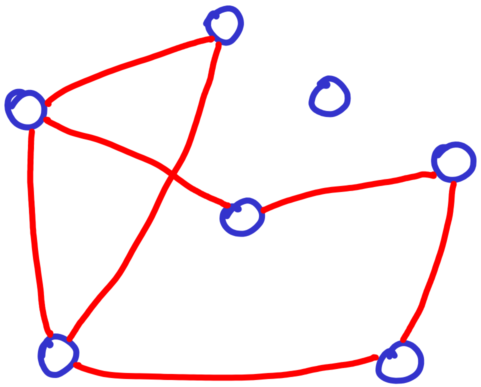
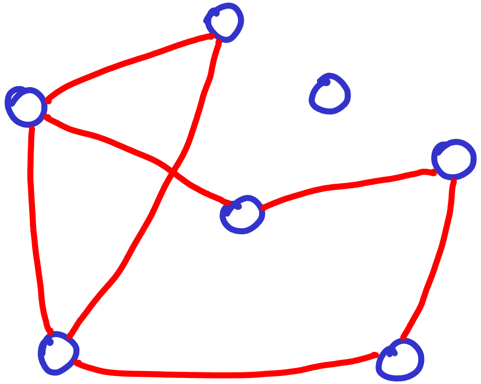
GRAPHS

