the MASTER METHOD

↳ a tool for solving recurrences of this form:

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

the MASTER METHOD

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$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

$$\text{assume} \begin{cases} a \geqslant 1 \\ b > 1 \quad \text{otherwise you get } \infty \\ f(n) > 0 \text{ for } n > n_0 \end{cases}$$

MASTER METHOD

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

<u>a</u> branches

$$f(n)$$

$$T\left(\frac{n}{b}\right) \;\cdots\cdots\; T\left(\frac{n}{b}\right)$$

MASTER METHOD

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

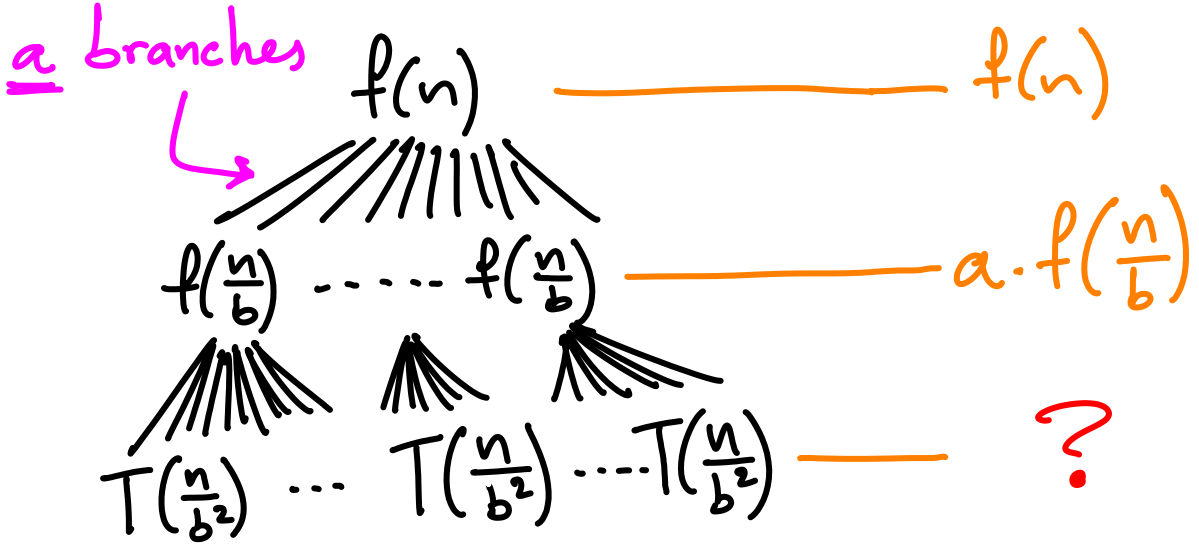
a branches

$$f(n) \text{ ———————— } f(n)$$

$$f\left(\frac{n}{b}\right) \;\cdots\cdots\; f\left(\frac{n}{b}\right) \text{ ———————— } a \cdot f\left(\frac{n}{b}\right)$$

MASTER METHOD

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$



MASTER METHOD

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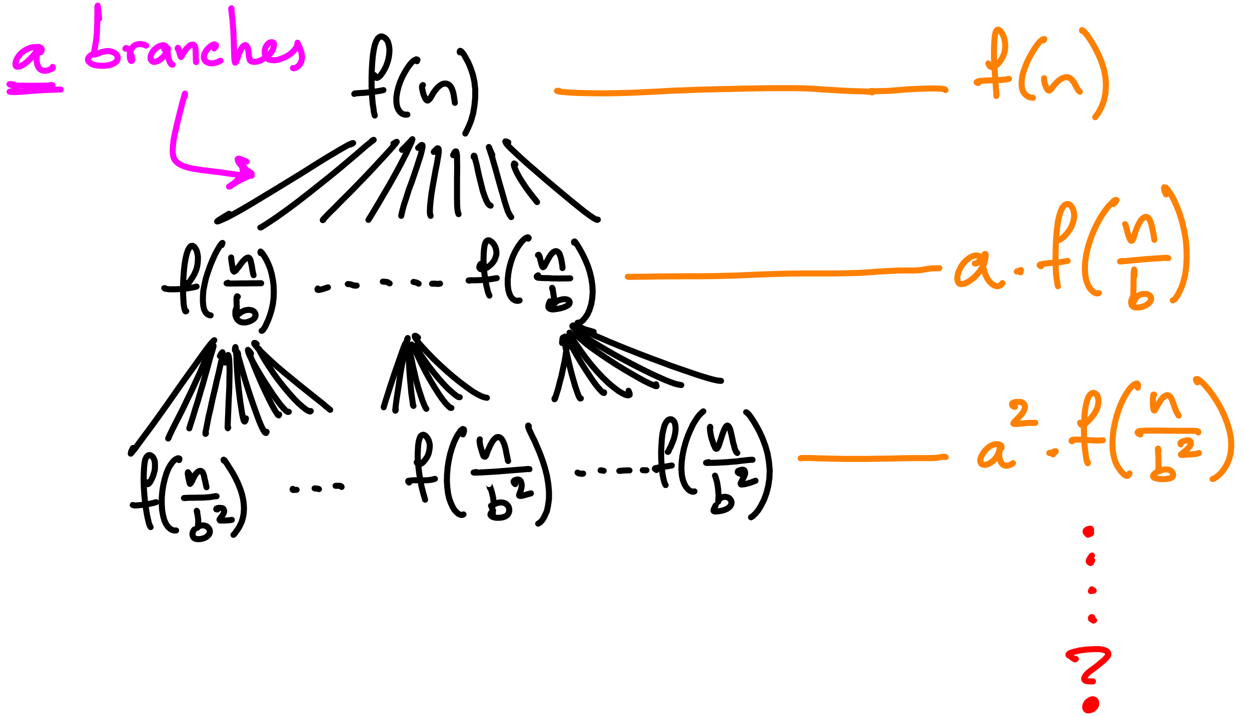

