlog, assume
pivot is
leftmost

10	8	3	12	7	15	20	30	7	7	29	14
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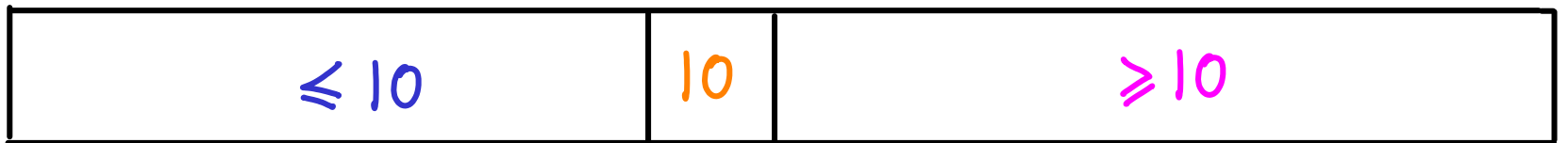
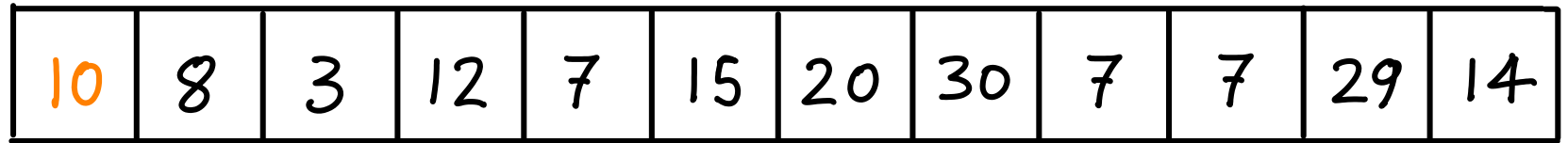
PARTITION

Input: an array (or linked list) \longleftrightarrow

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wlog, assume
pivot is
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- Elements smaller than pivot go to its left

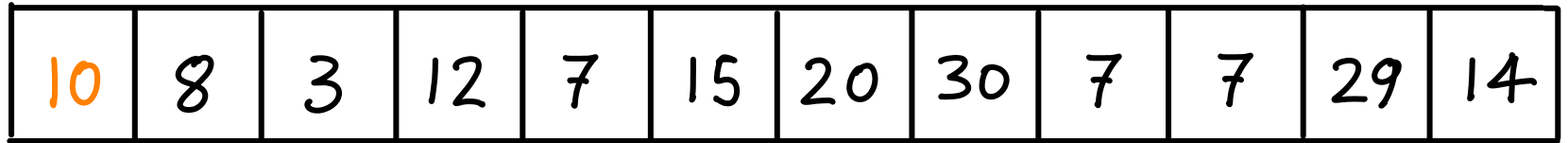
PARTITION

Input: an array (or linked list) \longleftrightarrow

PIVOT - An element in the array.

(either given as input, or random, depending on context)

wlog, assume
pivot is
leftmost



Output:



- Elements smaller than pivot go to its left
- Arbitrary handling of duplicates. (or use some consistent rule, e.g., <10 vs ≥ 10)

10	8	3	12	7	15	20	30	7	7	29	14
----	---	---	----	---	----	----	----	---	---	----	----



Grow "prefix" of smaller elements

Grow suffix of larger elements

10	8	3	12	7	15	20	30	7	7	29	14
----	---	---	----	---	----	----	----	---	---	----	----

