Resolving collisions w/ open addressing assuming $n \leq m$

1	36
2	43
3	
4	78
5	
6	5
7	103
8	
9	2014
10	
11	

RESOLVING COLLISIONS w/ OPEN ADDRESSING assuming $n \leq m$

The point is to avoid auxilliary linked lists. Use that space for table.
(and pointers)

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RESOLVING COLLISIONS w/ OPEN ADDRESSING assuming $n \leq m$

The point is to avoid auxilliary linked lists. Use that space for table.

Instead, create a probe sequence as a function of key value.

↳ permutation of slots to try.

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ex: $h(64) \rightarrow 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.$

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ex: $h(64) \rightarrow$ 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.

Insert(64): Try T[9]: full

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ex: $h(64) \rightarrow$ 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.

Insert(64): Try T[9]: full

Try T[2]: full

RESOLVING COLLISIONS w/ OPEN ADDRESSING assuming $n \leq m$

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Instead, create a probe sequence as a function of key value.

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ex: $h(64) \rightarrow 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.$

Insert(64): Try T[9]: full

Try T[2]: full

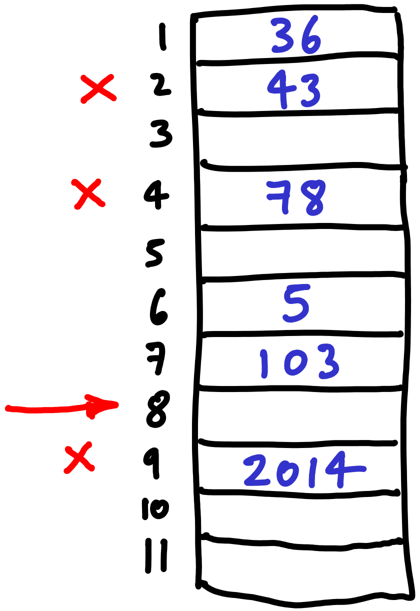
Try T[4]: full

RESOLVING COLLISIONS w/ OPEN ADDRESSING assuming $n \leq m$

The point is to avoid auxilliary linked lists. Use that space for table.

Instead, create a **probe sequence** as a function of key value.

↳ permutation of slots to try.



ex: $h(64) \to$ 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.

Insert(64):
Try T[9]: full
Try T[2]: full
Try T[4]: full
Try T[8]: OK

Resolving collisions w/ open addressing assuming $n \leq m$

The point is to avoid auxilliary linked lists. Use that space for table.

Instead, create a probe sequence as a function of key value.

↳ permutation of slots to try.

1	36
2	43
3	
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ex: $h(64) \rightarrow 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.$

Insert(64):
- Try T[9]: full
- Try T[2]: full
- Try T[4]: full
- Try T[8]: ok

Search(64) follows same sequence.
Would return "not found" after 4 attempts.

ex: $h(64) \rightarrow$ 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.

Really this is $h(k, i)$

1	36
2	43
3	
4	78
5	
6	5
7	103
8	64
9	2014
10	
11	

ex: $h(64) \rightarrow 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.$

Really this is $h(k, i)$

$h(64, 1) = 9$

1	36
2	43
3	
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ex: $h(64) \rightarrow 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.$

Really this is $h(k, i)$

$h(64,1) = 9$ / $h(64,2) = 2$

1	36
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ex: $h(64) \rightarrow 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.$

Really this is $h(k, i)$ $\quad h(64,1) = 9$ / $h(64,2) = 2$ / $h(64,3) = 4$ / etc

ex: $h(64) \rightarrow$ 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.

Really this is $h(k, i)$ $h(64,1) = 9$ / $h(64,2) = 2$ / $h(64,3) = 4$ / etc

1	36
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Delete(64) : ?

ex: $h(64) \rightarrow$ 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.

