

• SEARCH
 • INSERT
 • DELETE

} The 3 main operations that we perform on data structures
 } How fast can we do these?

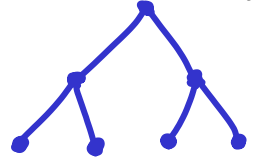
array



linked list



BST (RB, AVL, etc)



• SEARCH $O(n) \rightarrow [O(\log n) \text{ sorted}]$
 ↳ by key
 $O(1)$
 ↳ by index

$O(n)$

$O(\log n)$

• INSERT $O(n)$
 • DELETE $O(n)$

} $[O(1) \text{ amortized}]$
 } $[\text{if not sorted}]$

search + $O(1)$

$O(1) [O(n) \text{ sorted}]$

$O(\log n)$

$O(n)$
 search + $O(1)$

$O(\log n)$

basic HASHING

- SEARCH
- INSERT
- DELETE



$O(1)$ expected

with assumptions



not "expected worst-case",
it's just average time.

For some hashing methods, some operations can be $O(1)$ worst-case.

The simplest form of hashing → Direct access table

Assume keys are distinct
and come from a small set of possible values, U
Universe

e.g., $U = \{0, 1, 2, \dots, m-1\}$

$m = 74$

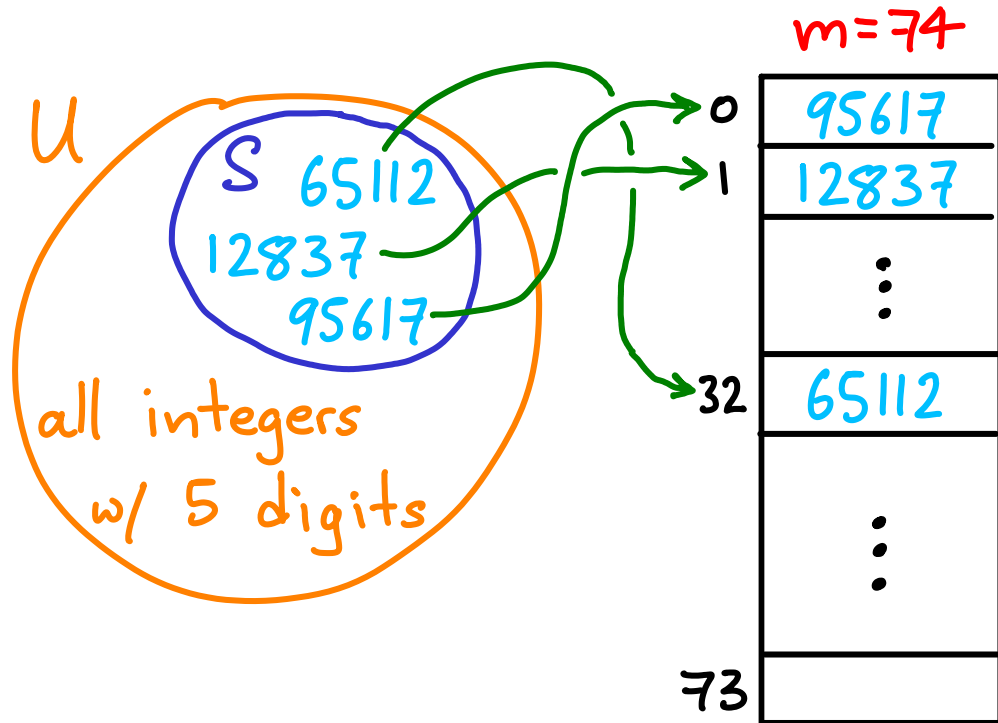
T

0	1	/	3							/	73
0	1	2	3							m-2	m-1

$\text{search}(T, 3) = 3$ // $\text{insert}(T, k) \rightarrow T[k] = k$ // $\text{delete}(T, k) \rightarrow T[k] = \emptyset$