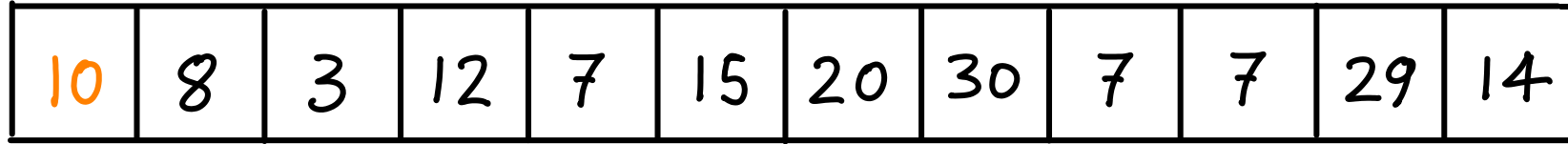


Grow "prefix" of smaller elements

Grow suffix of larger elements

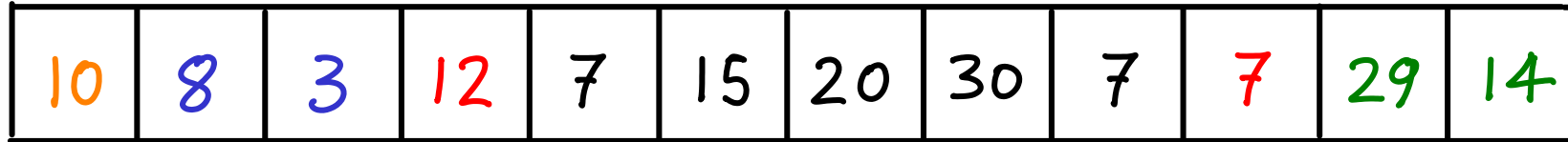
10	8	3	12	7	15	20	30	7	7	29	14
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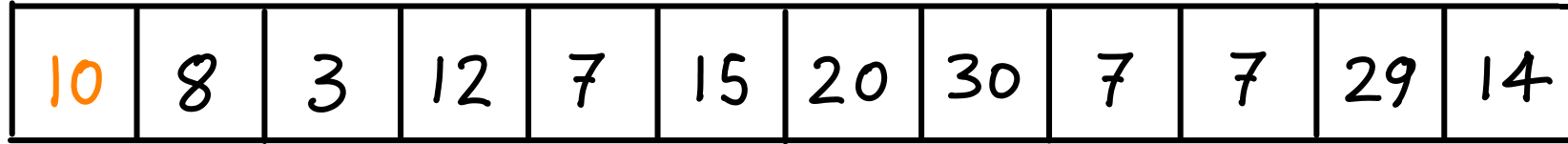