Really this is $h(k, i)$ $h(64,1) = 9$ / $h(64,2) = 2$ / $h(64,3) = 4$ / etc

1	36
2	43
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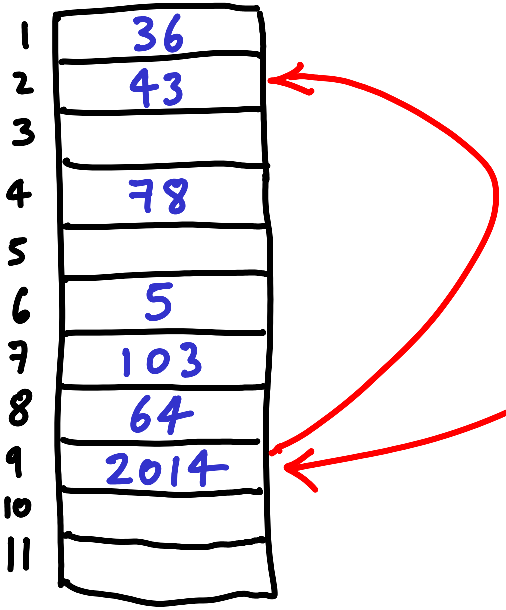
Delete(64) : $h(64,1) = 9$, occupied by 2014

ex: $h(64) \rightarrow 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.$

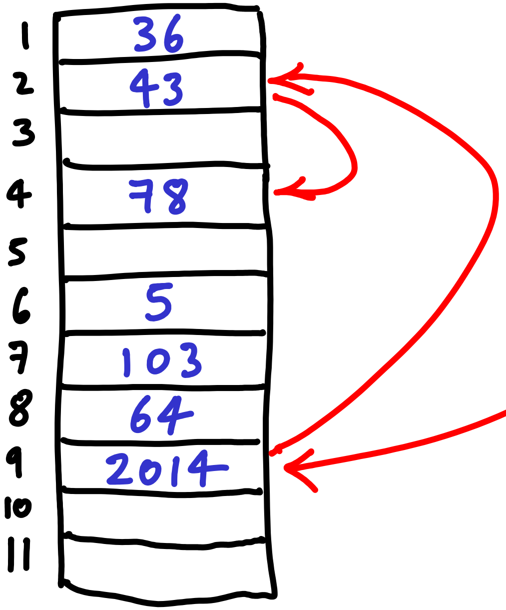
Really this is $h(k, i)$ $h(64,1) = 9$ / $h(64,2) = 2$ / $h(64,3) = 4$ / etc



Delete(64) : $h(64,1) = 9$, occupied by 2014
$h(64,2) = 2$, occupied by 43

ex: $h(64) \rightarrow 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.$

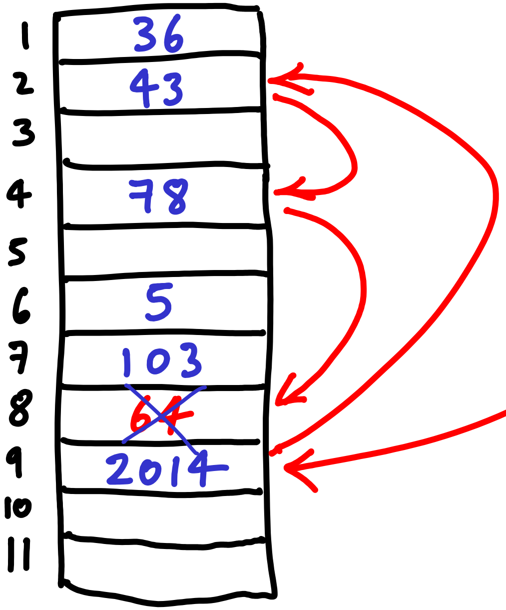
Really this is $h(k, i)$ $\quad h(64,1) = 9 \;/\; h(64,2) = 2 \;/\; h(64,3) = 4 \;/$ etc



Delete(64) : $h(64,1) = 9$, occupied by 2014
$h(64,2) = 2$, occupied by 43
$h(64,3) = 4$, occupied by 78

ex: $h(64) \rightarrow 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.$

Really this is $h(k, i)$ $\quad h(64,1) = 9$ / $h(64,2) = 2$ / $h(64,3) = 4$ / etc



Delete(64) : $h(64,1) = 9$, occupied by 2014

$h(64,2) = 2$, occupied by 43

$h(64,3) = 4$, occupied by 78

$h(64,4) = 8$, found 64, DELETE IT.