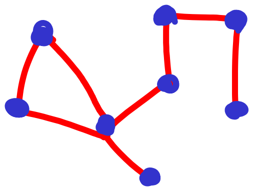


GRAPHS

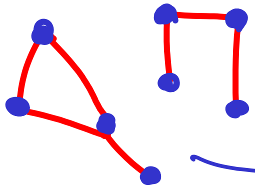


& vertices
edges

Connected

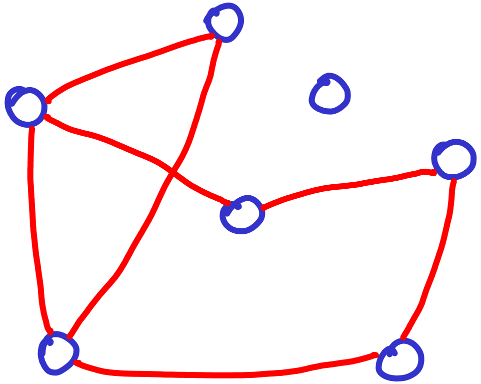


not connected

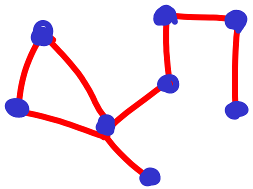


connected components

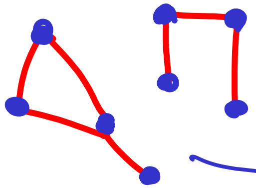
GRAPHS



connected

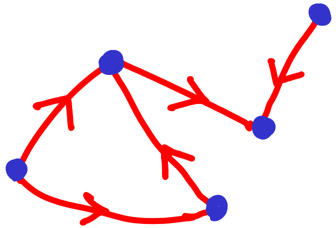


not connected

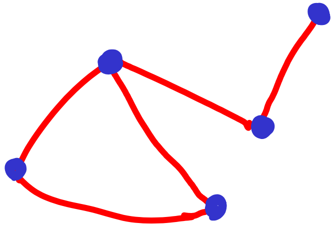


connected components

directed

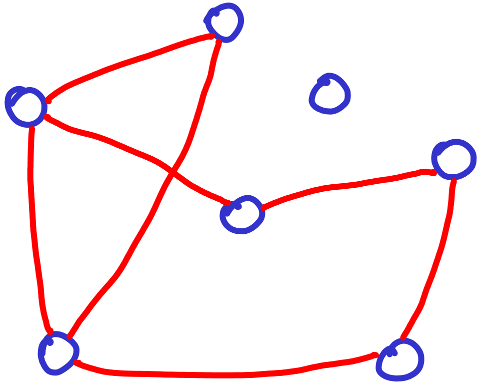


not directed



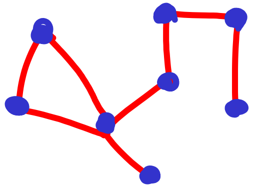
& vertices
edges

GRAPHS

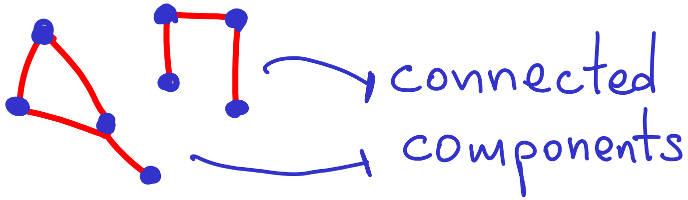


& vertices
edges

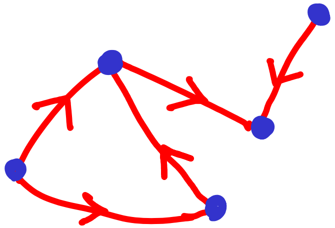
connected



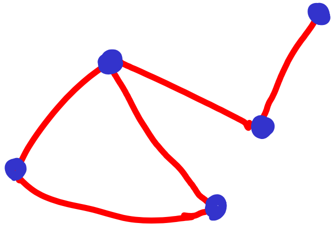
not connected



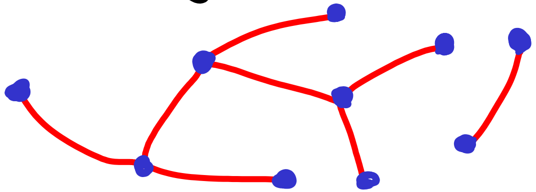
directed



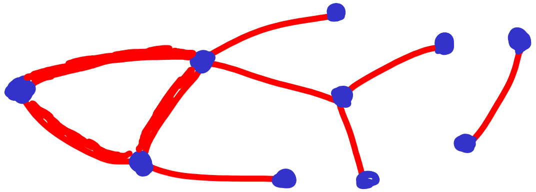
not directed



acyclic

