# the MASTER METHOD

Ga tool for solving recurrences of this form:

$$T(n) = aT(\frac{n}{b}) + f(n)$$

# the MASTER METHOD

Ga tool for solving recurrences of this form:

$$T(n) = aT(\frac{n}{b}) + f(n)$$

assume 
$$\begin{cases} a > 1 \\ b > 1 \end{cases}$$
 otherwise you get  $\infty$ 

$$f(n) > 0 \text{ for } n > n.$$

$$T(n) = aT(\frac{n}{b}) + f(n)$$

$$T(n) = aT(\frac{n}{b}) + f(n)$$

a branches 
$$f(n)$$
  $f(n)$   $f(n)$ 

$$T(n) = aT(\frac{n}{b}) + f(n)$$

a branches 
$$f(n)$$

$$f(\frac{n}{b}) - \cdots + f(\frac{n}{b})$$

$$T(n) = aT(\frac{n}{b}) + f(n)$$

a branches 
$$f(n)$$

$$f(\frac{n}{b}) - \cdots + f(\frac{n}{b})$$

$$f(\frac{n}{b^2}) \cdots f(\frac{n}{b^2}) \cdots f(\frac{n}{b^2})$$

$$\vdots$$

$$T(n) = aT(\frac{n}{b}) + f(n)$$

a branches 
$$f(n)$$

$$f(\frac{n}{b}) \dots f(\frac{n}{b})$$

$$f(\frac{n}{b}) \dots f(\frac{n}{b^2}) \dots f(\frac{n}{b^2})$$

$$= a \cdot f(\frac{n}{b^2})$$

$$T(n) = aT(\frac{n}{b}) + f(n)$$

height

$$\frac{a \text{ branches}}{f(n)} = \frac{f(n)}{h(n)} - \frac{f(n)}{h(n)} - \frac{f(n)}{h(n)} = \frac{a \cdot f(n)}{h(n)} - \frac{a \cdot f(n)}{h(n)} = \frac{a^2 \cdot$$