Grow "prefix" of smaller elements

Grow suffix of larger elements

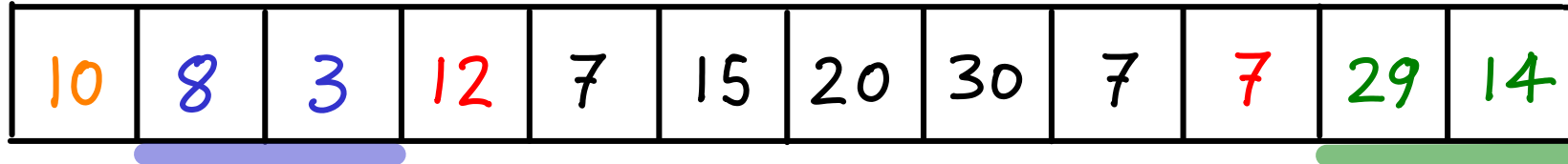


Now either the two sides meet or we can **SWAP**

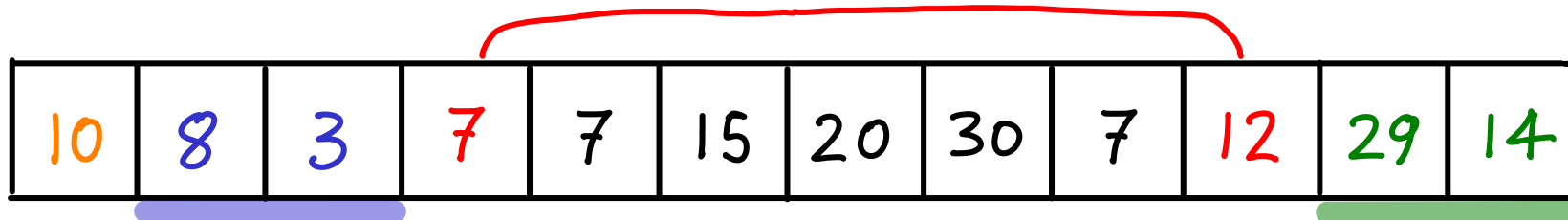


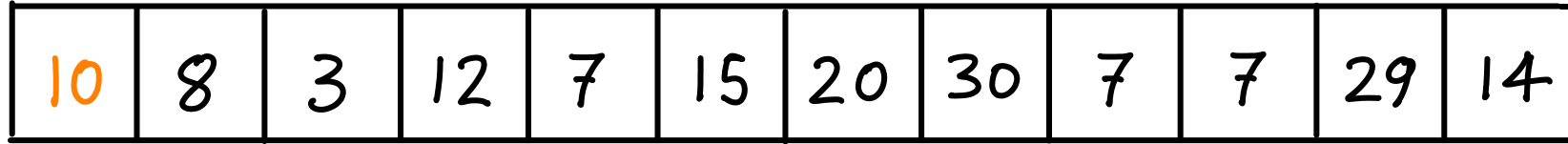
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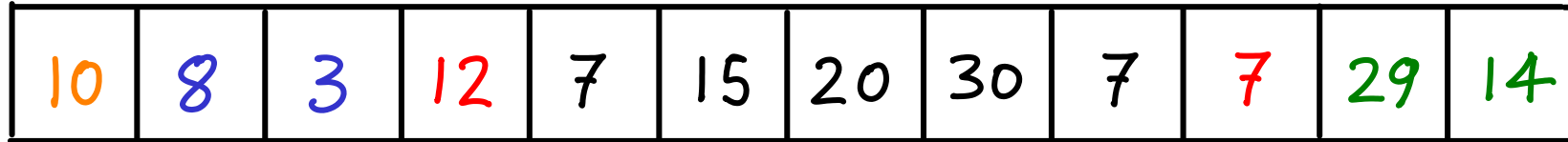
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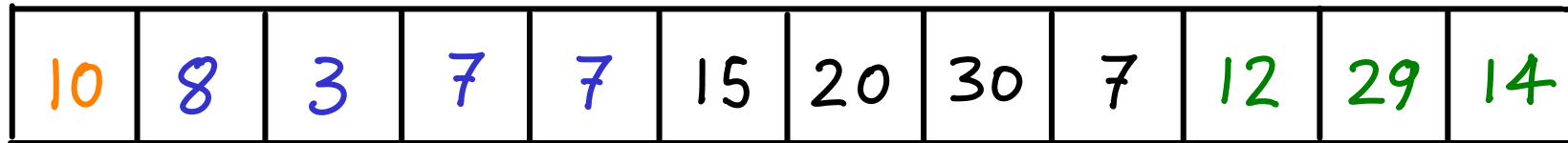


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... and continue

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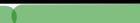
Grow suffix of larger elements

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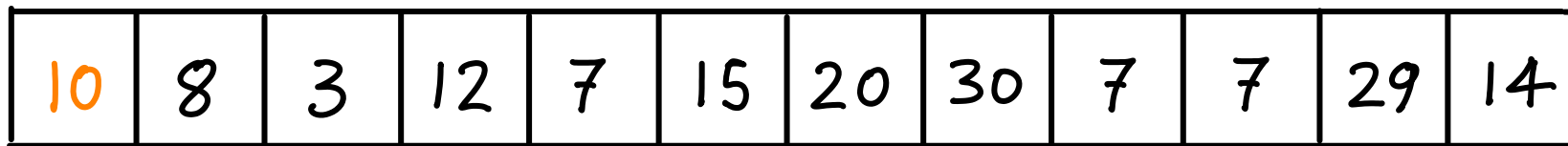


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10	8	3	7	7	15	20	30	7	12	29	14
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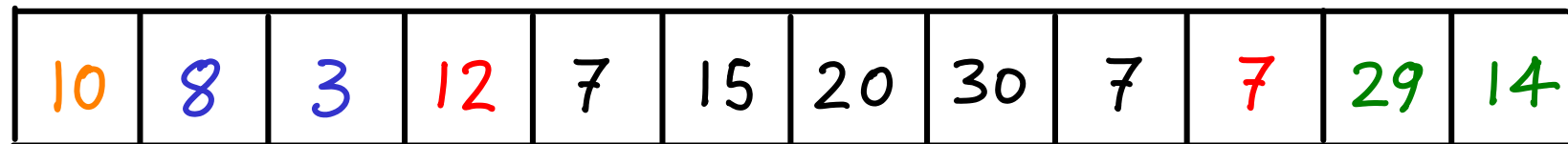