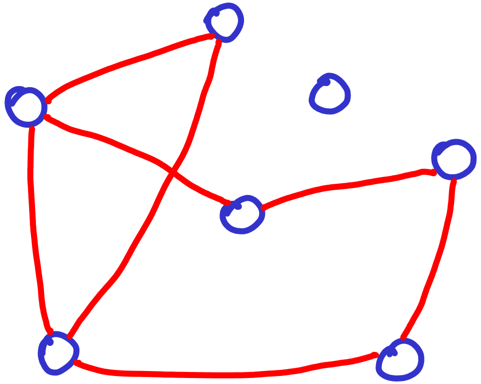


not acyclic



etc

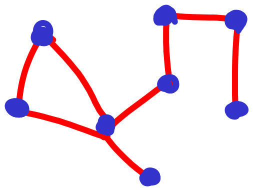
GRAPHS



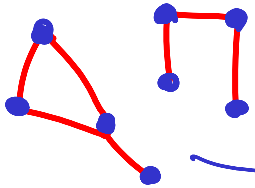
$G = \{V, E\}$

& vertices edges

connected

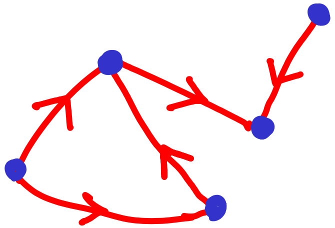


not connected

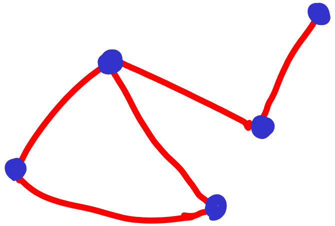


connected components

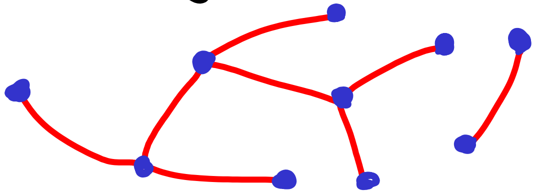
directed



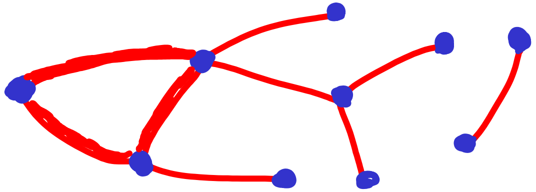
not directed



acyclic

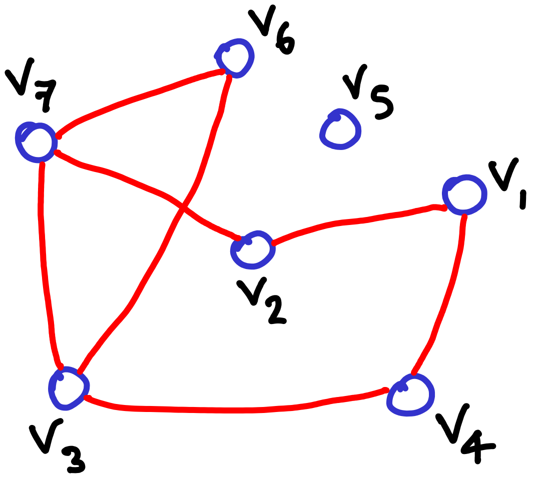


not acyclic



etc

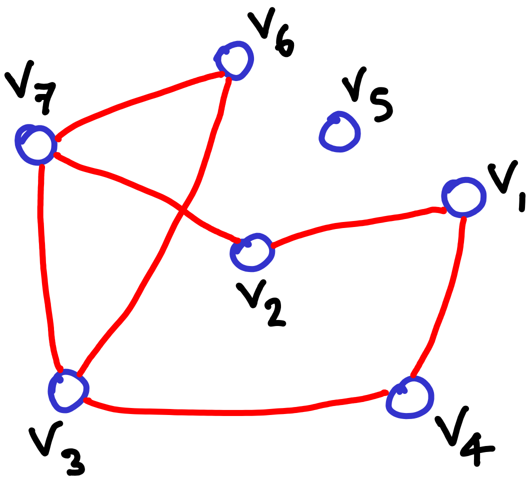
GRAPHS



$$G = \{V, E\}$$

& vertices
edges

GRAPHS

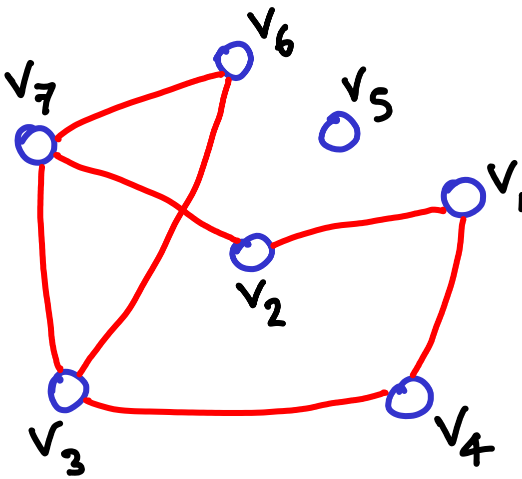


$$\left.\begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 4 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 7 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \end{array}\right\}$$ Adjacency matrix