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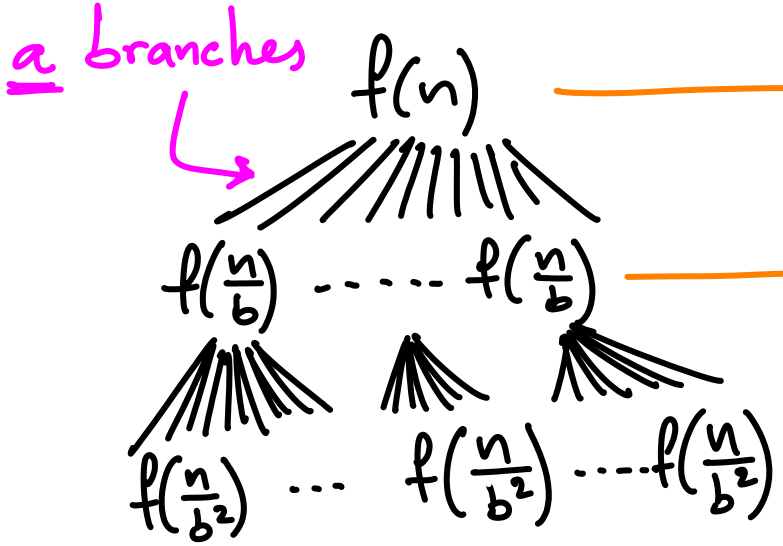


$$f(n)$$

$$a \cdot f\left(\frac{n}{b}\right)$$

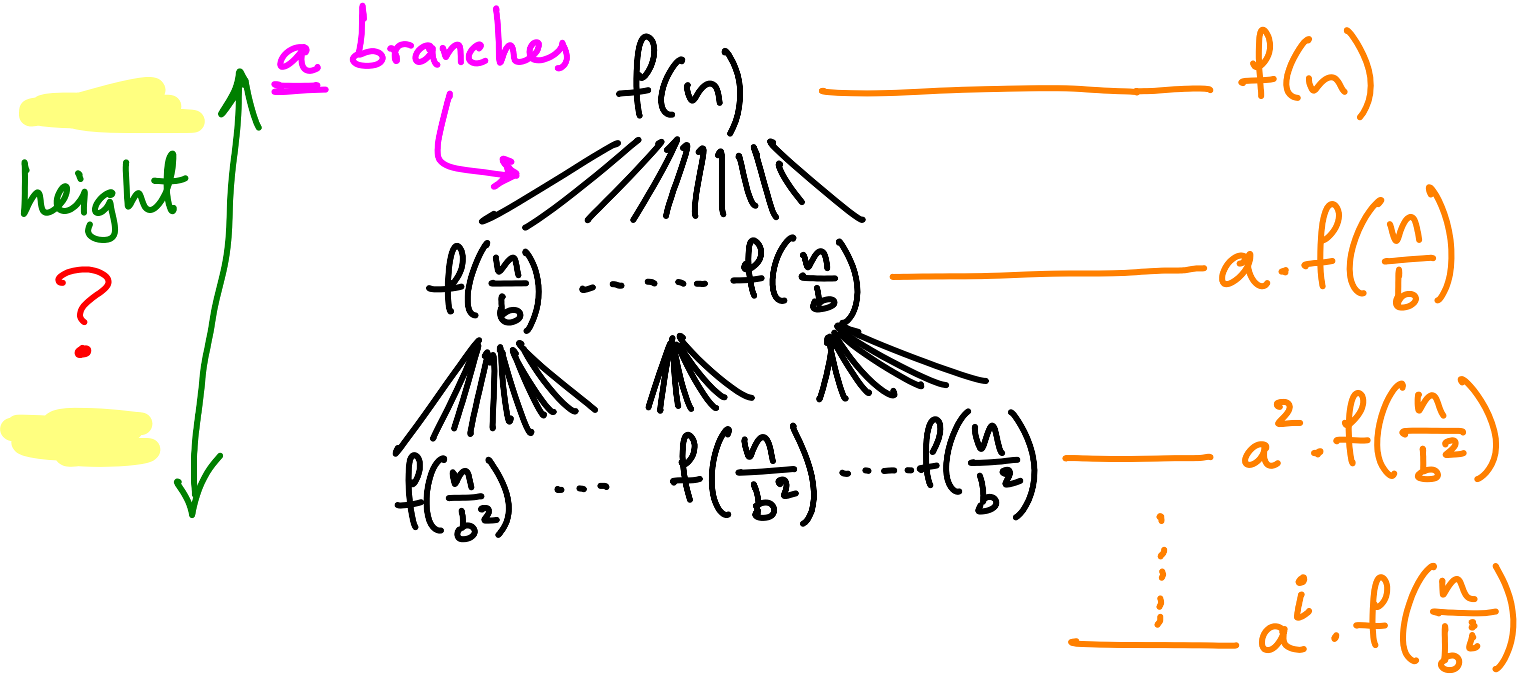
$$a^2 \cdot f\left(\frac{n}{b^2}\right)$$

$$\vdots$$

$$a^i \cdot f\left(\frac{n}{b^i}\right)$$

MASTER METHOD

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$



MASTER METHOD

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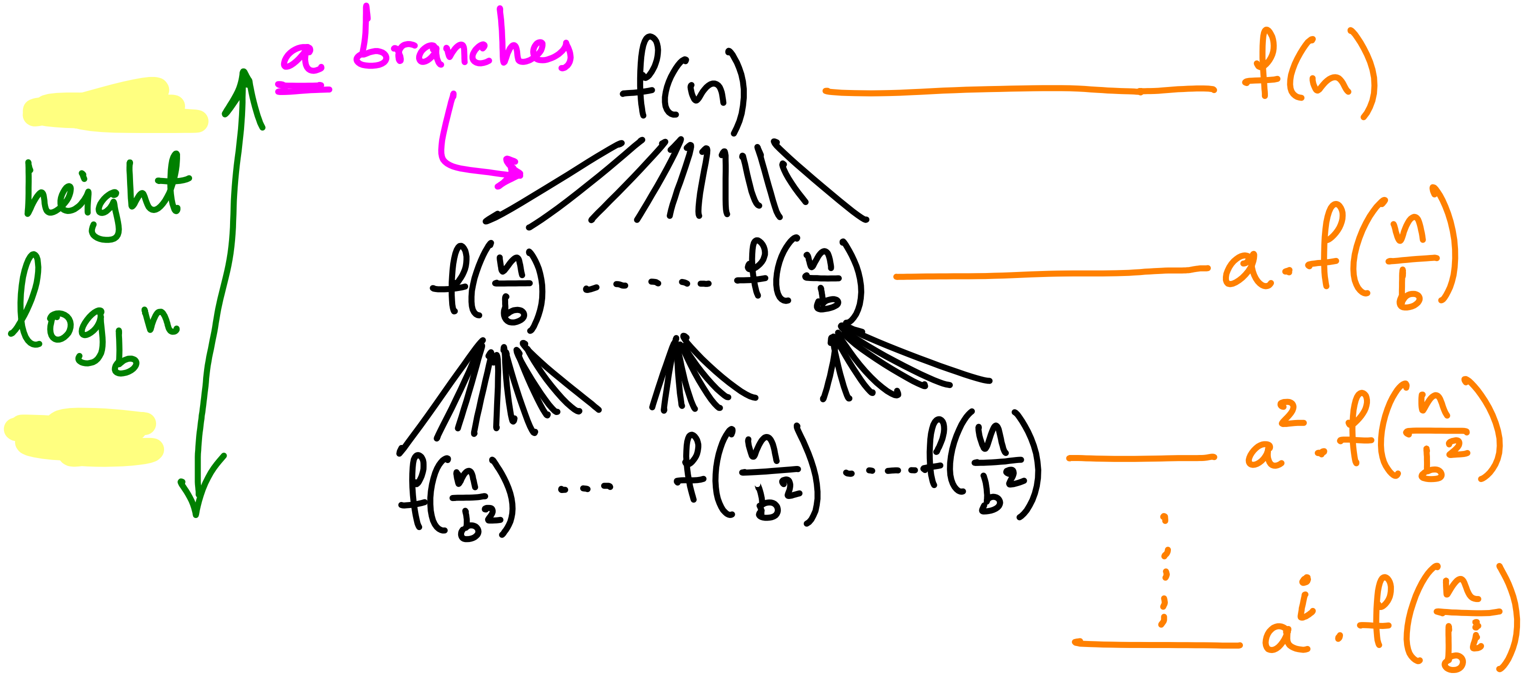