... and continue

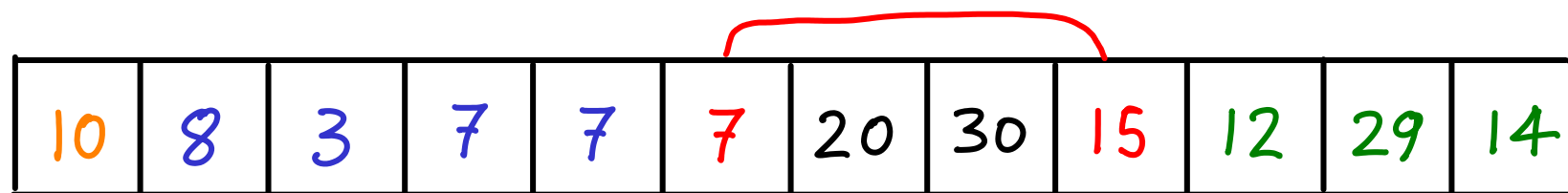


Grow "prefix" of smaller elements

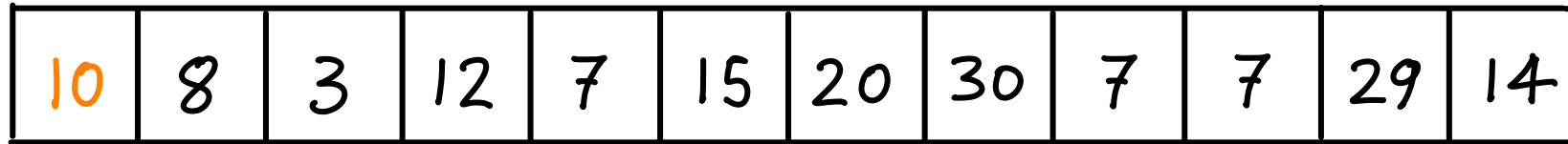
Grow suffix of larger elements



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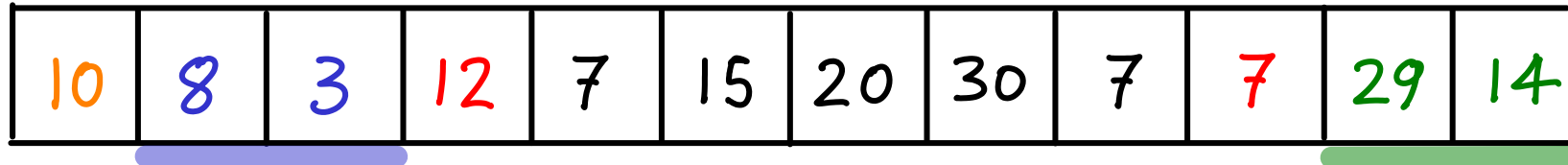


... and continue

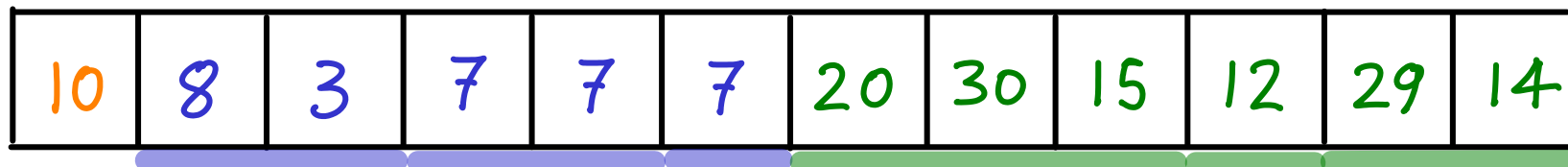


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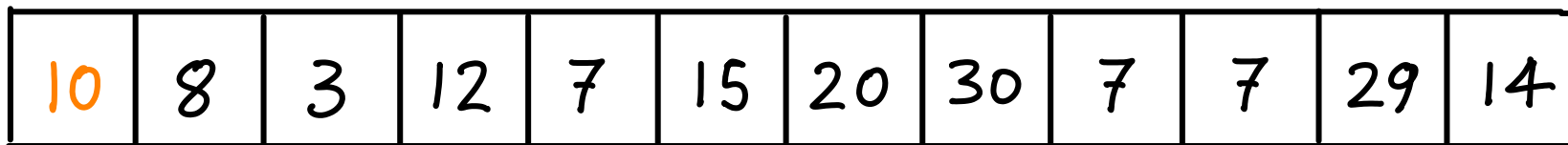
Grow suffix of larger elements



