# OPEN ADDRESSING

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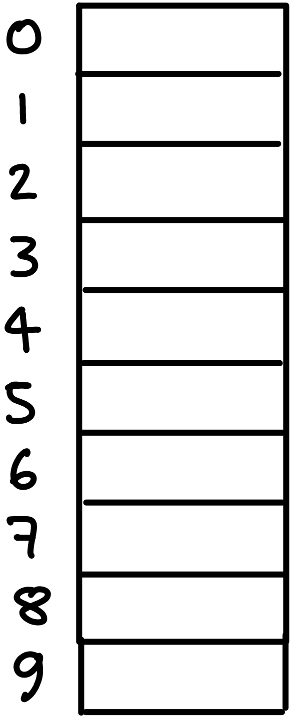
Avoid using pointers in linked lists. Use that space for a larger table.



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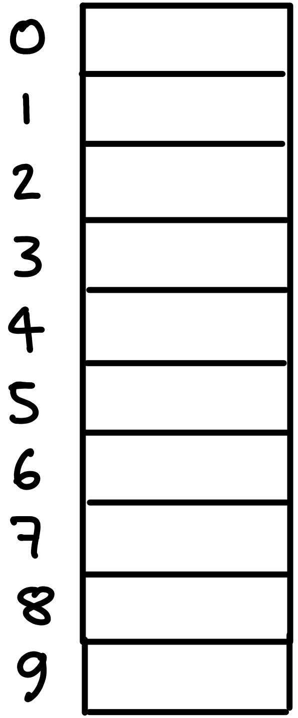
Avoid using pointers in linked lists. Use that space for a larger table.

To be clear, for the same number of keys, chaining uses extra pointers that take more space



# OPEN ADDRESSING

Require $n \le m$

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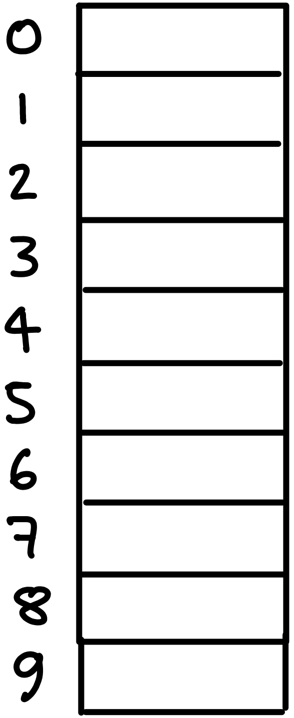


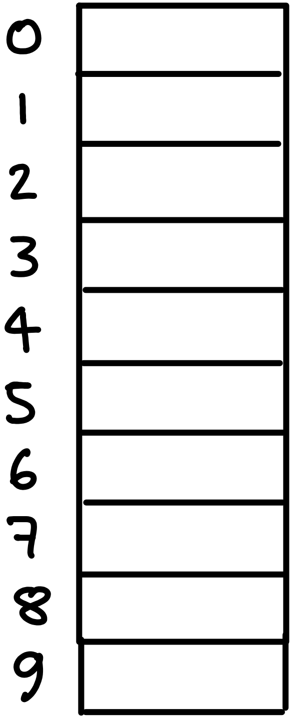
# OPEN ADDRESSING

Require $n \leq m$

Avoid using pointers in linked lists. Use that space for a larger table.