$$T(n) = aT(\frac{n}{b}) + f(n)$$

height 
$$f(n)$$

$$f(\frac{n}{b}) - \dots + f(\frac{n}{b})$$

$$f(\frac{n}{b^2}) \dots + f(\frac{n}{b^2}) \dots + f(\frac{n}{b^2})$$

$$a \cdot f(\frac{n}{b^2}) \dots + f(\frac{n}{b^2}) \dots + f(\frac{n}{b^2})$$

$$a \cdot f(\frac{n}{b^2}) \dots + f(\frac{n}{b^2}) \dots + f(\frac{n}{b^2})$$

$$a \cdot f(\frac{n}{b^2}) \dots + f(\frac{n$$

$$T(n) = aT(\frac{n}{b}) + f(n)$$

height 
$$f(n)$$

$$f(n)$$

$$f(\frac{n}{b}) - \cdots + f(\frac{n}{b})$$

$$f(\frac{n}{b}) - \cdots + f(\frac{n}{b})$$

$$f(\frac{n}{b^2}) - \cdots + f(\frac{n}{b^2}) - \cdots + f(\frac{n}{b^2})$$

$$a \cdot f(\frac{n}{b^2})$$

$$a \cdot f(\frac{n}{b^2})$$

$$a \cdot f(\frac{n}{b^2})$$

$$T(n) = aT(\frac{n}{b}) + f(n)$$

height 
$$f(n)$$

$$f(\frac{n}{b}) - \cdots + f(\frac{n}{b})$$

$$f(\frac{n}{b}) - \cdots + f(\frac{n}{b})$$

$$f(\frac{n}{b^2}) - \cdots + f(\frac{n}{b^2}) - \cdots + f(\frac{n}{b^2})$$

$$a \cdot f(\frac{n}{b^2})$$

$$a \cdot f(\frac{n}{b^2})$$

$$a \cdot f(\frac{n}{b^2})$$

$$a \cdot f(\frac{n}{b^2})$$

$$T(n) = aT(\frac{n}{b}) + f(n)$$

height 
$$f(n)$$

$$f(\frac{n}{b}) - \cdots + f(\frac{n}{b})$$

$$f(\frac{n}{b}) - \cdots + f(\frac{n}{b})$$

$$f(\frac{n}{b^2}) - \cdots + f(\frac{n}{b^2}) - \cdots + f(\frac{n}{b^2})$$

$$a \cdot f(\frac{n}{b^2})$$

$$a \cdot f(\frac{n}{b^2})$$

$$a \cdot f(\frac{n}{b^2})$$

$$T(n) = aT(\frac{n}{b}) + f(n)$$

height 
$$f(n)$$

$$T(1) = \Theta(1)$$

$$T(n) = aT(\frac{n}{b}) + f(n)$$

height 
$$f(n)$$
  $f(n)$   $f(n)$ 

MASTER METHOD 
$$T(n) = aT(\frac{n}{b}) + f(n)$$

MASTER METHOD  $T(n) = aT(\frac{n}{b}) + f(n)$  compare f(n) to  $n^{\log_b a}$ 

MASTER METHOD  $T(n) = aT(\frac{n}{b}) + f(n)$  compare f(n) to  $n^{\log_b a}$ 

#leaves = 
$$\Omega(f(n) \cdot n^{\varepsilon})$$
 leaf level dominates polynomially

MASTER METHOD  $T(n) = aT(\frac{n}{b}) + f(n)$  compare f(n) to  $n^{log_ba}$ 

#leaves = 
$$\Omega(f(n) \cdot n^{\varepsilon})$$
 leaf level dominates  
 $f(n) = O(\#leaves/n^{\varepsilon})$  polynomial

MASTER METHOD  $T(n) = aT(\frac{n}{b}) + f(n)$  compare f(n) to  $n^{\log_b a}$ 

1) 
$$f(n) = O(n(\log_b a) - \epsilon)$$
 #leaves =  $\Omega(f(n) \cdot n^{\epsilon})$  leaf level dominates polynomially

MASTER METHOD  $T(n) = aT(\frac{n}{b}) + f(n)$  compare f(n) to  $n^{log_ba}$ 

1) 
$$f(n) = O(n^{(\log_b a) - \epsilon})$$
 #leaves =  $\Omega(f(n) \cdot n^{\epsilon})$  leaf level dominates polynomially

2) 
$$f(n)$$
 =  $\Theta(\# leaves)$  all levels ~ same

MASTER METHOD  $T(n) = aT(\frac{n}{b}) + f(n)$  compare f(n) to  $n^{\log_b a}$ 

1) 
$$f(n) = O(n(\log_b a) - \epsilon)$$
 #leaves =  $\Omega(f(n) \cdot n^{\epsilon})$  leaf level dominates polynomially

=  $\Theta(\# \text{leaves. log}^{k}n)$  all levels ~ same 2) f(n)

not mentioned in CLRS

MASTER METHOD 
$$T(n) = aT(\frac{n}{b}) + f(n)$$
 compare  $f(n)$  to  $n^{logba}$ 

1) 
$$f(n) = O(n^{(\log_b a) - \epsilon})$$
 #leaves =  $\Omega(f(n) \cdot n^{\epsilon})$  leaf level dominates polynomially

2) 
$$f(n) = \Theta(n^{\log ba} \cdot \log^k n) = \Theta(\# \text{leaves} \cdot \log^k n)$$
 all levels ~ same

MASTER METHOD 
$$T(n) = aT(\frac{n}{b}) + f(n)$$
 compare  $f(n)$  to  $n^{logba}$ 

1) 
$$f(n) = O(n^{(\log_b a) - \epsilon})$$
 #leaves =  $\Omega(f(n) \cdot n^{\epsilon})$  leaf level dominates polynomially

2) 
$$f(n) = \Theta(n^{\log ba} \cdot \log^k n) = \Theta(\# \text{leaves} \cdot \log^k n)$$
 all levels ~ same

3) 
$$f(n)$$
 =  $\Omega(\# \text{leaves} \cdot n^{\epsilon})$  root dominates polynomially

