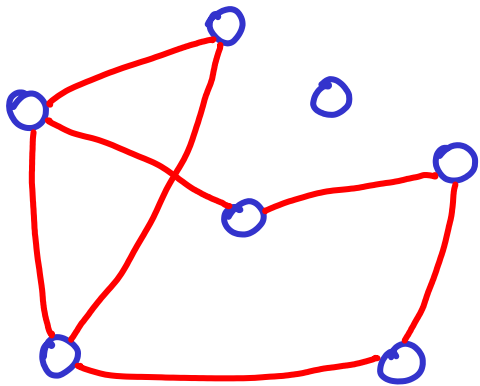


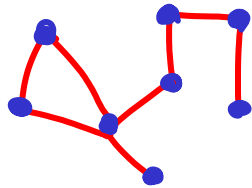
# GRAPHS



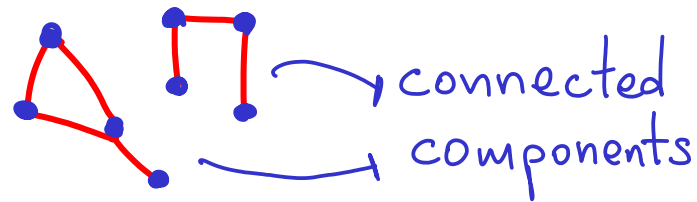
$$G = \{V, E\}$$

& vertices  
& edges

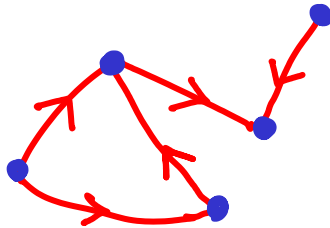
connected



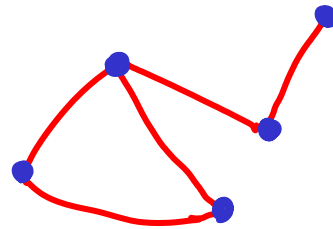
not connected



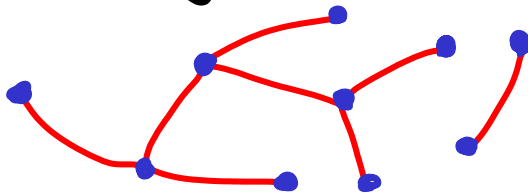
directed



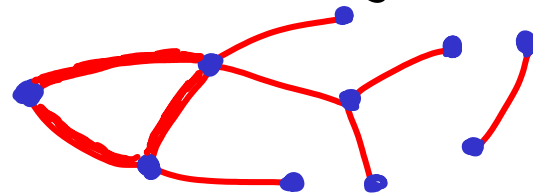
not directed



acyclic

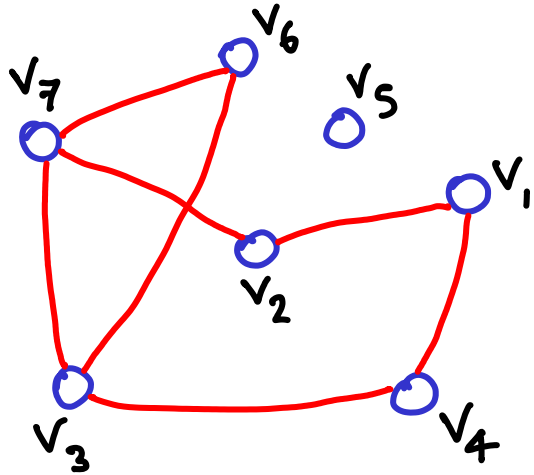


not acyclic



etc

# GRAPHS



$$G = \{V, E\}$$

& vertices  
& edges

	1	2	3	4	5	6	7
1	0	1	0	1	0	0	0
2	1	0	0	0	0	0	1
3	0	0	0	1	0	1	1
4	1	0	1	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	1	0	0	0	1
7	0	1	1	0	0	1	0