We use a probe sequence → permutation of all slots.

e.g., $h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$



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$m = 10$

| | |
|---|---|
| 0 | |
| 1 | 36 |
| 2 | 43 |
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| 8 | |
| 9 | 2014 |

Search(64)

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---

Search(64)

Try T[9] : not 64

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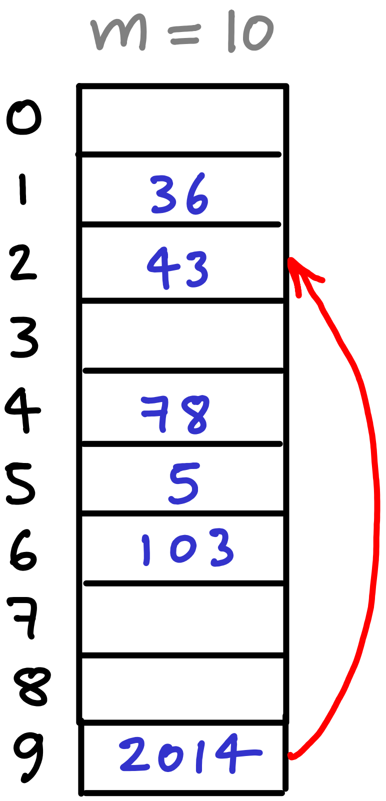


Search(64)

Try T[9]: not 64

Try T[2]: not 64

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e.g., $h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$

Search(64)

Try T[9]: not 64

Try T[2]: not 64

Try T[4]: not 64

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| 9 | 2014 |

Search(64)

Try T[9] : not 64

Try T[2] : not 64

Try T[4] : not 64

Try T[8] : "not found"

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| 4 | 78 |
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| 8 | |
| 9 | 2014 |

Search(64)

Try T[9] : not 64

Try T[2] : not 64

Try T[4] : not 64

Try T[8] : "not found"

Insert(64)

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Avoid using pointers in linked lists. Use that space for a larger table.

We use a probe sequence → permutation of all slots.

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$m = 10$

| | |
|---|---|
| 0 | |
| 1 | 36 |
| 2 | 43 |
| 3 | |
| 4 | 78 |
| 5 | 5 |
| 6 | 103 |
| 7 | |
| 8 | |
| 9 | 2014 |

**Search(64)**

Try T[9]: not 64

Try T[2]: not 64

Try T[4]: not 64

Try T[8]: "not found"

**Insert(64)**

Try T[9]: full

Try T[2]: full

Try T[4]: full

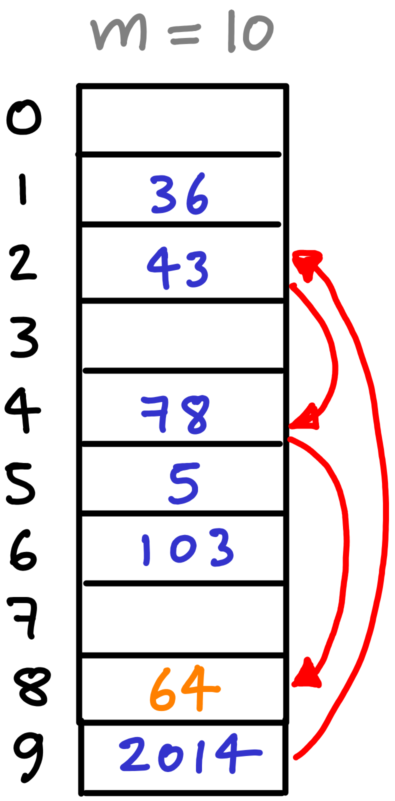
# OPEN ADDRESSING

Require $n \le m$

Avoid using pointers in linked lists. Use that space for a larger table.

We use a probe sequence → permutation of all slots.

e.g., $h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$




| Search(64) | Insert(64) |
|---|---|
| Try T[9] : not 64 | Try T[9] : full |
| Try T[2] : not 64 | Try T[2] : full |
| Try T[4] : not 64 | Try T[4] : full |
| Try T[8] : "not found" | Try T[8] : OK |

$h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$ $\quad i =$ iteration. Then $h(k) \to h(k,i)$

Remember, the probe sequence has to be generated somehow

via function $h(k,i)$

$h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$ $i$ = iteration. Then $h(k) \rightarrow h(k,i)$