MASTER METHOD 
$$T(n) = aT(\frac{n}{b}) + f(n)$$
 compare  $f(n)$  to  $n^{\log_b a}$ 

i) 
$$f(n) = O(n^{(\log_b a) - \epsilon})$$
 #leaves =  $\Omega(f(n) \cdot n^{\epsilon})$  leaf level dominates polynomially

2) 
$$f(n) = \Theta(n^{\log ba} \cdot \log^{k} n) = \Theta(\# \text{leaves} \cdot \log^{k} n)$$
 all levels ~ same

3) 
$$f(n) = \Omega(n(\log_b a) + \varepsilon) = \Omega(\# \text{leaves} \cdot n^{\varepsilon})$$
 root dominates polynomially

MASTER METHOD 
$$T(n) = aT(\frac{n}{b}) + f(n)$$
 compare  $f(n)$  to  $n^{\log_b a}$ 

\*\*Leaves =  $\Omega(f(n) \cdot n^{\epsilon})$  leaf level dominates

1) 
$$f(n) = O(n^{(\log_b a) - \epsilon})$$
 #leaves =  $\Omega(f(n) \cdot n^{\epsilon})$  leaf level dominates polynomially

all levels ~ same

3) 
$$f(n) = \Omega(n(\log_b a) + \epsilon) = \Omega(\# \text{leaves} \cdot n^{\epsilon})$$
 root dominates polynomially AND  $af(\frac{n}{b}) \le \delta \cdot f(n)$  work reduced by constant fraction in each level

2)  $f(n) = \Theta(n^{\log ba} \cdot \log^{k} n) = \Theta(\# \text{leaves} \cdot \log^{k} n)$ 

(E>0, k>0, 0<5<1)

MASTER METHOD 
$$T(n) = aT(\frac{n}{b}) + f(n)$$
 compare  $f(n)$  to  $n^{\log_b a}$ 

1) 
$$f(n) = O(n^{(\log ba)-\epsilon})$$
 #leaves =  $\Omega(f(n) \cdot n^{\epsilon})$  leaf level dominates polynomially

$$T(n) = \Theta(n^{\log ba})$$
2)  $f(n) = \Theta(n^{\log ba} \cdot \log^{k} n) = \Theta(\# \text{leaves} \cdot \log^{k} n)$  all levels ~ same

all levels ~ same

$$l() = O((lea, a) + E) = O(\#lea, a)$$

Toot dominates

root dominates polynomially 3)  $f(n) = \Omega(n^{(\log_b a) + \epsilon})$ AND  $af(\frac{n}{b}) \leq \delta \cdot f(n)$ =  $\Omega$  (#leaves ·  $n^{\varepsilon}$ ) > work reduced by constant fraction in each level (ε>0, k>0, 0<δ<1)

MASTER METHOD 
$$T(n) = aT(\frac{n}{b}) + f(n)$$
 compare  $f(n)$  to  $n^{log_ba}$ 

\*\*Leaves =  $\Omega(f(n) \cdot n^{\epsilon})$  loof level dominates

1) 
$$f(n) = O(n(\log_b a) - \epsilon)$$
 #leaves =  $\Omega(f(n) \cdot n^{\epsilon})$  leaf level dominates polynomially
$$T(n) = \Theta(n\log_b a)$$

all levels ~ same

 $T(n) = \Theta(f(n) \cdot logn)$ 3)  $f(n) = \Omega(n(log_ba) + E) = \Omega(\#leaves \cdot n^E)$ Foot dominates polynomially polynomially work reduced by constant fraction in each level

2)  $f(n) = \Theta(n^{\log ba} \cdot \log^{k} n) = \Theta(\# \text{leaves} \cdot \log^{k} n)$ 

(E>0, k>0, 0<8<1)

MASTER METHOD 
$$T(n) = aT(\frac{n}{b}) + f(n)$$
 compare  $f(n)$  to  $n^{\log_b a}$ 