Often U is larger than the available space, m

but we only need to deal with a subset S of U , where $|S| \leq m$



h : hash function

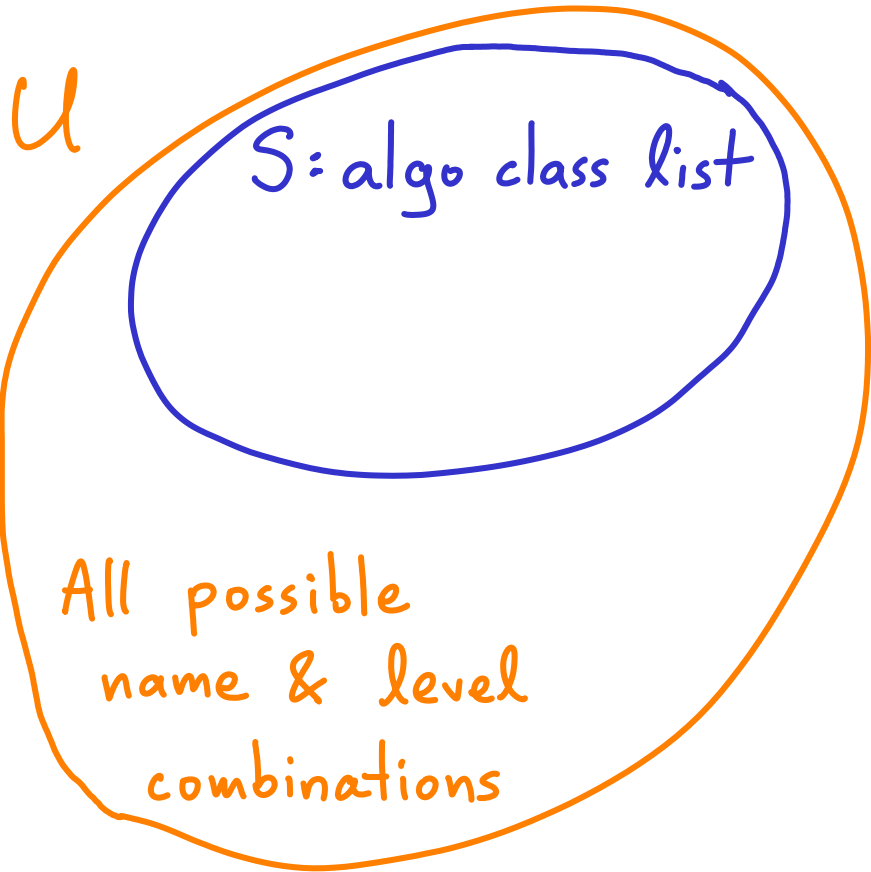
maps keys to T

$$h(95617) = 0$$

$$h(12837) = 1$$

$$h(65112) = 32$$

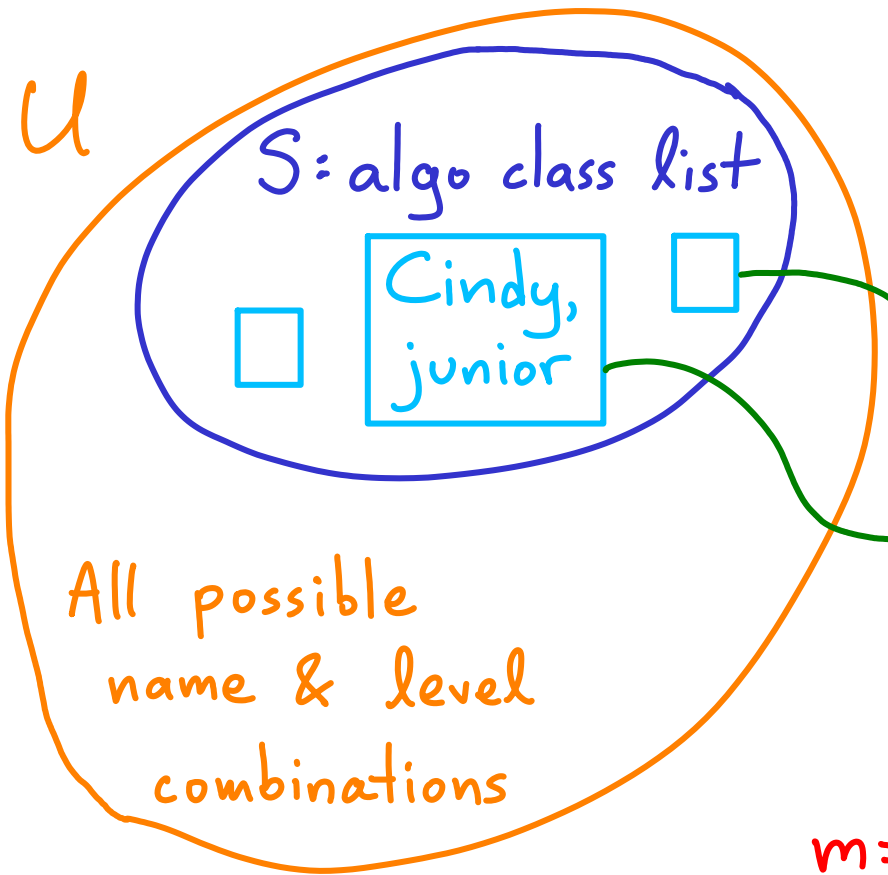
Example: look up this semester's ALGO students
using only their first name & academic level



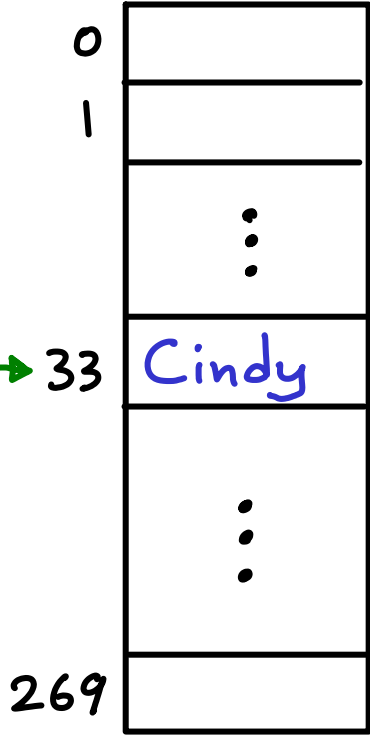
- take 1st letter of name, map to number $\rightarrow N = \{1 \dots 26\}$
- map level similarly: junior=3, PhD=7, etc $\rightarrow L = \{0 \dots 9\}$

$$h(\text{student}) = h(L, N) = 10 \cdot N + L$$

L unique per pair N, L



$$m = 270$$



PROBLEMS?

- (1) some permanently empty slots
- (2) collisions

Could use more of the given info to design a more complicated $h()$

↳ might reduce collisions

But that involves costly processing

and will need to be repeated if S changes (next semester)

We want to use a simple $h()$ and deal with collisions