Now either the two sides meet or we can **SWAP**

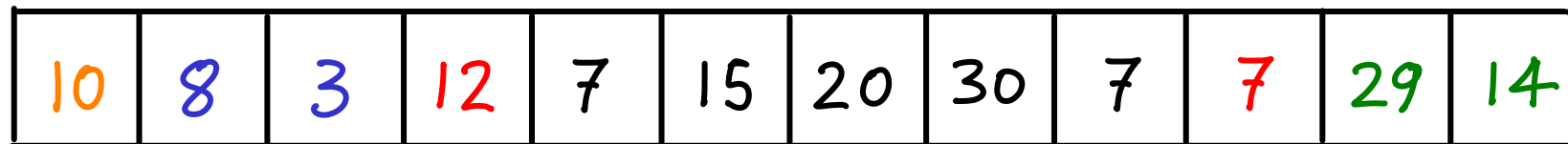


... and continue

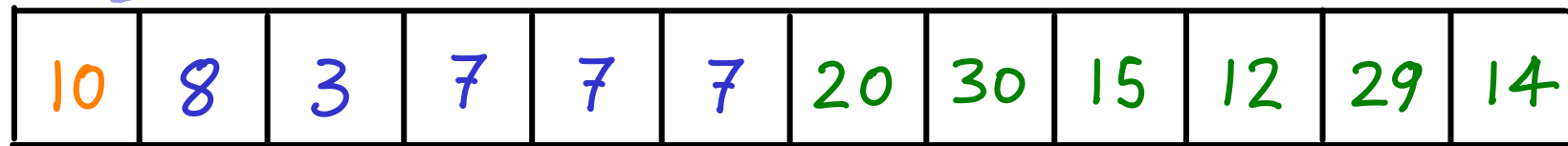


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Finally place pivot at correct spot

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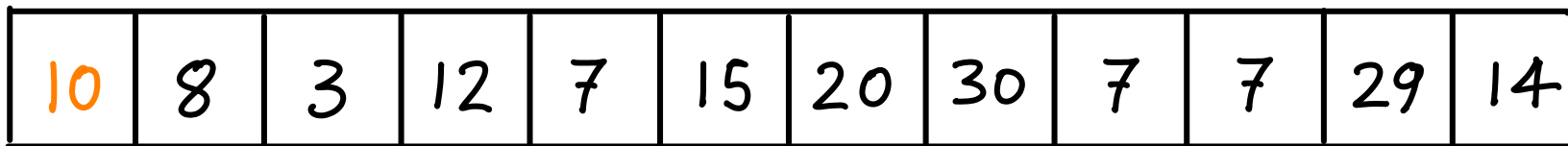


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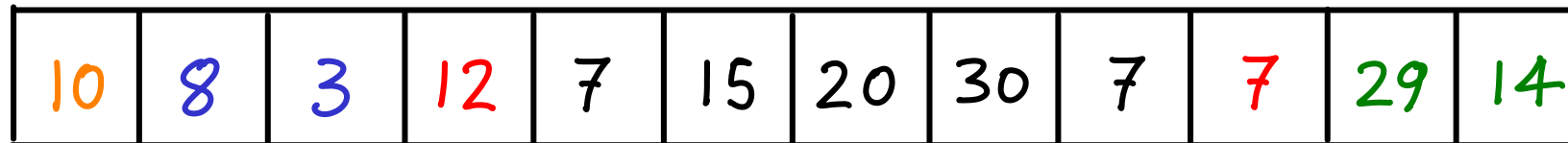


Finally place pivot at correct spot

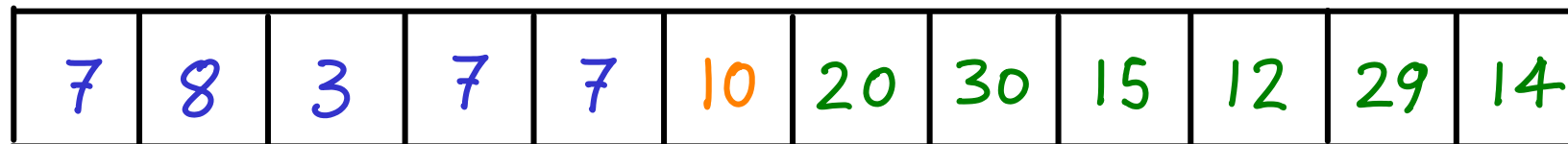


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Finally place pivot at correct spot

$\Theta(n)$ time

Partition was done IN-PLACE : only $O(1)$ extra space.