Adjacency matrix

size:  $|V|^2$

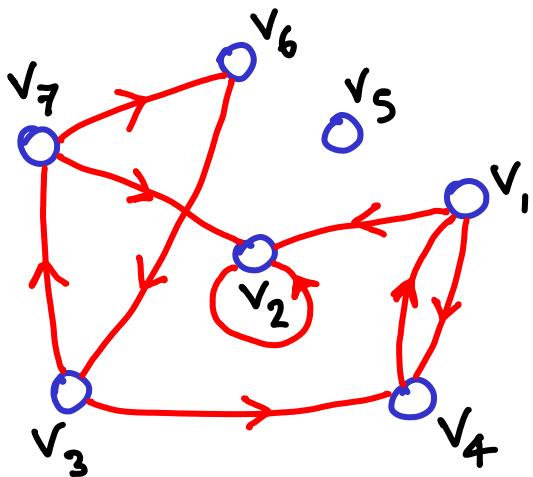
(symmetric for  
undirected)

Adjacency list

size:  $|V| + 2|E|$   
(undirected)

1  $\rightarrow$  2  $\rightarrow$  4  
2  $\rightarrow$  1  $\rightarrow$  7  
3  $\rightarrow$  4  $\rightarrow$  6  $\rightarrow$  7  
4  $\rightarrow$  1  $\rightarrow$  3  
5  
6  $\rightarrow$  3  $\rightarrow$  7  
7  $\rightarrow$  2  $\rightarrow$  3  $\rightarrow$  6

# GRAPHS



$$G = \{V, E\}$$

& vertices  
& edges

	1	2	3	4	5	6	7
1	0	1	0	1	0	0	0
2	0	1	0	0	0	0	0
3	0	0	0	1	0	0	1
4	1	0	0	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	1	0	0	0	0
7	0	1	0	0	0	1	0

Adjacency matrix

size:  $|V|^2$

(directed or not)

Adjacency list

size:  $|V| + |E|$   
(directed)

1  $\rightarrow$  2  $\rightarrow$  4

2  $\rightarrow$  2

3  $\rightarrow$  4  $\rightarrow$  7

4  $\rightarrow$  1

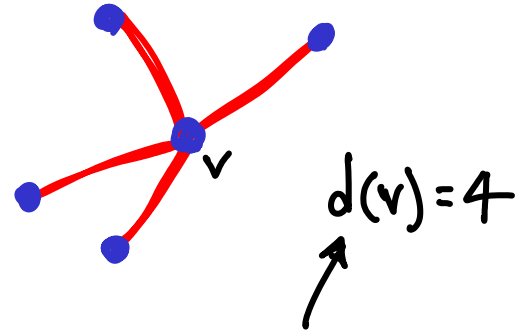
5

6  $\rightarrow$  3

7  $\rightarrow$  2  $\rightarrow$  6

Adjacency matrix size:  $O(|V|^2)$   
Adjacency list size:  $O(|V| + |E|)$  } directed or not

Same for "dense" graphs, i.e.  $|E| \sim |V|^2$



Query adjacency :  
(is  $v_j$  my neighbor ?)