---

Delete(64)

| | |
|---|---|
| 0 | |
| 1 | 36 |
| 2 | 43 |
| 3 | |
| 4 | 78 |
| 5 | 5 |
| 6 | 103 |
| 7 | |
| 8 | 64 |
| 9 | 2014 |

$h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$    $i$ = iteration. Then $h(k) \rightarrow h(k, i)$

| | |
|---|---|
| 0 | |
| 1 | 36 |
| 2 | 43 |
| 3 | |
| 4 | 78 |
| 5 | 5 |
| 6 | 103 |
| 7 | |
| 8 | 64 |
| 9 | 2014 |

Delete(64) :

$h(64, 1) = 9$

Occupied

$h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$ $i$ = iteration. Then $h(k) \rightarrow h(k, i)$

| | |
|---|---|
| 0 | |
| 1 | 36 |
| 2 | 43 |
| 3 | |
| 4 | 78 |
| 5 | 5 |
| 6 | 103 |
| 7 | |
| 8 | 64 |
| 9 | 2014 |

Delete(64) :

$h(64,1) = 9$ $h(64,2) = 2$

$h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$ $\quad$ $i$ = iteration. Then $h(k) \to h(k, i)$

| | |
|---|---|
| 0 | |
| 1 | 36 |
| 2 | 43 |
| 3 | |
| 4 | 78 |
| 5 | 5 |
| 6 | 103 |
| 7 | |
| 8 | 64 |
| 9 | 2014 |

Delete(64) :

$h(64,1) = 9 \qquad h(64,2) = 2 \qquad h(64,3) = 4$

$h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$   $i$ = iteration. Then $h(k) \rightarrow h(k,i)$

---

| | |
|---|---|
| 0 | |
| 1 | 36 |
| 2 | 43 |
| 3 | |
| 4 | 78 |
| 5 | 5 |
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| 7 | |
| 8 | 64 |
| 9 | 2014 |

Delete(64) :

$h(64,1) = 9$   $h(64,2) = 2$   $h(64,3) = 4$   $h(64,4) = 8$

found 64, can delete

$h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$ $i$ = iteration. Then $h(k) \to h(k,i)$

| | |
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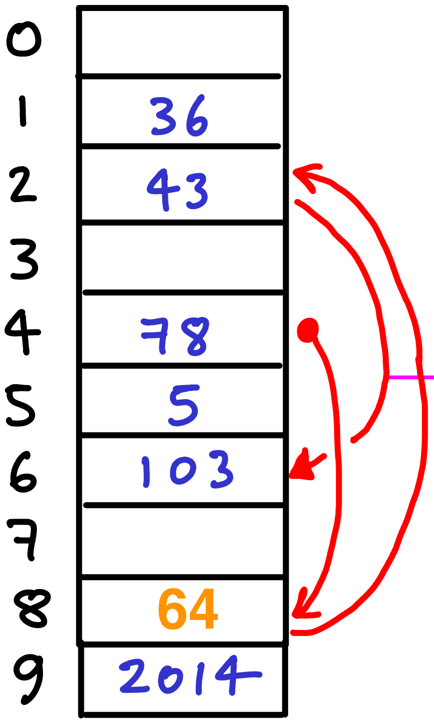
Delete(64) :

$h(64,1) = 9$   $h(64,2) = 2$   $h(64,3) = 4$   $h(64,4) = 8$

found 64, can delete

Problem: what if $h(103) = \{4, 8, 2, 6, \ldots$

$h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$ $i$ = iteration. Then $h(k) \to h(k,i)$




Delete(64) :

$h(64,1) = 9$ $h(64,2) = 2$ $h(64,3) = 4$ $h(64,4) = 8$

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Problem: what if $h(103) = \{4, 8, 2, 6, \ldots$

(103 was inserted after 64)

$h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$ $i$ = iteration. Then $h(k) \rightarrow h(k,i)$

| | |
|---|---|
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Delete(64) :