1. (1)  $a \in \mathcal{O}(f(n) \cdot n^{\epsilon})$  leaf level dominates

1)  $f(n) = O(n^{(\log_b a) - \epsilon})$  #leaves =  $\Omega(f(n) \cdot n^{\epsilon})$  leaf level dominates polynomially  $T(n) = O(n^{(\log_b a) - \epsilon})$ 

$$T(n) = \Theta(n \log_b a)$$
2)  $f(n) = \Theta(n \log_b a) + O(n \log_b a)$  all levels ~ same

 $T(n) = \Theta(f(n) \cdot logn)$   $f(n) = \Omega(n^{(logba) + \epsilon}) = \Omega(\# leaves \cdot n^{\epsilon})$ root dominates polynomially

3) 
$$f(n) = \Omega(n(\log_b a) + \epsilon) = \Omega(\# \text{ leaves} \cdot n^{\epsilon})$$

AND  $af(\frac{n}{b}) \leq \delta \cdot f(n)$ 

work reduced by constant fraction in each level

 $T(n) = \Theta(f(n))$ 

$$T(n) = 4T\left(\frac{n}{2}\right) + n$$

$$a \qquad b \qquad f(n)$$

$$T(n) = 4T(\frac{n}{2}) + n$$

$$n \log_{b} a = n^{\log_{2} 4} = n^{2}$$

$$T(n) = 4T(\frac{n}{2}) + n$$

$$n \log_{b} a = n \log_{2} 4 = n^{2}$$

$$f(n) = n = O(n) = O(n^{2-1})$$
: case 1

$$T(n) = 4T(\frac{n}{2}) + n$$

$$n \log_{b} a = n \log_{2} 4 = n^{2}$$

$$f(n) = n = O(n) = O(n^{2-1})$$

$$so n \log_{b} a \text{ dominates } f(n) : answer = n \log_{b} a = O(n^{2})$$

$$T(n) = 4T(\frac{n}{2}) + n$$

$$f(n) = n = O(n) = O(n^{2-1})$$

$$so n^{\log_b a} dominates f(n) : answer = n^{\log_b a} = O(n^2)$$

$$T(n) = 4T(\frac{n}{2}) + n^2 // n^{\log_b a} = n^2$$

$$T(n) = 4T(\frac{n}{2}) + n$$

$$n\log_{2}4 = n^{2}$$

$$f(n) = n = O(n) = O(n^{2-1})$$

$$case 1$$

$$so n\log_{2}a \text{ dominates } f(n) : answer = n\log_{2}a = O(n^{2})$$

$$T(n) = 4T(\frac{n}{2}) + n^{2}$$

$$f(n) = n^{2} = O(n\log_{2}a - \log_{2}a) : case 2$$

$$T(n) = 4T(\frac{n}{2}) + n$$

$$n^{\log_2 4} = n^2$$

$$f(n) = n = O(n) = O(n^{2-1})$$

$$so n^{\log_2 a} dominates f(n) : answer = n^{\log_2 a} = O(n^2)$$

$$T(n) = 4T(\frac{n}{2}) + n^2$$

$$n^{\log_2 a} = n^2$$

$$T(n) = 4T(\frac{n}{2}) + n^2 // n^{\log b^2} = n^2$$

$$f(n) = n^2 = \Theta(n^{\log b^2} \cdot \log n) : case 2$$

$$ANS: \Theta(n^{\log b^2} \cdot \log^{k+1} n) = \Theta(n^2 \log n)$$

$$T(n) = 4T(\frac{n}{2}) + n$$

$$n\log_{b} a = n\log_{2} 4 = n^{2}$$

$$f(n) = n = O(n) = O(n^{2-1})^{\epsilon} : case 1$$

$$so n\log_{b} a dominates f(m) : answer = n\log_{b} a = O(n^{2})$$

$$T(n) = 4T(\frac{n}{2}) + n^{2} // n\log_{b} a = n^{2}$$

$$f(n) = n^{2} = O(n\log_{b} a \cdot \log_{b} a) : case 2$$