MASTER METHOD

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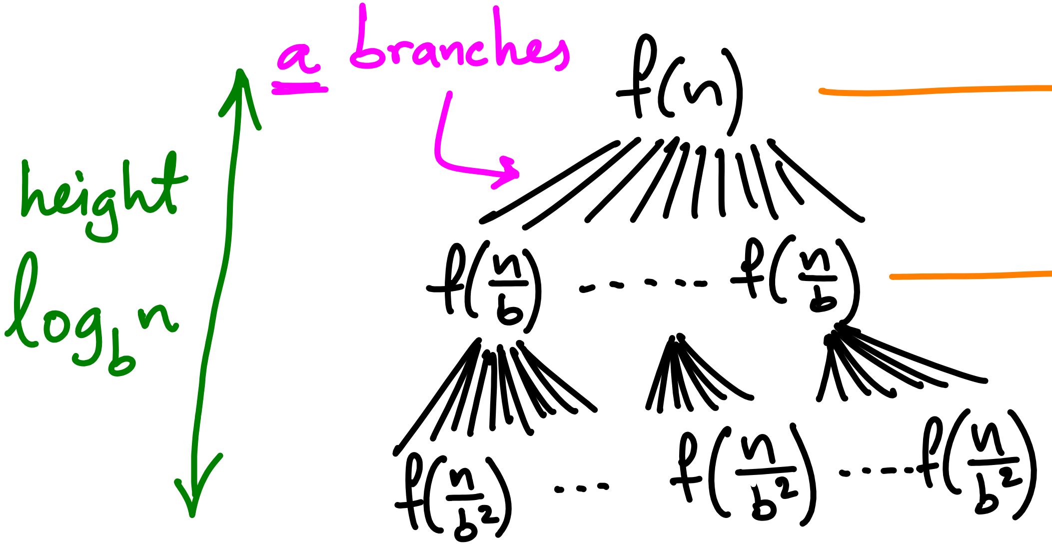

$f(n)$

$a \cdot f\left(\frac{n}{b}\right)$

$a^2 \cdot f\left(\frac{n}{b^2}\right)$

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$a^i \cdot f\left(\frac{n}{b^i}\right)$

leaf level:

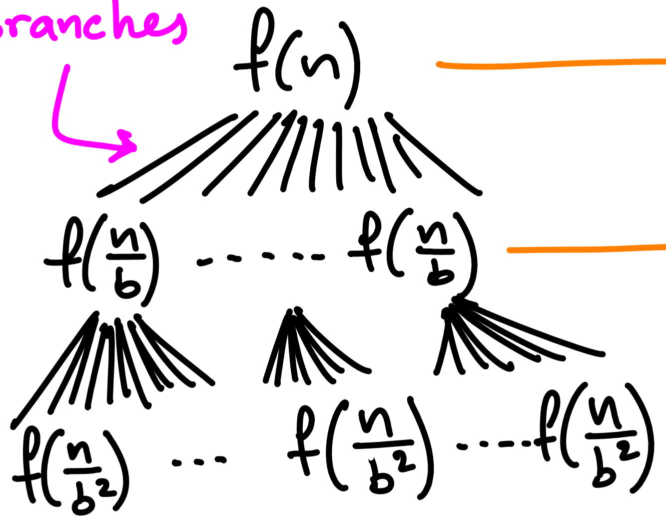
$\#\text{leaves} = a^h$

MASTER METHOD

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

a branches

height $\log_b n$



$f(n)$

$a \cdot f\left(\frac{n}{b}\right)$

$a^2 \cdot f\left(\frac{n}{b^2}\right)$

$\vdots$

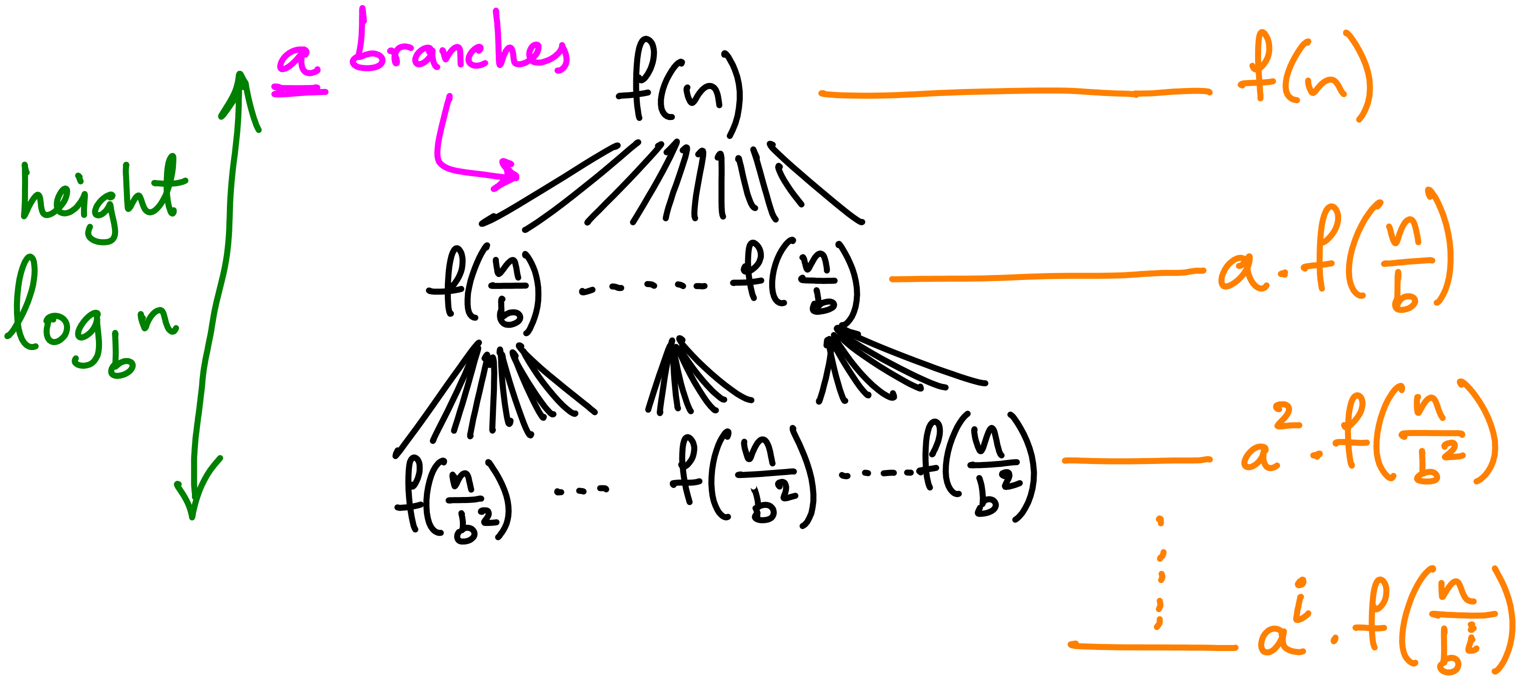
$a^i \cdot f\left(\frac{n}{b^i}\right)$

leaf level:

$\#\text{leaves} = a^h = a^{\log_b n}$

MASTER METHOD

$$T(n) = a\,T\left(\frac{n}{b}\right) + f(n)$$

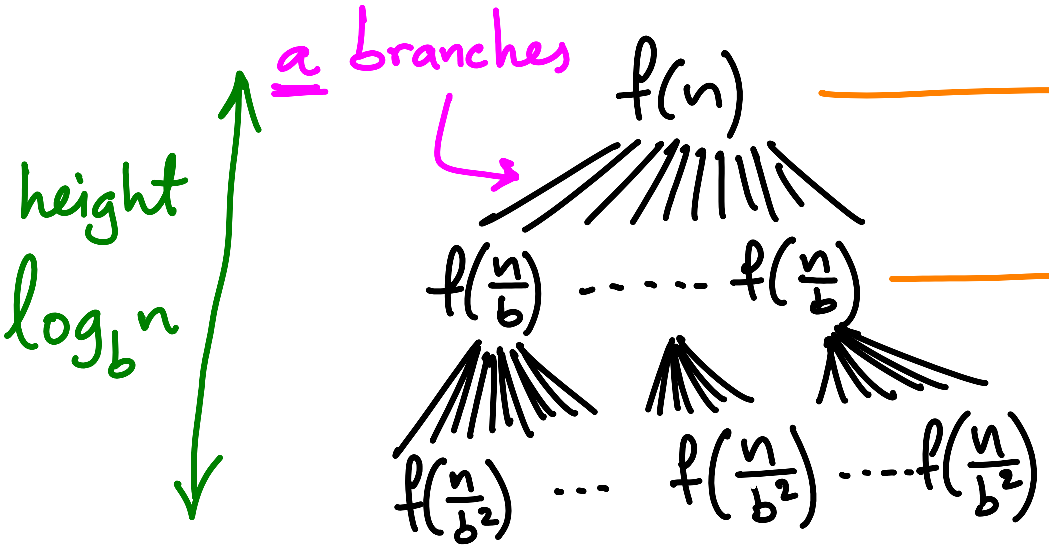


leaf level:

$$\#\text{leaves} = a^h = a^{\log_b n} = n^{\log_b a}$$

MASTER METHOD

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$



$f(n)$

$a \cdot f\left(\frac{n}{b}\right)$

$a^2 \cdot f\left(\frac{n}{b^2}\right)$

$\vdots$

$a^i \cdot f\left(\frac{n}{b^i}\right)$

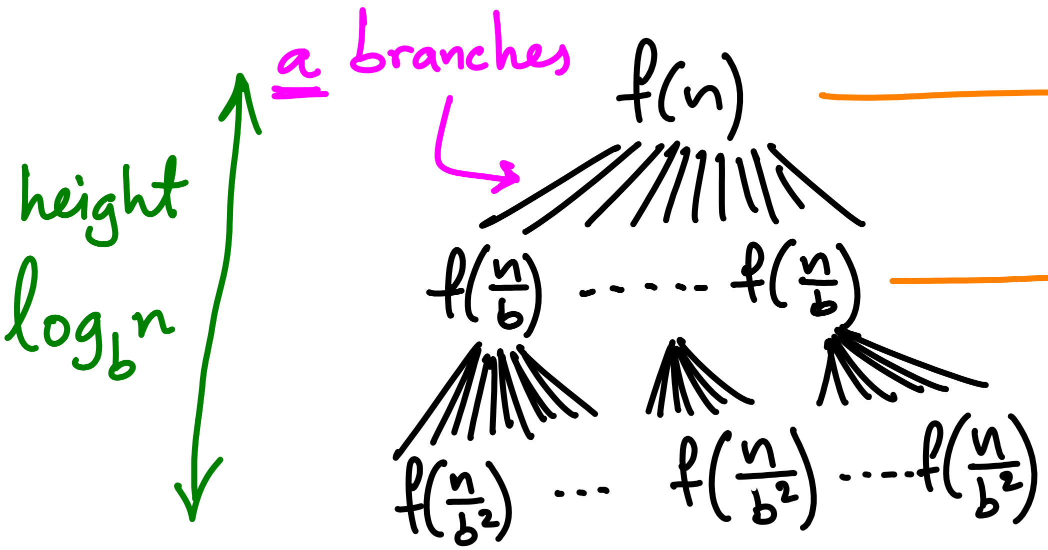
leaf level:

$\#\text{leaves} = a^h = a^{\log_b n} = n^{\log_b a}$ — $\Theta\left(n^{\log_b a}\right)$

$T(1) = \Theta(1)$

MASTER METHOD

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$



$f(n)$

$a \cdot f\left(\frac{n}{b}\right)$

$a^2 \cdot f\left(\frac{n}{b^2}\right)$

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leaf level:

$\#\text{leaves} = a^h = a^{\log_b n} = n^{\log_b a}$ — $\Theta\left(n^{\log_b a}\right)$

CASE 3: root level "dominates"

CASE 2: all levels ≈ same

CASE 1: leaf level "dominates"

MASTER METHOD $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

MASTER METHOD $T(n) = aT\left(\frac{n}{b}\right) + f(n)$ compare $f(n)$ to $n^{\log_b a}$

MASTER METHOD $T(n) = aT\left(\frac{n}{b}\right) + f(n)$ compare $f(n)$ to $n^{\log_b a}$

1) # leaves $= \Omega(f(n) \cdot n^{\varepsilon})$ leaf level dominates polynomially

$\boxed{\varepsilon > 0}$

MASTER METHOD $T(n) = aT\left(\frac{n}{b}\right) + f(n)$ compare $f(n)$ to $n^{\log_b a}$

1) $\#\text{leaves} = \Omega(f(n) \cdot n^{\varepsilon})$ leaf level dominates polynomially

$f(n) = O(\#\text{leaves} / n^{\varepsilon})$

$\varepsilon > 0$

MASTER METHOD $T(n) = aT\left(\frac{n}{b}\right) + f(n)$ compare $f(n)$ to $n^{\log_b a}$

1) $\boxed{f(n) = O\left(n^{(\log_b a) - \varepsilon}\right)}$ $\#\text{leaves} = \Omega\left(f(n) \cdot n^{\varepsilon}\right)$ leaf level dominates polynomially

$f(n) = O\left(\#\text{leaves} / n^{\varepsilon}\right)$

$\varepsilon > 0$

MASTER METHOD $T(n) = aT\left(\frac{n}{b}\right) + f(n)$ compare $f(n)$ to $n^{\log_b a}$

1) $f(n) = O\left(n^{(\log_b a)-\varepsilon}\right)$ $\#\text{leaves} = \Omega\left(f(n) \cdot n^{\varepsilon}\right)$ leaf level dominates polynomially

$f(n) = O\left(\#\text{leaves}/n^{\varepsilon}\right)$

2) $f(n) = \Theta(\#\text{leaves})$ all levels ~ same

$\boxed{\varepsilon > 0}$

MASTER METHOD $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

compare $f(n)$ to $n^{\log_b a}$

1) $f(n) = O\left(n^{(\log_b a)-\varepsilon}\right)$

$\#\text{leaves} = \Omega\left(f(n) \cdot n^{\varepsilon}\right)$

$\hookrightarrow f(n) = O\left(\#\text{leaves}/n^{\varepsilon}\right)$

leaf level dominates polynomially

2) $f(n) = \Theta\left(\#\text{leaves} \cdot \log^k n\right)$

all levels ~ same

[still considered ≈ same]

not mentioned in CLRS

$\varepsilon > 0,\ k \geq 0$

MASTER METHOD $T(n) = aT\left(\frac{n}{b}\right) + f(n)$ compare $f(n)$ to $n^{\log_b a}$

1) $f(n) = O\left(n^{(\log_b a) - \varepsilon}\right)$ $\#\text{leaves} = \Omega\left(f(n) \cdot n^{\varepsilon}\right)$ leaf level dominates polynomially

$\Leftrightarrow f(n) = O\left(\#\text{leaves} / n^{\varepsilon}\right)$

2) $f(n) = \Theta\left(n^{\log_b a} \cdot \log^k n\right) = \Theta\left(\#\text{leaves} \cdot \log^k n\right)$ all levels ~ same

$\varepsilon > 0$, $k \geq 0$

MASTER METHOD $T(n) = aT\left(\frac{n}{b}\right) + f(n)$ compare $f(n)$ to $n^{\log_b a}$

---

1) $f(n) = O\left(n^{(\log_b a) - \varepsilon}\right)$ $\#\text{leaves} = \Omega\left(f(n) \cdot n^{\varepsilon}\right)$ leaf level dominates polynomially

$\Leftrightarrow f(n) = O\left(\#\text{leaves} / n^{\varepsilon}\right)$

---

2) $f(n) = \Theta\left(n^{\log_b a} \cdot \log^k n\right) = \Theta\left(\#\text{leaves} \cdot \log^k n\right)$ all levels $\sim$ same

---

3) $f(n) = \Omega\left(\#\text{leaves} \cdot n^{\varepsilon}\right)$ root dominates polynomially

$\boxed{\varepsilon > 0,\ k \geq 0}$

MASTER METHOD $T(n) = aT\left(\frac{n}{b}\right) + f(n)$ compare $f(n)$ to $n^{\log_b a}$

1) $f(n) = O\left(n^{(\log_b a)-\varepsilon}\right)$ $\#\text{leaves} = \Omega\left(f(n) \cdot n^{\varepsilon}\right)$ leaf level dominates polynomially

$\Leftrightarrow f(n) = O\left(\#\text{leaves}/n^{\varepsilon}\right)$

2) $f(n) = \Theta\left(n^{\log_b a} \cdot \log^k n\right) = \Theta\left(\#\text{leaves} \cdot \log^k n\right)$ all levels ~ same

3) $f(n) = \boxed{\Omega\left(n^{(\log_b a)+\varepsilon}\right)} = \Omega\left(\#\text{leaves} \cdot n^{\varepsilon}\right)$ root dominates polynomially

$\varepsilon > 0$, $k \geq 0$

MASTER METHOD $T(n) = aT\left(\frac{n}{b}\right) + f(n)$ compare $f(n)$ to $n^{\log_b a}$

1) $f(n) = O\left(n^{(\log_b a) - \varepsilon}\right)$ $\#\text{leaves} = \Omega\left(f(n) \cdot n^{\varepsilon}\right)$ leaf level dominates polynomially

$\Leftrightarrow f(n) = O\left(\#\text{leaves} / n^{\varepsilon}\right)$

2) $f(n) = \Theta\left(n^{\log_b a} \cdot \log^k n\right) = \Theta\left(\#\text{leaves} \cdot \log^k n\right)$ all levels ~ same

3) $f(n) = \Omega\left(n^{(\log_b a) + \varepsilon}\right) = \Omega\left(\#\text{leaves} \cdot n^{\varepsilon}\right)$ root dominates polynomially

AND $af\left(\frac{n}{b}\right) \leq \delta \cdot f(n)$ → work reduced by constant fraction in each level

$\boxed{(\varepsilon > 0,\ k \geq 0,\ 0 < \delta < 1)}$

MASTER METHOD $T(n) = aT\left(\frac{n}{b}\right) + f(n)$ compare $f(n)$ to $n^{\log_b a}$