$h(64,1) = 9$ $h(64,2) = 2$ $h(64,3) = 4$ $h(64,4) = 8$

found 64, can delete

Problem: what if $h(103) = \{4, 8, 2, 6, \ldots$

(103 was inserted after 64)

Now, search(103)

$h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$  $i$ = iteration. Then $h(k) \rightarrow h(k,i)$

| | |
|---|---|
| 0 | |
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Delete(64) :

$h(64,1) = 9$  $h(64,2) = 2$  $h(64,3) = 4$  $h(64,4) = 8$

found 64, can delete

Problem: what if $h(103) = \{4, 8, 2, 6, \ldots$

(103 was inserted after 64)

Now, search(103) :  $h(103,1) = 4$

$h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$ $i$ = iteration. Then $h(k) \to h(k,i)$

| | |
|---|---|
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Delete(64) :

$h(64,1) = 9$ $h(64,2) = 2$ $h(64,3) = 4$ $h(64,4) = 8$

found 64, can delete

Problem: what if $h(103) = \{4, 8, 2, 6, \ldots$

(103 was inserted after 64)

Now, search(103) : $h(103,1) = 4$, $h(103,2) = 8$

"not found"

$h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$ $\quad$ $i$ = iteration. Then $h(k) \to h(k, i)$

| | |
|---|---|
| 0 | |
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| 8 | DEL |
| 9 | 2014 |

Delete(64) :

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Problem: what if $h(103) = \{4, 8, 2, 6, \ldots$

(103 was inserted after 64)

Now, search(103) : $\quad h(103,1) = 4, \quad h(103,2) = 8$

"not found"

Could use special "deleted" markers, but search becomes inefficient.

$h(64) = \{9, 2, 4, 8, 1, 3, 0, 7, 5, 6\}$  $i$ = iteration. Then $h(k) \to h(k,i)$

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Delete(64) :

$h(64,1) = 9$  $h(64,2) = 2$  $h(64,3) = 4$  $h(64,4) = 8$

found 64, can delete

Problem: what if $h(103) = \{4, 8, 2, 6, \ldots$

(103 was inserted after 64)

Now, search(103) :  $h(103,1) = 4$,  $h(103,2) = 8$

"not found"

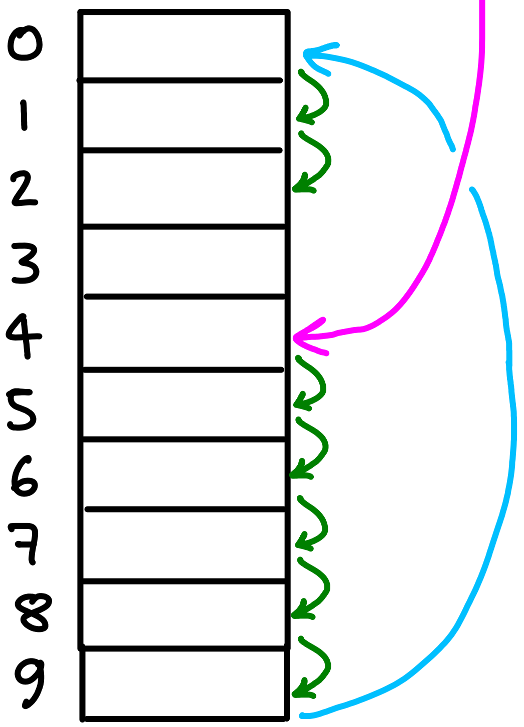
Could use special "deleted" markers, but search becomes inefficient.

e.g., insert n elements, delete n-1, search for last remaining.