Matrix  $O(1)$

List  $O(|V|)$

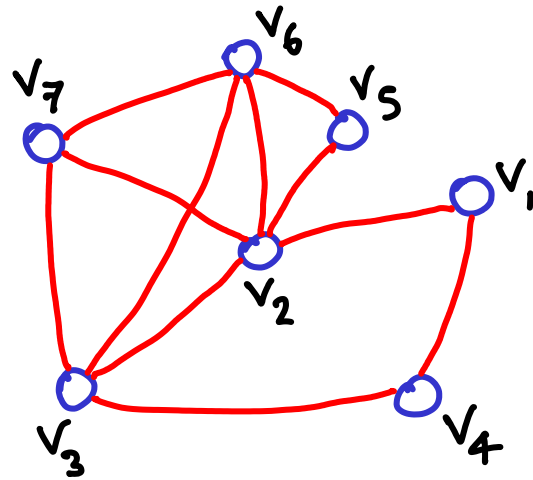
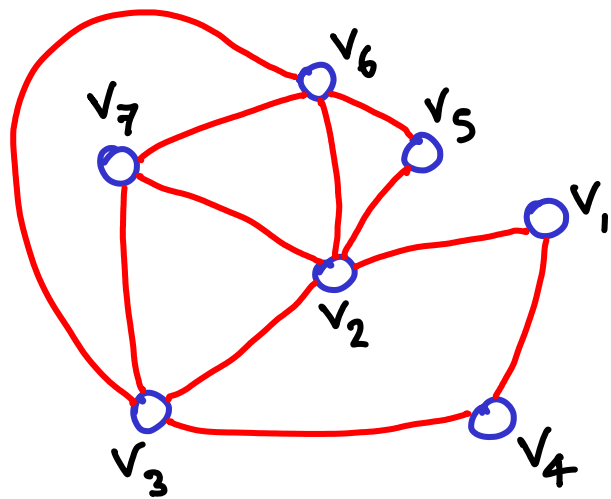
but really  $O(\text{degree}(v))$

Enumerate neighbors :  
(of one vertex)

List  $O(\text{degree})$

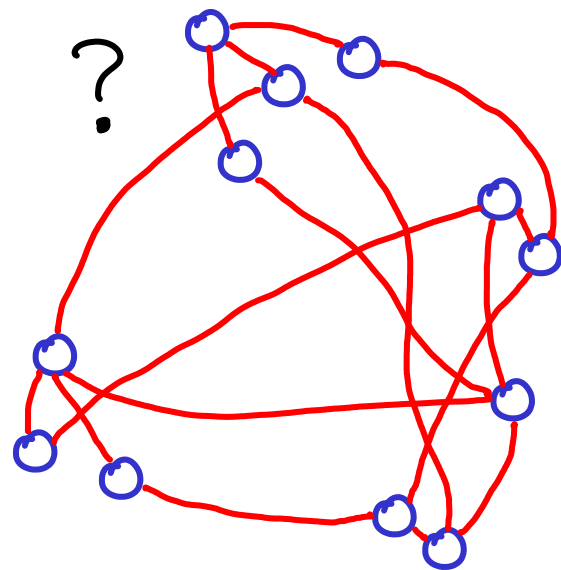
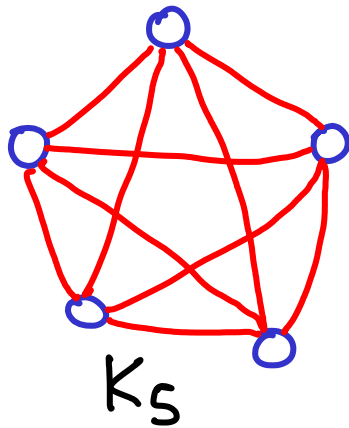
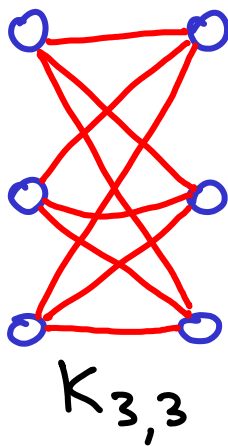
Matrix  $O(|V|)$