10	8	3	12	7	15	20	30	7	7	29	14
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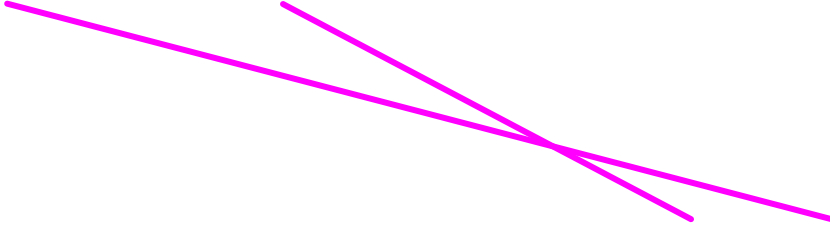
10	8	3	7	7	7	20	30	15	12	29	14
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Partition was done IN-PLACE : only $O(1)$ extra space.

But it was not STABLE : order of elements might change.

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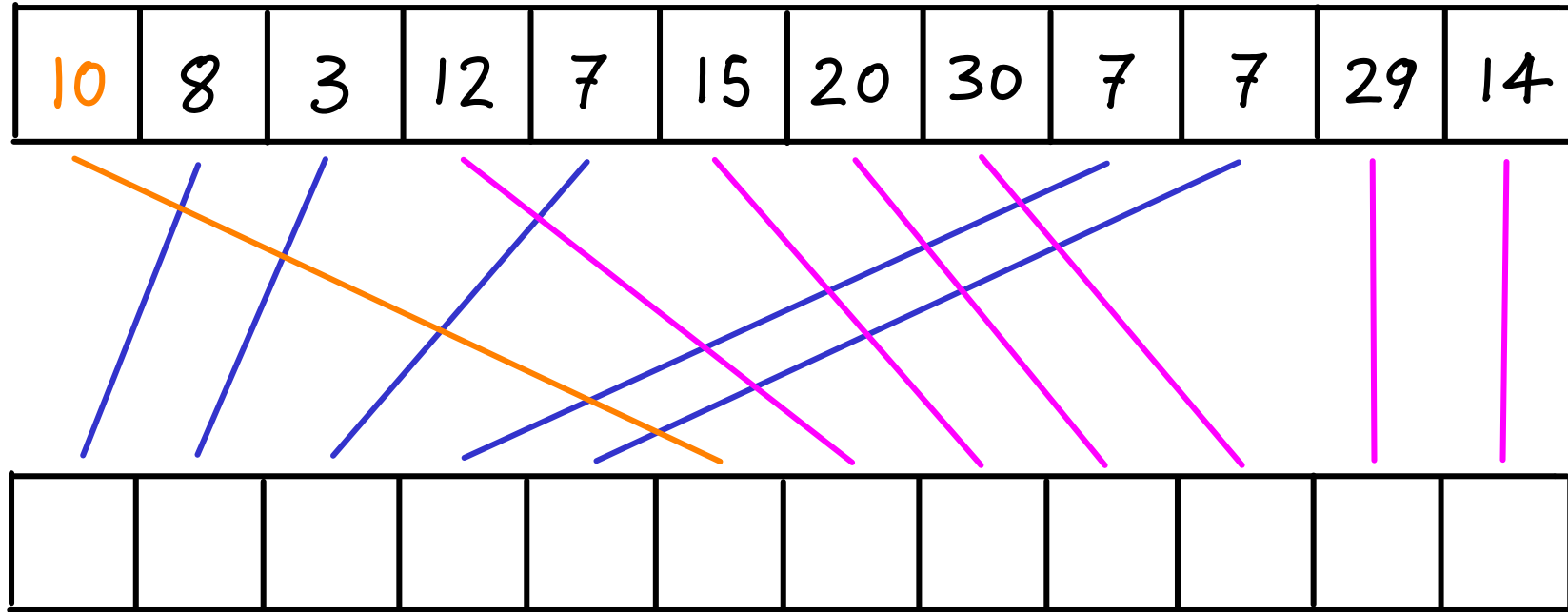
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Trivial stable version, using extra space



$\Theta(n)$ output space

Stable AND in-place? → Not $O(n)$ time