# Typical probing sequences

# Typical probing sequences

Linear probing: $h(k,i) = (h(k,0) + i) \bmod m$ ~ h(k) & scan



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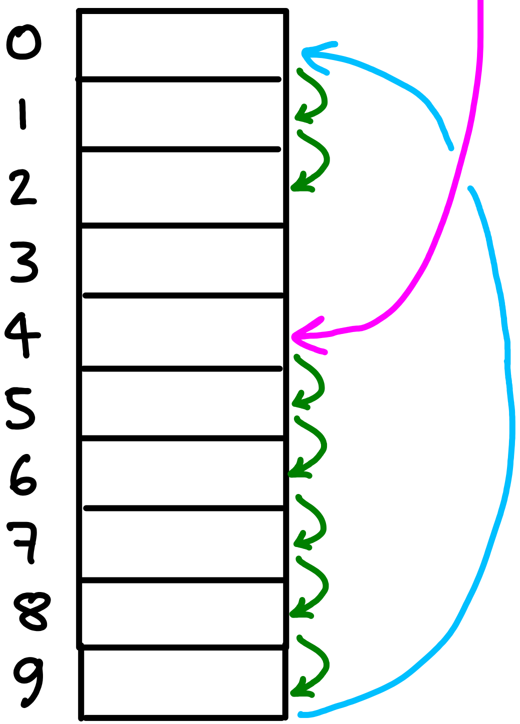


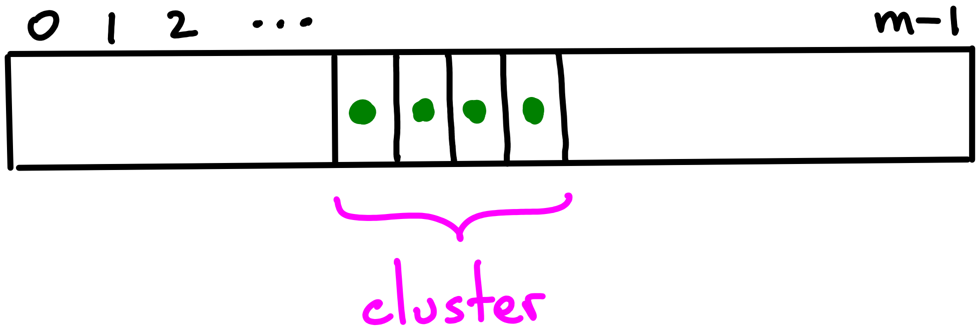

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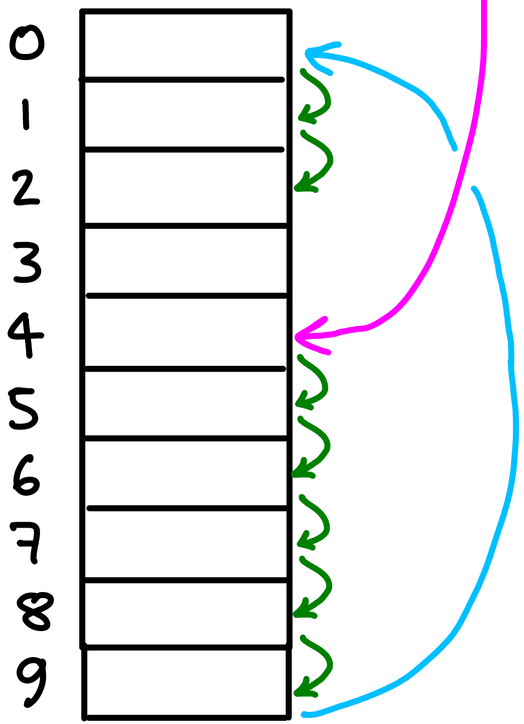