1) $f(n) = O\left(n^{(\log_b a) - \varepsilon}\right)$

$\#\text{leaves} = \Omega\left(f(n) \cdot n^{\varepsilon}\right)$

$\Leftrightarrow f(n) = O\left(\#\text{leaves} / n^{\varepsilon}\right)$

leaf level dominates polynomially

$$\boxed{T(n) = \Theta\left(n^{\log_b a}\right)}$$

2) $f(n) = \Theta\left(n^{\log_b a} \cdot \log^k n\right) = \Theta\left(\#\text{leaves} \cdot \log^k n\right)$

all levels ~ same

3) $f(n) = \Omega\left(n^{(\log_b a) + \varepsilon}\right) = \Omega\left(\#\text{leaves} \cdot n^{\varepsilon}\right)$

root dominates polynomially

AND $af\left(\frac{n}{b}\right) \leq \delta \cdot f(n)$

$\rightarrow$ work reduced by constant fraction in each level

$(\varepsilon > 0, k \geq 0, 0 < \delta < 1)$

MASTER METHOD $T(n) = aT\left(\frac{n}{b}\right) + f(n)$ compare $f(n)$ to $n^{\log_b a}$

---

1) $f(n) = O\left(n^{(\log_b a) - \varepsilon}\right)$ $\#\text{leaves} = \Omega\left(f(n) \cdot n^{\varepsilon}\right)$

$\Leftrightarrow f(n) = O\left(\#\text{leaves} / n^{\varepsilon}\right)$

leaf level dominates polynomially

$T(n) = \Theta\left(n^{\log_b a}\right)$

---

2) $f(n) = \Theta\left(n^{\log_b a} \cdot \log^k n\right) = \Theta\left(\#\text{leaves} \cdot \log^k n\right)$

all levels ~ same

$\boxed{T(n) = \Theta\left(f(n) \cdot \log n\right)}$

---

3) $f(n) = \Omega\left(n^{(\log_b a) + \varepsilon}\right) = \Omega\left(\#\text{leaves} \cdot n^{\varepsilon}\right)$

root dominates polynomially

AND $af\left(\frac{n}{b}\right) \leq \delta \cdot f(n)$

$\rightarrow$ work reduced by constant fraction in each level

$(\varepsilon > 0,\ k \geq 0,\ 0 < \delta < 1)$

MASTER METHOD $T(n) = aT\left(\frac{n}{b}\right) + f(n)$ compare $f(n)$ to $n^{\log_b a}$

---

1) $f(n) = O\left(n^{(\log_b a)-\varepsilon}\right)$ $\quad \#\text{leaves} = \Omega\left(f(n)\cdot n^{\varepsilon}\right)$

$\Leftrightarrow f(n) = O\left(\#\text{leaves}/n^{\varepsilon}\right)$

leaf level dominates polynomially

$T(n) = \Theta\left(n^{\log_b a}\right)$

---

2) $f(n) = \Theta\left(n^{\log_b a}\cdot \log^k n\right) = \Theta\left(\#\text{leaves}\cdot \log^k n\right)$

all levels ~ same

$T(n) = \Theta\left(f(n)\cdot \log n\right)$

---

3) $f(n) = \Omega\left(n^{(\log_b a)+\varepsilon}\right) = \Omega\left(\#\text{leaves}\cdot n^{\varepsilon}\right)$

root dominates polynomially

AND $af\left(\frac{n}{b}\right) \leq \delta \cdot f(n)$ $\rightarrow$ work reduced by constant fraction in each level

$\boxed{T(n) = \Theta(f(n))}$

$(\varepsilon > 0,\ k \geq 0,\ 0 < \delta < 1)$

$$T(n) = \underbrace{4}_{a}\,T\Big(\frac{n}{\underbrace{2}_{b}}\Big) + \underbrace{n}_{f(n)}$$

$$T(n) = \underset{a}{4}\,T\left(\frac{n}{\underset{b}{2}}\right) + \underset{f(n)}{n}$$

$$n^{\log_b a} = n^{\log_2 4} = n^2$$

$$T(n) = \underbrace{4}_{a}\,T\Big(\frac{n}{\underbrace{2}_{b}}\Big) + \underbrace{n}_{f(n)}$$

$$n^{\log_b a} = n^{\log_2 4} = n^2$$

$$f(n) = n = O(n) = O(n^{2-\overbrace{1}^{\varepsilon}})$$: case 1

$T(n) = \underset{a}{4}\,T\left(\frac{n}{\underset{b}{2}}\right) + \underset{f(n)}{n}$ // $n^{\log_b a} = n^{\log_2 4} = n^2$

$f(n) = n = O(n) = O(n^{2-\overset{\varepsilon}{1}})$: case 1

so $n^{\log_b a}$ dominates $f(n)$: answer $= n^{\log_b a} = \Theta(n^2)$

$$T(n) = \underset{a}{4}T\left(\frac{n}{\underset{b}{2}}\right) + \underset{f(n)}{n} \qquad // \quad n^{\log_b a} = n^{\log_2 4} = n^2$$

$$f(n) = n = O(n) = O(n^{2-\overset{\varepsilon}{1}}) \quad : \text{case } 1$$

so $n^{\log_b a}$ dominates $f(n)$: answer $= n^{\log_b a} = \Theta(n^2)$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2 \qquad // \quad n^{\log_b a} = n^2$$

$T(n) = \underbrace{4}_{a}T\left(\frac{n}{\underbrace{2}_{b}}\right) + \underbrace{n}_{f(n)}$ // $n^{\log_b a} = n^{\log_2 4} = n^2$

$f(n) = n = O(n) = O(n^{2-\overset{\varepsilon}{1}})$: case 1

so $n^{\log_b a}$ dominates $f(n)$: answer $= n^{\log_b a} = \Theta(n^2)$

$T(n) = 4T\left(\frac{n}{2}\right) + n^2$ // $n^{\log_b a} = n^2$

$f(n) = n^2 = \Theta(n^{\log_b a} \cdot \log^{\overset{k}{0}} n)$: case 2

$$T(n) = \underbrace{4}_{a}\,T\left(\frac{n}{\underbrace{2}_{b}}\right) + \underbrace{n}_{f(n)} \quad // \quad n^{\log_b a} = n^{\log_2 4} = n^2$$

$$f(n) = n = O(n) = O(n^{2-\overbrace{1}^{\varepsilon}}) \quad : \text{case } 1$$

so $n^{\log_b a}$ dominates $f(n)$: answer $= n^{\log_b a} = \Theta(n^2)$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2 \quad // \quad n^{\log_b a} = n^2$$

$$f(n) = n^2 = \Theta(n^{\log_b a} \cdot \log^{\overbrace{0}^{k}} n) \quad : \text{case } 2$$