OK?

ex: $h(64) \rightarrow$ 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.

Really this is $h(k,i)$ $h(64,1)=9$ / $h(64,2)=2$ / $h(64,3)=4$ / etc

1	36
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Delete(64) : $h(64,1) = 9$, occupied by 2014

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$h(64,3) = 4$, occupied by 78

$h(64,4) = 8$, found 64, DELETE IT.

what if $h(103) \rightarrow$ 4, 8, 2, 7, 1, 10, 11, 3, 5, 9, 6 ?

ex: $h(64) \rightarrow$ 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.

Really this is $h(k, i)$ $h(64,1) = 9$ / $h(64,2) = 2$ / $h(64,3) = 4$ / etc

1	36
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Delete(64) : $h(64,1) = 9$, occupied by 2014
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what if $h(103) \rightarrow$ 4, 8, 2, 7, 1, 10, 11, 3, 5, 9, 6 ?

Search(103) : $h(103,1) = 4$, occupied by 78

ex: $h(64) \rightarrow 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.$

Really this is $h(k, i)$ $\quad h(64,1) = 9$ / $h(64,2) = 2$ / $h(64,3) = 4$ / etc

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Delete(64) : $h(64,1) = 9$, occupied by 2014
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$h(64,3) = 4$, occupied by 78
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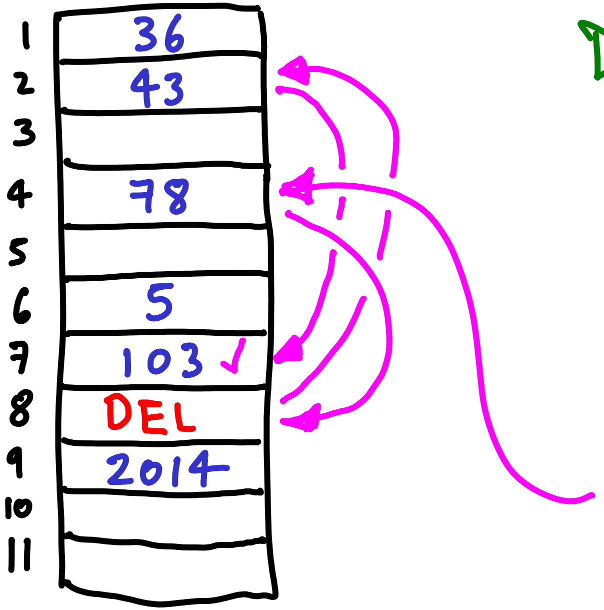
what if $h(103) \rightarrow 4, 8, 2, 7, 1, 10, 11, 3, 5, 9, 6$?

Search(103) : $h(103,1) = 4$, occupied by 78
$h(103,2) = 8$, empty : declare 103 not in T.

ex: $h(64) \to 9, 2, 4, 8, 1, 3, 11, 7, 10, 5, 6.$

Really this is $h(k, i)$ $h(64,1) = 9$ / $h(64,2) = 2$ / $h(64,3) = 4$ / etc



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$h(64,4) = 8$, found 64, DELETE IT.

what if $h(103) \to 4, 8, 2, 7, 1, 10, 11, 3, 5, 9, 6$?

Search(103) : $h(103,1) = 4$, occupied by 78
$h(103,2) = 8$, empty : declare 103 not in T.