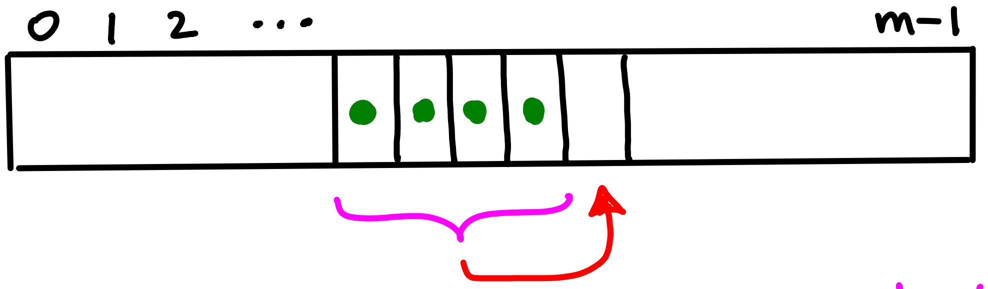


$$\underbrace{\text{probability of extending a cluster} = \frac{|\text{cluster}|}{m} \gg \frac{1}{m}}_{\text{slows down search}}$$

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... tends to generate clusters

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... tends to generate clusters

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Quadratic probing: $h(k,i) = (h(k,0) + \underbrace{c \cdot i}_{\text{linear}} + \underbrace{d \cdot i^2}_{\text{make it look more random}}) \bmod m$

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Less clustering, need to make sure sequence hits all slots

↳ (number theory)

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...tends to generate clusters

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Both generate $m$ probe sequences

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Double hashing: $h(k,i) = (h_1(k) + \underbrace{i \cdot h_2(k)}_{\text{each } k \text{ has its own "random" offset}}) \bmod m$

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Double hashing: $h(k,i) = (h_1(k) + \underbrace{i \cdot h_2(k)}_{\text{each } k \text{ has its own "random" offset}}) \bmod m$

Can generate up to $m^2$ probe sequences: better

# ANALYSIS of OPEN ADDRESSING

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ASSUMPTION: UNIFORM HASHING

Don't confuse with Simple Uniform Hashing

(assumption for Chaining)

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ASSUMPTION: UNIFORM HASHING

↳ Every key is equally likely to have any of the $m!$ permutations as a probe sequence

(and all probe sequences are independent)

---

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ASSUMPTION: UNIFORM HASHING

↳ Every key is equally likely to have any of the $m!$ permutations as a probe sequence

(and all probe sequences are independent)

- The common probing methods that we saw don't even come close

---

Don't confuse with Simple Uniform Hashing
(assumption for Chaining)

# ANALYSIS of OPEN ADDRESSING

## with UNIFORM HASHING ASSUMPTION

# ANALYSIS of OPEN ADDRESSING with UNIFORM HASHING ASSUMPTION

Recall, $n < m$, so $\alpha < 1$

Claim: Expected #probes when searching $\leq \frac{1}{1-\alpha}$ $\left(\frac{m}{m-n}\right)$

# ANALYSIS of OPEN ADDRESSING with UNIFORM HASHING ASSUMPTION

Recall, $n < m$, so $\alpha < 1$

Claim: Expected #probes when searching $\leq \frac{1}{1-\alpha}$ $\left(\frac{m}{m-n}\right)$

If true, then for $n \ll m$ we get $E[\#\text{probes}] = O(1)$

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If true, then for $n \ll m$ we get $E[\#\text{probes}] = O(1)$

↳ $n = \frac{1}{2}m \rightarrow 2$ probes

↳ 90% full table → 10 probes

Claim: $E[\#\text{probes}] \leq \frac{1}{1-\alpha}$

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e.g., consider unsuccessful search

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$P[\text{1st probe collides}] = \frac{n}{m}$ $\rightarrow$ need 2nd probe

$P[\text{2nd probe collides}] = \frac{n-1}{m-1}$

Claim: $E[\#\text{probes}] \le \frac{1}{1-\alpha}$ e.g., consider unsuccessful search

$P[\text{1st probe collides}] = \frac{n}{m}$ → need 2nd probe

$P[\text{2nd probe collides}] = \frac{n-1}{m-1}$ → need 3rd probe

Claim: $E[\#\text{probes}] \leq \frac{1}{1-\alpha}$  e.g., consider unsuccessful search