ANS: $\Theta(n^{\log_b a} \cdot \log^{k+1} n) = \Theta(n^2 \log n)$

$$T(n) = \underset{a}{4}\,T\left(\frac{n}{\underset{b}{2}}\right) + \underset{f(n)}{n} \quad // \quad n^{\log_b a} = n^{\log_2 4} = n^2$$

$$f(n) = n = O(n) = O(n^{2-\overset{\varepsilon}{1}}) \quad : \text{case } 1$$

so $n^{\log_b a}$ dominates $f(n)$: answer $= n^{\log_b a} = \Theta(n^2)$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2 \quad // \quad n^{\log_b a} = n^2$$

$$f(n) = n^2 = \Theta(n^{\log_b a} \cdot \log^{\overset{k}{0}} n) \quad : \text{case } 2$$

ANS : $\Theta(n^{\log_b a} \cdot \log^{k+1} n) = \Theta(n^2 \log n)$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^3 \quad // \quad f(n) = n^3 = \Omega(n^{2+\varepsilon})$$

$T(n) = 4T\left(\frac{n}{2}\right) + n$ // $n^{\log_b a} = n^{\log_2 4} = n^2$

$a = 4$, $b = 2$, $f(n) = n$

$f(n) = n = O(n) = O(n^{2-\varepsilon})$, $\varepsilon = 1$: case 1

so $n^{\log_b a}$ dominates $f(n)$: answer $= n^{\log_b a} = \Theta(n^2)$

$T(n) = 4T\left(\frac{n}{2}\right) + n^2$ // $n^{\log_b a} = n^2$

$f(n) = n^2 = \Theta(n^{\log_b a} \cdot \log^0 n)$, $k = 0$: case 2

ANS: $\Theta(n^{\log_b a} \cdot \log^{k+1} n) = \Theta(n^2 \log n)$

$T(n) = 4T\left(\frac{n}{2}\right) + n^3$ // $f(n) = n^3 = \Omega(n^{2+\varepsilon})$ AND $af\left(\frac{n}{b}\right) = 4 \cdot \left(\frac{n}{2}\right)^3 = \frac{1}{2} n^3 = \frac{1}{2} \cdot f(n)$

$\delta = \frac{1}{2}$

$$T(n) = \underset{a}{4}\,T\left(\frac{n}{\underset{b}{2}}\right) + \underset{f(n)}{n} \quad // \quad n^{\log_b a} = n^{\log_2 4} = n^2$$

$$f(n) = n = O(n) = O(n^{2-\overset{\varepsilon}{1}}) \quad : \text{case } 1$$

so $n^{\log_b a}$ dominates $f(n)$: answer $= n^{\log_b a} = \Theta(n^2)$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2 \quad // \quad n^{\log_b a} = n^2$$

$$f(n) = n^2 = \Theta(n^{\log_b a} \cdot \log^{\overset{k}{0}} n) \quad : \text{case } 2$$

ANS: $\Theta(n^{\log_b a} \cdot \log^{k+1} n) = \Theta(n^2 \log n)$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^3 \quad // \quad f(n) = n^3 = \Omega(n^{2+\varepsilon}) \text{ AND } af\left(\frac{n}{b}\right) = 4\cdot\left(\frac{n}{2}\right)^3 = \frac{1}{2}n^3 = \frac{1}{2}\cdot f(n)$$

$\hookrightarrow$ case 3 : ANS: $\Theta(n^3)$

$$T(n) = \underset{a}{4}\,T\left(\frac{n}{\underset{b}{2}}\right) + \underset{f(n)}{n} \quad // \quad n^{\log_b a} = n^{\log_2 4} = n^2$$

$$f(n) = n = O(n) = O\left(n^{2-1}\right)^{\varepsilon} \quad : \text{case } 1$$

so $n^{\log_b a}$ dominates $f(n)$: answer $= n^{\log_b a} = \Theta(n^2)$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2 \quad // \quad n^{\log_b a} = n^2$$

$$f(n) = n^2 = \Theta\left(n^{\log_b a} \cdot \log^{0} n\right) \quad : \text{case } 2 \qquad (0 = k)$$

ANS: $\Theta\left(n^{\log_b a} \cdot \log^{k+1} n\right) = \Theta(n^2 \log n)$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^3 \quad // \quad f(n) = n^3 = \Omega\left(n^{2+\varepsilon}\right) \text{ AND } af\left(\frac{n}{b}\right) = 4 \cdot \left(\frac{n}{2}\right)^3 = \frac{1}{2}n^3 = \frac{1}{2} \cdot f(n)$$

$\hookrightarrow$ case 3 : ANS: $\Theta(n^3)$

$$T(n) = 4T\left(\frac{n}{2}\right) + \frac{n^2}{\log n} \quad ?$$

$$T(n) = \underset{a}{4}\,T\left(\frac{n}{\underset{b}{2}}\right) + \underset{f(n)}{n} \qquad // \quad n^{\log_b a} = n^{\log_2 4} = n^2$$

$$f(n) = n = O(n) = O(n^{2-\overset{\varepsilon}{1}}) \quad : \text{case } 1$$

so $n^{\log_b a}$ dominates $f(n)$: answer $= n^{\log_b a} = \Theta(n^2)$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2 \qquad // \quad n^{\log_b a} = n^2$$

$$f(n) = n^2 = \Theta(n^{\log_b a} \cdot \log^{\overset{k}{0}} n) \quad : \text{case } 2$$

ANS : $\Theta(n^{\log_b a} \cdot \log^{k+1} n) = \Theta(n^2 \log n)$

$$T(n) = 4T\left(\frac{n}{2}\right) + n^3 \qquad // \quad f(n) = n^3 = \Omega(n^{2+\varepsilon}) \text{ AND } af\left(\frac{n}{b}\right) = 4 \cdot \left(\frac{n}{2}\right)^3 = \frac{1}{2} n^3 = \frac{1}{2} \cdot f(n)$$

$\hookrightarrow$ case 3 : ANS : $\Theta(n^3)$

$$T(n) = 4T\left(\frac{n}{2}\right) + \frac{n^2}{\log n} \quad \Big\} \quad \text{N/A}$$

But in fact there is now yet another extension of case 2

(see last page)