$$Ans: O(n\log_{b} a \cdot \log_{b} a \cdot \log_{b} a) = O(n^{2}\log_{b} a)$$

 $T(n) = 4T(\frac{n}{2}) + n^3$  /  $f(n) = n^3 = \Omega(n^{2+\epsilon})$ 

$$T(n) = 4T(\frac{n}{2}) + n$$

$$n \log_{b} a = n \log_{2} 4 = n^{2}$$

$$f(n) = n = O(n) = O(n^{2-1})^{\epsilon} : case 1$$

$$so n \log_{b} a dominates f(n) : answer = n \log_{b} a = O(n^{2})$$

$$T(n) = 4T(\frac{n}{2}) + n^{2} / n \log_{b} a = n^{2}$$

$$f(n) = n^{2} = O(n \log_{b} a \cdot \log_{n} a) : case 2$$

$$Ans: O(n \log_{b} a \cdot \log_{n} a \cdot \log_{n} a) = O(n^{2} \log_{n} a \cdot \log_{n}$$

$$T(n) = 4T(\frac{n}{2}) + n^3$$
 /  $f(n) = n^3 = \Omega(n^{2+\epsilon})$  AND  $af(\frac{n}{6}) = 4 \cdot (\frac{n}{2})^3 = \frac{1}{2} n^3 = \frac{1}{2} \cdot f(n)$ 

$$T(n) = 4T(\frac{n}{2}) + n$$

$$f(n) = n = O(n) = O(n^{2-1})^{\epsilon} : case 1$$

$$so n logba dominates f(m) : answer = n logba = O(n^{2})$$

$$T(n) = 4T(\frac{n}{2}) + n^{2}$$

$$f(n) = n^{2} = O(n^{\log_{2} a} \cdot \log^{n}) : case 2$$

$$Ans: O(n^{\log_{2} a} \cdot \log^{n}) = O(n^{2}\log^{n})$$

$$T(n) = 4T(\frac{n}{2}) + n^3 // f(n) = n^3 = \Omega(n^{2+\epsilon}) \text{ AND } af(\frac{n}{6}) = 4 \cdot (\frac{n}{2})^3 = \frac{1}{2} \cdot f(n)$$

$$4 + Case 3 : ANS: \Theta(n^3)$$

$$T(n) = 4T(\frac{n}{2}) + n$$

$$n \log_{b} a = n \log_{2} 4 = n^{2}$$

$$f(n) = n = O(n) = O(n^{2-1}) : case 1$$

$$so n \log_{b} a dominates f(n) : answer = n \log_{b} a = O(n^{2})$$

$$T(n) = 4T(\frac{n}{2}) + n^{2} / n \log_{b} a = n^{2}$$

$$f(n) = n^{2} = O(n \log_{b} a \cdot \log_{n} a) : case 2$$

$$Ans: O(n \log_{b} a \cdot \log_{n} a) = O(n^{2} \log_{n} a)$$

$$T(n) = 4T(\frac{n}{2}) + n^{3} // f(n) = n^{3} = \Omega(n^{24\xi}) \text{ AND af}(\frac{n}{2}) = 4 \cdot (\frac{n}{2})^{3} = \frac{1}{2} \cdot f(n)$$

$$\Rightarrow case 3 : ANS: \Theta(n^{3})$$

$$T(n) = 4T(\frac{n}{2}) + \frac{n^{2}}{logn}$$
?

$$T(n) = 4T(\frac{n}{2}) + n$$

$$f(n) = n = O(n) = O(n^{2-1})$$

$$so \quad n^{\log_b a} \quad dominates \quad f(n) : answer = n^{\log_b a} = O(n^2)$$

$$T(n) = 4T(\frac{n}{2}) + n^2 \quad // n^{\log_b a} = n^2$$

$$f(n) = n^2 = O(n^{\log_b a} \cdot \log_n) : case 2$$

$$Ans: O(n^{\log_b a} \cdot \log_n) = O(n^{2\log_n})$$

$$T(n) = 4T(\frac{n}{2}) + n^3 \qquad f(n) = n^3 = \Omega(n^{2+\epsilon}) \text{ AND af}(\frac{n}{6}) = 4 \cdot (\frac{n}{2})^3 = \frac{1}{2} \cdot f(n)$$