PLANE GRAPH  
no crossings

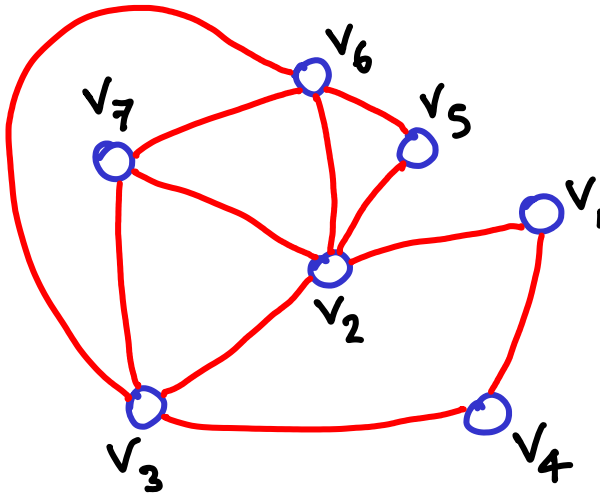


PLANAR GRAPH  
can redraw  
without crossings

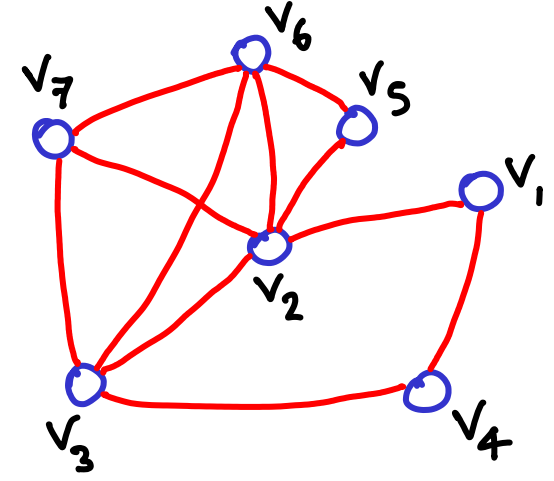
Non-planar graphs  
(can't redraw)



PLANE GRAPH  
no crossings

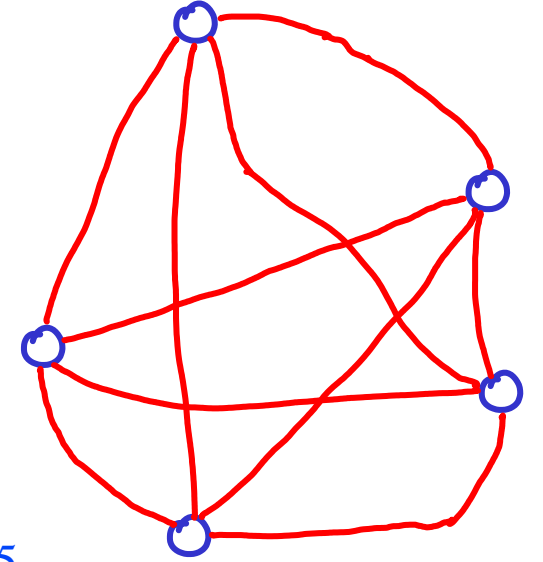
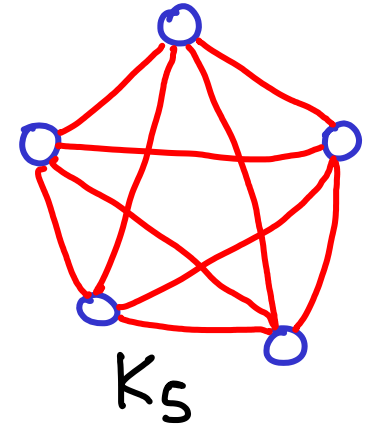
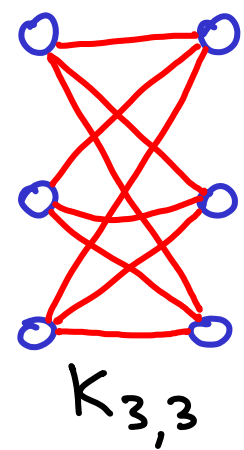


PLANAR GRAPH  
can redraw  
without crossings



obtained by successive contractions

Non-planar graphs  
(can't redraw)



A graph is non-planar if and only if it "contains" a  $K_{3,3}$  or  $K_5$