Could use special "deleted" markers, but search time increases

(Consider deleting all but one element, and then searching for it)

Typical probing sequences

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Linear probing : $h(k,i) = \left(h(k,0) + i\right) \bmod m$

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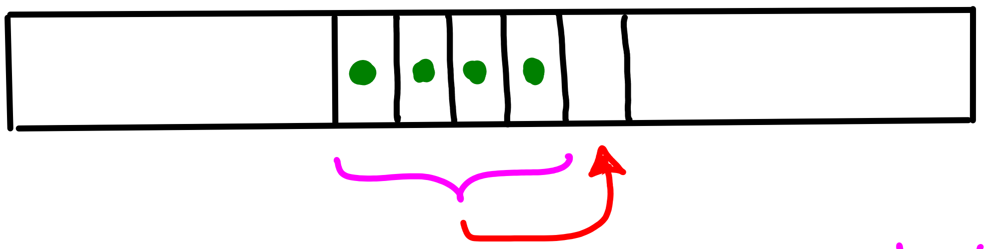
... tends to generate clusters.



$$\underbrace{\text{probability of extending a cluster} = \frac{|\text{cluster}|}{m}}_{\text{slows down search}}$$

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Quadratic probing : $h(k,i) = \left(h(k,0) + c \cdot i + d \cdot i^2\right) \bmod m$

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Both generate m probe sequences in total

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Less clustering, need to make sure sequence hits all slots

Both generate m probe sequences in total

Double hashing : $h(k,i) = \left(h_1(k) + i \cdot h_2(k)\right) \bmod m$

Typical probing sequences

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Double hashing : $h(k,i) = \left(h_1(k) + \underbrace{i \cdot h_2(k)}_{\text{each } k \text{ has "random" offset}}\right) \bmod m$

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Generates $O(m^2)$ probe sequences: better

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Generates $O(m^2)$ probe sequences: better

Heuristic : choose $m = 2^r$ & $h_2(k)$: odd.

ANALYSIS OF OPEN ADDRESSING

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ASSUMING UNIFORM HASHING : each key is equally likely to have any of the $m!$ permutations as probe sequence (independent of other keys)

$\neq$

simple uniform hashing

For a random h, every slot is equally likely

ANALYSIS OF OPEN ADDRESSING

ASSUMING UNIFORM HASHING : each key is equally likely to have any of the m! permutations as probe sequence (independent of other keys)

even though all we have so far is $O(m^2)$

$\neq$

simple uniform hashing

For a random h, every slot is equally likely

ANALYSIS OF OPEN ADDRESSING

ASSUMING UNIFORM HASHING : each key is equally likely to have any of the $m!$ permutations as probe sequence (independent of other keys)

Recall $n<m$, so $\alpha<1$. $\boxed{\text{Claim:}}$ $E[\#\text{probes}] \leq \frac{1}{1-\alpha}$ $\left(\frac{m}{m-n}\right)$

(search)

Analysis of open addressing

ASSUMING UNIFORM HASHING : each key is equally likely to have any of the $m!$ permutations as probe sequence (independent of other keys)

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If true, then for $n \ll m$ $E[\#\text{probes}] = O(1)$

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If true, then for $n \ll m$ $E[\#\text{probes}] = O(1)$

↳ $n = \frac{1}{2}m \rightarrow$ 2 probes

↳ 90% full table $\rightarrow$ 10 probes

ANALYSIS OF OPEN ADDRESSING

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(search)

If true, then for $n \ll m$ $E[\#\text{probes}] = O(1)$

↳ $n = \frac{1}{2}m \rightarrow 2$ probes

↳ 90% full table → 10 probes

Works well if you can afford a table ~ data x2

but keep in mind: we're using a very strong assumption

Claim: $E[\#\text{probes}] \leq \frac{1}{1-\alpha}$

Look at unsuccessful search

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Remember, probe sequence is a permutation.

Never check one slot twice.

Claim: $E[\#\text{probes}] \leq \frac{1}{1-\alpha}$

Look at unsuccessful search

$P[\text{1st probe collides}] = \frac{n}{m}$

Remember, probe sequence is a permutation.

Never check one slot twice.

Claim: $E[\#\text{probes}] \leq \frac{1}{1-\alpha}$

Look at unsuccessful search

$P[\text{1st probe collides}] = \frac{n}{m}$ → need 2nd probe

Remember, probe sequence is a permutation.
Never check one slot twice.