Divide & Conquer

$$T(n) = 2T\left(\frac{n}{2}\right) + f(n)$$

$$n^{\log_b a} = n$$

Divide & Conquer

$$T(n) = 2T\left(\frac{n}{2}\right) + f(n)$$

if $f(n) = n \Rightarrow$

$$n^{\log_b a} = n$$

Divide & Conquer

$$T(n) = 2T\left(\frac{n}{2}\right) + f(n)$$

$$n^{\log_b a} = n$$

if $f(n) = n \Rightarrow$ case2 $\Rightarrow$

Divide & Conquer

$$T(n) = 2T\left(\frac{n}{2}\right) + f(n)$$

if $f(n) = n \Rightarrow$ case2 $\Rightarrow \Theta(n \cdot \log n)$

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if $f(n) = n\log^k n \Rightarrow$

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if $f(n) = n \Rightarrow$ case 2 $\Rightarrow \Theta(n \cdot \log n)$

$(k=0)$

if $f(n) = n \log^k n \Rightarrow$ case 2 $\Rightarrow \Theta(n \log^{k+1} n)$

Divide & Conquer

$$T(n) = 2T\left(\frac{n}{2}\right) + f(n)$$

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if $f(n) = \log^c n \Rightarrow$

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if $f(n) = \log^c n \Rightarrow$ case 1

$f(n) = O(n^{1-\varepsilon})$

Divide & Conquer

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$f(n) = O(n^{1-\varepsilon})$

if $f(n) = n^c$

~~for~~ $c > 1$

Divide & Conquer

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if $f(n) = n \Rightarrow$ case 2 $\Rightarrow \Theta(n \cdot \log n)$

$(k=0)$

if $f(n) = n \log^k n \Rightarrow$ case 2 $\Rightarrow \Theta(n \log^{k+1} n)$

if $f(n) = \log^c n \Rightarrow$ case 1 $\Rightarrow \Theta(n)$

$f(n) = O(n^{1-\varepsilon})$

if $f(n) = n^c$ for $c > 1$ $\Big\}$ $f(n) = \Omega(n^{1+\varepsilon})$

Divide & Conquer

$$T(n) = 2T\left(\frac{n}{2}\right) + f(n)$$

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if $f(n) = \log^c n \Rightarrow$ case 1 $\Rightarrow \Theta(n)$

$f(n) = O(n^{1-\varepsilon})$

if $f(n) = n^c$ for $c > 1$ $\left\{ f(n) = \Omega(n^{1+\varepsilon}) \right.$

AND $2f\left(\frac{n}{2}\right) = \frac{2}{2^c} n^c = \frac{1}{2^{c-1}} \cdot f(n)$

Divide & Conquer

$$T(n) = 2T\left(\frac{n}{2}\right) + f(n)$$

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AND $2f\left(\frac{n}{2}\right) = \frac{2}{2^c} n^c = \frac{1}{2^{c-1}} \cdot f(n)$

... case 3 $\Rightarrow$

Divide & Conquer

$$T(n) = 2T\left(\frac{n}{2}\right) + f(n)$$

$$n^{\log_b a} = n$$

if $f(n) = n \Rightarrow$ case 2 $\Rightarrow \Theta(n \cdot \log n)$

$(k=0)$

if $f(n) = n \log^k n \Rightarrow$ case 2 $\Rightarrow \Theta(n \log^{k+1} n)$

if $f(n) = \log^c n \Rightarrow$ case 1 $\Rightarrow \Theta(n)$

$f(n) = O(n^{1-\varepsilon})$

if $f(n) = n^c$ for $c > 1$ $\left\{ f(n) = \Omega(n^{1+\varepsilon}) \right.$

AND $2f\left(\frac{n}{2}\right) = \frac{2}{2^c} n^c = \frac{1}{2^{c-1}} \cdot f(n)$

... case 3 $\Rightarrow \Theta(f(n))$

Divide & Conquer

$$T(n) = 2T\left(\frac{n}{2}\right) + f(n)$$

$$n^{\log_b a} = n$$

if $f(n) = n \Rightarrow$ case 2 $\Rightarrow \Theta(n \cdot \log n)$

$(k=0)$

if $f(n) = n \log^k n \Rightarrow$ case 2 $\Rightarrow \Theta(n \log^{k+1} n)$

if $f(n) = \log^c n \Rightarrow$ case 1 $\Rightarrow \Theta(n)$

$f(n) = O(n^{1-\varepsilon})$

if $f(n) = n^c$ for $c > 1$ $\Big\}$ $f(n) = \Omega(n^{1+\varepsilon})$

AND $2f\left(\frac{n}{2}\right) = \frac{2}{2^c} n^c = \frac{1}{2^{c-1}} \cdot f(n)$

... case 3 $\Rightarrow \Theta(f(n))$

$$T(n) = 4T\left(\frac{n}{4}\right) + f(n)$$

?

Divide & Conquer

$$T(n) = 2T\left(\frac{n}{2}\right) + f(n)$$

$$n^{\log_b a} = n$$

if $f(n) = n \Rightarrow$ case2 $\Rightarrow \Theta(n \cdot \log n)$
$(k=0)$

if $f(n) = n \log^k n \Rightarrow$ case 2 $\Rightarrow \Theta(n \log^{k+1} n)$

if $f(n) = \log^c n \Rightarrow$ case 1 $\Rightarrow \Theta(n)$
$f(n) = O(n^{1-\varepsilon})$

if $f(n) = n^c$
~~for~~ $c > 1$
$\Big\}\ f(n) = \Omega(n^{1+\varepsilon})$

AND $2f\left(\frac{n}{2}\right) = \frac{2}{2^c} n^c = \frac{1}{2^{c-1}} \cdot f(n)$

... case 3 $\Rightarrow \Theta(f(n))$

$$T(n) = 4T\left(\frac{n}{4}\right) + f(n)$$

SAME

FYI - EXTRA-EXTENDED CASE 2 $f(n) = \Theta(n^{\log_b a} \cdot \log^k n)$

(Doesn't come up in any algorithms that we will see)

Standard extended case 2

$k \geq 0$ $T(n) = \Theta(n^{\log_b a} \cdot \log^{k+1} n)$ $T(n) = \Theta(f(n) \cdot \log n)$

$k = -1$ $T(n) = \Theta(n^{\log_b a} \cdot \log\log n)$ $T(n) = \Theta(f(n) \cdot \log n \cdot \log\log n)$

e.g., $T(n) = 8T(\frac{n}{2}) + \frac{n^3}{\log n}$ $= n^3 \log\log n$

$k \leq -2$ $T(n) = \Theta(n^{\log_b a})$ almost like an extended case 1:

Leaf level dominates by a "large" poly-log factor.