$$\Rightarrow case 3 : \text{ANS}: \Theta(n^3)$$

$$T(n) = 4T(\frac{n}{2}) + \frac{n^2}{\log n} \qquad \text{But in fact there is now yet another extension of case 2}$$

$$\text{(see last page)}$$

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

if 
$$f(n)=n \Rightarrow cose2 \Rightarrow$$

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

if 
$$f(n)=n \Rightarrow case2 \Rightarrow \Theta(n \cdot logn)$$

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

if 
$$f(n)=n \Rightarrow cose2 \Rightarrow \Theta(n \cdot logn)$$
(k=0)

if 
$$f(n) = n \log n \Rightarrow case 2 \Rightarrow \Theta(n \log^{k+1} n)$$

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

if 
$$f(n)=n \Rightarrow case2 \Rightarrow \Theta(n \cdot logn)$$
(k=0)

if 
$$f(n) = n \log n \Rightarrow case 2 \Rightarrow \Theta(n \log^{k+1} n)$$

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

if 
$$f(n)=n \Rightarrow cose2 \Rightarrow \Theta(n \cdot logn)$$
(k=0)

if 
$$f(n) = n \log^k n \Rightarrow case 2 \Rightarrow \Theta(n \log^{k+1} n)$$

if 
$$f(n) = \log^{c} n \Rightarrow \text{case } 1$$
  
 $f(n) = O(n^{1-\epsilon})$ 

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

if 
$$f(n)=n \Rightarrow case2 \Rightarrow \Theta(n \cdot logn)$$
(k=0)

if 
$$f(n) = n \log n \Rightarrow case 2 \Rightarrow \Theta(n \log^{k+1} n)$$

if 
$$f(n) = \log^{c} n \Rightarrow \cos 1 \Rightarrow \Theta(n)$$
  
 $f(n) = O(n^{1-\epsilon})$ 

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

if 
$$f(n)=n \Rightarrow case2 \Rightarrow \Theta(n \cdot logn)$$
(k=0)

if 
$$f(n) = n \log n \Rightarrow case 2 \Rightarrow \Theta(n \log^{k+1} n)$$

if 
$$f(n) = \log^{c} n \Rightarrow \cos 1 \Rightarrow \Theta(n)$$
  
 $f(n) = O(n^{1-\epsilon})$ 

if 
$$f(n)=n^{c}$$
for  $c>1$ 

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

if 
$$f(n)=n \Rightarrow case2 \Rightarrow \Theta(n \cdot logn)$$
(k=0)

if 
$$f(n) = n \log n \Rightarrow case 2 \Rightarrow \Theta(n \log^{k+1} n)$$

if 
$$f(n) = \log^{c} n \Rightarrow \cos 1 \Rightarrow \Theta(n)$$
  
 $f(n) = O(n^{1-\epsilon})$ 

if 
$$f(n)=n^{\epsilon}$$
for  $c>1$ 

$$f(n)=\Omega(n^{1+\epsilon})$$

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

if 
$$f(n)=n \Rightarrow case2 \Rightarrow \Theta(n.logn)$$
(k=0)

if 
$$f(n) = n \log n \Rightarrow case 2 \Rightarrow \Theta(n \log^{k+1} n)$$

if 
$$f(n) = \log^{c} n \Rightarrow \cos 1 \Rightarrow \Theta(n)$$
  
 $f(n) = O(n^{1-\epsilon})$ 

if 
$$f(n)=n^{c}$$
  $\int f(n) = \Omega(n^{1+c})$   
for  $c>1$   $\int f(n) = \Omega(n^{1+c})$   
AND  $2f(\frac{n}{2}) = \frac{2}{2^{c}}n^{c} = \frac{1}{2^{c-1}} \cdot f(n)$ 