Claim: $E[\#\text{probes}] \leq \frac{1}{1-\alpha}$

Look at unsuccessful search

$P[\text{1st probe collides}] = \frac{n}{m}$ → need 2nd probe

$P[\text{2nd probe collides}] = \frac{n-1}{m-1}$ → need 3rd probe

Remember, probe sequence is a permutation.
Never check one slot twice.

Claim: $E[\#\text{probes}] \le \frac{1}{1-\alpha}$

Look at unsuccessful search

$P[\text{1st probe collides}] = \frac{n}{m}$ → need 2nd probe

$P[\text{2nd probe collides}] = \frac{n-1}{m-1}$ → need 3rd probe

$$\vdots$$

$$\frac{n-i}{m-i}$$

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$\frac{n-i}{m-i} < \frac{n}{m} = \alpha$

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$*$ $P[\text{1st probe collides}] = \frac{n}{m}$ → need 2nd probe

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$\vdots$

$\frac{n-i}{m-i} < \frac{n}{m} = \alpha$

Remember, probe sequence is a permutation.

Never check one slot twice.

$E[\#\text{probes}] = 1 + \frac{n}{m}\Big($ need at least a 2nd probe $\Big)$

$\uparrow$

Claim: $E[\#\text{probes}] \leq \frac{1}{1-\alpha}$

Look at unsuccessful search

$P[\text{1st probe collides}] = \frac{n}{m}$ → need 2nd probe

* $P[\text{2nd probe collides}] = \frac{n-1}{m-1}$ → need 3rd probe

Remember, probe sequence is a permutation. Never check one slot twice.

$$\vdots$$

$$\frac{n-i}{m-i} < \frac{n}{m} = \alpha$$

$E[\#\text{probes}] = 1 + \frac{n}{m}\Big(1 + \frac{n-1}{m-1}\big(\text{ need a 3rd probe }\big)\Big)$

Claim: $E[\#\text{probes}] \leq \frac{1}{1-\alpha}$

Look at unsuccessful search

$P[\text{1st probe collides}] = \frac{n}{m}$ → need 2nd probe

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$\vdots$

$\frac{n-i}{m-i} < \frac{n}{m} = \alpha$

Remember, probe sequence is a permutation.
Never check one slot twice.

$E[\#\text{probes}] = 1 + \frac{n}{m}\left(1 + \frac{n-1}{m-1}\left(1 + \frac{n-2}{m-2}\left(\qquad \right)\right)\right)$

Claim: $E[\#\text{probes}] \leq \frac{1}{1-\alpha}$

Look at unsuccessful search

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$$\leq 1 + \alpha\left(1 + \alpha\left(1 + \alpha\left(\cdots\left(1 + \alpha\right)\right)\right)\right) \quad \text{... n terms}$$

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Look at unsuccessful search

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Remember, probe sequence is a permutation.
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$$E[\#\text{probes}] = 1 + \frac{n}{m}\left(1 + \frac{n-1}{m-1}\left(1 + \frac{n-2}{m-2}\left(\cdots\left(1 + \frac{0}{m-n}\right)\right)\right)\right)$$

$$\leq 1 + \alpha\left(1 + \alpha\left(1 + \alpha\left(\cdots\left(1 + \alpha\right)\right)\right)\right) \quad \text{... n terms}$$

$$\leq 1 + \alpha + \alpha^2 + \alpha^3 + \cdots \quad \text{.... } \infty \text{ terms}$$

Claim: $E[\#\text{probes}] \leq \frac{1}{1-\alpha}$ Look at unsuccessful search

$P[\text{1st probe collides}] = \frac{n}{m}$ → need 2nd probe

$P[\text{2nd probe collides}] = \frac{n-1}{m-1}$ → need 3rd probe

$\vdots$

$\frac{n-i}{m-i} < \frac{n}{m} = \alpha$

Remember, probe sequence is a permutation.
Never check one slot twice.

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see CLRS for alternate analysis incl. successful search

Suggested reading:

perfect hashing — Family of hash functions, pick one randomly. Beats adversaries.

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cuckoo hashing ! — Cuckoos Use Mafia Tactics, And They Work

April 18, 2014 | by Stephen Luntz