$P[\text{1st probe collides}] = \frac{n}{m}$ → need 2nd probe

$P[\text{2nd probe collides}] = \frac{n-1}{m-1}$ → need 3rd probe

$$\vdots$$

$$\frac{n-i}{m-i}$$

Claim: $E[\#\text{probes}] \le \frac{1}{1-\alpha}$

e.g., consider unsuccessful search

$P[\text{1st probe collides}] = \frac{n}{m}$ → need 2nd probe

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$$\vdots$$

$$\frac{n-i}{m-i} < \frac{n}{m} = \alpha$$

Claim: $E[\#\text{probes}] \le \frac{1}{1-\alpha}$ e.g., consider unsuccessful search

$P[\text{1st probe collides}] = \frac{n}{m}$ → need 2nd probe

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$\vdots$

$\frac{n-i}{m-i} < \frac{n}{m} = \alpha$

---

$E[\#\text{probes}] = 1 + \cdots$

must probe at least once

Claim: $E[\#\text{probes}] \le \frac{1}{1-\alpha}$ e.g., consider unsuccessful search

$P[\text{1st probe collides}] = \frac{n}{m}$ → need 2nd probe

$P[\text{2nd probe collides}] = \frac{n-1}{m-1}$ → need 3rd probe

$\vdots$

$\frac{n-i}{m-i} < \frac{n}{m} = \alpha$

---

$E[\#\text{probes}] = 1 + \frac{n}{m}(\cdots)$

probability of needing to probe more

Claim: $E[\#\text{probes}] \le \frac{1}{1-\alpha}$ e.g., consider unsuccessful search

$P[\text{1st probe collides}] = \frac{n}{m}$ → need 2nd probe

$P[\text{2nd probe collides}] = \boxed{\frac{n-1}{m-1}}$ → need 3rd probe

$\vdots$

$\frac{n-i}{m-i} < \frac{n}{m} = \alpha$

---

$$E[\#\text{probes}] = 1 + \frac{n}{m}\Big(1 + \underbrace{\frac{n-1}{m-1}}_{\text{probability of needing to probe more}}(\cdots)\Big)$$

2nd probe

Claim: $E[\#\text{probes}] \le \frac{1}{1-\alpha}$ e.g., consider unsuccessful search

$P[\text{1st probe collides}] = \frac{n}{m}$ → need 2nd probe

$P[\text{2nd probe collides}] = \frac{n-1}{m-1}$ → need 3rd probe

$\vdots$

$\frac{n-i}{m-i} < \frac{n}{m} = \alpha$

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$$E[\#\text{probes}] = 1 + \frac{n}{m}\left(1 + \frac{n-1}{m-1}\left(1 + \frac{n-2}{m-2}\left(1 + \cdots \quad \cdots\left(1 + \frac{0}{m-n}\right)\right)\right)\right)$$

Claim: $E[\#\text{probes}] \leq \frac{1}{1-\alpha}$    e.g., consider unsuccessful search

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$$\leq 1 + \alpha\left(1 + \alpha\left(1 + \alpha\left(1 + \alpha \cdots \quad \right)\right)\right)$$ ~ n terms

Claim: $E[\#\text{probes}] \leq \frac{1}{1-\alpha}$ e.g., consider unsuccessful search

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$$\leq 1 + \alpha\left(1 + \alpha\left(1 + \alpha\left(1 + \alpha \cdots \right)\right)\right)$$

$$< 1 + \alpha + \alpha^2 + \alpha^3 \cdots$$ $\infty$ terms

Claim: $E[\#\text{probes}] \leq \frac{1}{1-\alpha}$ e.g., consider unsuccessful search

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$$= \sum_{i=0}^{\infty} \alpha^i$$

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$$= \sum_{i=0}^{\infty} \alpha^i = \frac{1}{1-\alpha}$$

see CLRS for further analysis
including successful search