$$G = \{V, E\}$$

vertices & edges

GRAPHS



$$G = \{V, E\}$$

& vertices
edges

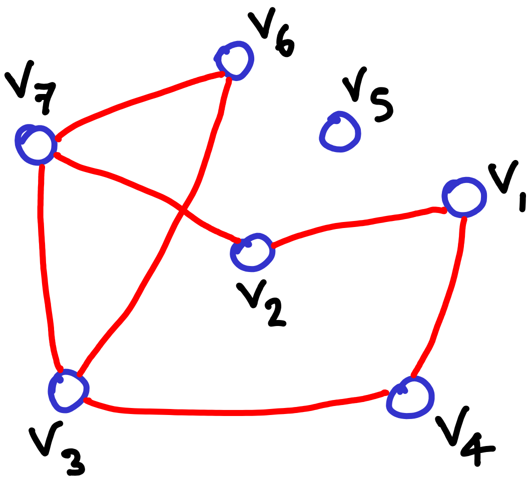
	1	2	3	4	5	6	7
1	0	1	0	1	0	0	0
2	1	0	0	0	0	0	1
3	0	0	0	1	0	1	1
4	1	0	1	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	1	0	0	0	1
7	0	1	1	0	0	1	0

Adjacency matrix

size: $|V|^2$

(symmetric for undirected)

GRAPHS



$G = \{V, E\}$

& vertices
edges

	1	2	3	4	5	6	7
1	0	1	0	1	0	0	0
2	1	0	0	0	0	0	1
3	0	0	0	1	0	1	1
4	1	0	1	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	1	0	0	0	1
7	0	1	1	0	0	1	0

Adjacency matrix

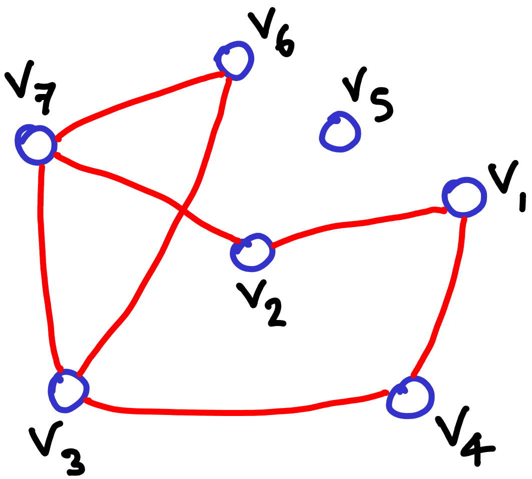
size: $|V|^2$

(symmetric for undirected)

Adjacency list

1 → 2 → 4
2 → 1 → 7
3 → 4 → 6 → 7
4 → 1 → 3
5
6 → 3 → 7
7 → 2 → 3 → 6

GRAPHS



$G = \{V, E\}$

& vertices
edges

	1	2	3	4	5	6	7
1	0	1	0	1	0	0	0
2	1	0	0	0	0	0	1
3	0	0	0	1	0	1	1
4	1	0	1	0	0	0	0
5	0	0	0	0	0	0	0
6	0	0	1	0	0	0	1
7	0	1	1	0	0	1	0

Adjacency matrix

size: $|V|^2$

(symmetric for undirected)

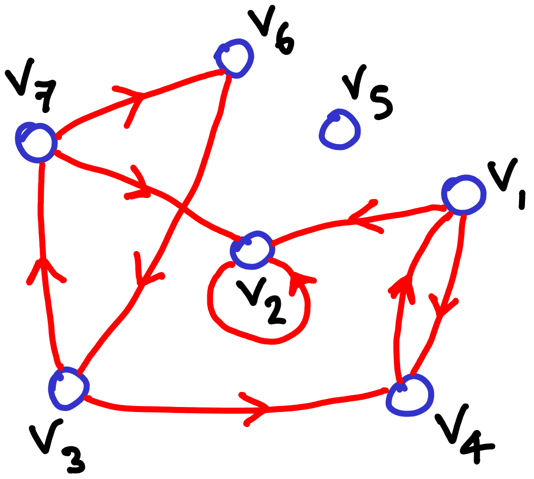
Adjacency list

size: $|V| + 2|E|$

(undirected)