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

if 
$$f(n)=n \Rightarrow case2 \Rightarrow \Theta(n \cdot logn)$$
(k=0)

if 
$$f(n) = n \log n \Rightarrow case 2 \Rightarrow \Theta(n \log^{k+1} n)$$

if 
$$f(n) = \log^{c} n \Rightarrow \text{cose } 1 \Rightarrow \Theta(n)$$

$$f(n) = O(n^{1-\epsilon})$$

if 
$$f(n) = n^{c}$$
  $\int f(n) = \Omega(n^{1+c})$   
for  $c > 1$   $\int f(n) = \Omega(n^{1+c})$   
AND  $2f(\frac{n}{2}) = \frac{2}{2^{c}}n^{c} = \frac{1}{2^{c-1}} \cdot f(n)$ 

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

if 
$$f(n)=n \Rightarrow case2 \Rightarrow \Theta(n.logn)$$
(k=0)

if 
$$f(n) = n \log^k n \Rightarrow case 2 \Rightarrow \Theta(n \log^{k+1} n)$$

if 
$$f(n) = \log^{c} n \Rightarrow \cos 1 \Rightarrow \Theta(n)$$
  
 $f(n) = O(n^{1-\epsilon})$ 

if 
$$f(n)=n^{c}$$
  $\int f(n) = \Omega(n^{1+c})$   
for  $c>1$   $\int f(n) = \Omega(n^{1+c})$   
AND  $2f(\frac{n}{2}) = \frac{2}{2^{c}}n^{c} = \frac{1}{2^{c-1}} \cdot f(n)$ 

... case  $3 \Rightarrow \Theta(f(n))$ 

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

if 
$$f(n)=n \Rightarrow cose2 \Rightarrow \Theta(n.logn)$$
 $(k=0)$ 

if  $f(n)=nlogn \Rightarrow cose2 \Rightarrow \Theta(nlog^{k+1}n)$ 

if 
$$f(n) = \log^{c} n \Rightarrow \cos 1 \Rightarrow \Theta(n)$$
  
 $f(n) = O(n^{1-\epsilon})$ 

$$T(n) = 4T(\frac{n}{4}) + f(n)$$

$$f(n) = n^{c} \int f(n) = \Omega(n^{1+c})$$

$$f(n) = 1$$

$$f(n) = n^{c} \int f(n) = \Omega(n^{1+c})$$

$$f(n) = 1$$

$$T(n) = 2T(\frac{n}{2}) + f(n)$$

if 
$$f(n)=n \Rightarrow cose2 \Rightarrow \Theta(n \cdot logn)$$
(k=0)

if 
$$f(n) = n \log^k n \Rightarrow case 2 \Rightarrow \Theta(n \log^{k+1} n)$$

if 
$$f(n) = \log^{c} n \Rightarrow \cos 1 \Rightarrow \Theta(n)$$
  
 $f(n) = O(n^{1-\epsilon})$ 

if 
$$f(n) = n^{c}$$
  $\int f(n) = \Omega(n^{1+c})$   
for  $c > 1$   $\int f(n) = \Omega(n^{1+c})$   
AND  $2f(\frac{n}{2}) = \frac{2}{2^{c}}n^{c} = \frac{1}{2^{c-1}} \cdot f(n)$ 

... case 
$$3 \Rightarrow \Theta(f(n))$$

$$T(n) = 4T(\frac{n}{4}) + f(n)$$
SAME

FYI - EXTRA-EXTENDED CASE 2 f(n) = \(\theta\) (n\logba \logka \logka) (Doesn't come up in any algorithms that we will see)

Standard extended case 2 k>0  $T(n) = \Theta(n^{\log_{\delta}a} \cdot \log^{k+1}n)$  $T(n) = \Theta(f(n) \cdot logn)$ 

 $\Rightarrow K=-1$   $T(n) = \Theta(n^{\log n} \cdot \log \log n)$   $T(n) = \Theta(f(n) \cdot \log n \cdot \log \log n)$ e.g.,  $T(n) = 8T(\frac{n}{2}) + \frac{n^3}{logn} = n^3 loglogn$ 

> k ≤ -2  $T(n) = \Theta(n^{\log_1 a})$  almost like an extended case 1:

Leaf level dominates by a "large" poly-log factor.