1 → 2 → 4
2 → 1 → 7
3 → 4 → 6 → 7
4 → 1 → 3
5
6 → 3 → 7
7 → 2 → 3 → 6

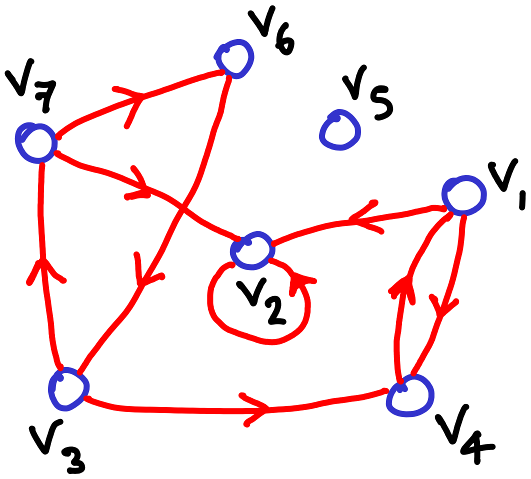
GRAPHS



$G = \{V, E\}$

& vertices
edges

GRAPHS



$$
\begin{array}{c|ccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
7 & 0 & 1 & 0 & 0 & 0 & 1 & 0
\end{array}
\Bigg\}
$$

Adjacency matrix

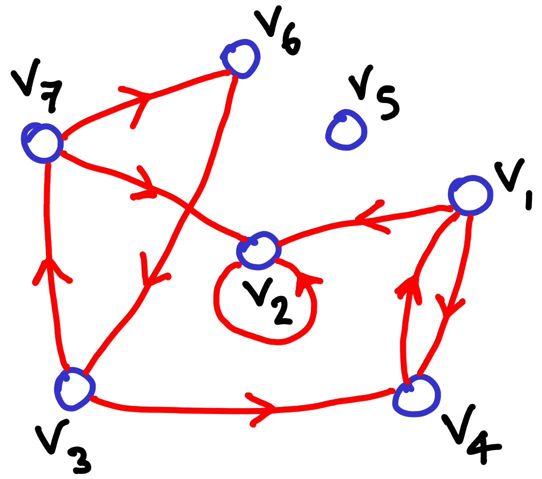
size: $|V|^2$

(directed or not)

$$G = \{V, E\}$$

& vertices
edges

GRAPHS



$$G = \{V, E\}$$

& vertices
edges

$$\begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 7 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{array}$$

Adjacency matrix

size: $|V|^2$

(directed or not)

Adjacency list

size: $|V| + |E|$

(directed)

$1 \to 2 \to 4$
$2 \to 2$
$3 \to 4 \to 7$
$4 \to 1$
5
$6 \to 3$
$7 \to 2 \to 6$

$$\left.\begin{array}{ll}\text{Adjacency matrix} & \text{size: } O(|V|^2) \\ \text{Adjacency list} & \text{size: } O(|V|+|E|)\end{array}\right\}\ \text{directed or not}$$

$$\left.\begin{array}{ll} \text{Adjacency matrix} & \text{size: } O(|V|^2) \\ \text{Adjacency list} & \text{size: } O(|V|+|E|) \end{array}\right\} \text{directed or not}$$

Same for "dense" graphs, i.e. $|E| \sim |V|^2$

Adjacency matrix size: $O(|V|^2)$
Adjacency list size: $O(|V|+|E|)$

$\left.\begin{array}{l} \text{Adjacency matrix size: } O(|V|^2) \\ \text{Adjacency list size: } O(|V|+|E|) \end{array}\right\}$ directed or not

Same for "dense" graphs, i.e. $|E| \sim |V|^2$

Query adjacency : Matrix ?
(is v_j my neighbor ?) List ?

$$\left.\begin{array}{ll}\text{Adjacency matrix} & \text{size: } O(|V|^2) \\ \text{Adjacency list} & \text{size: } O(|V|+|E|)\end{array}\right\}\ \text{directed or not}$$

Same for "dense" graphs, i.e. $|E| \sim |V|^2$

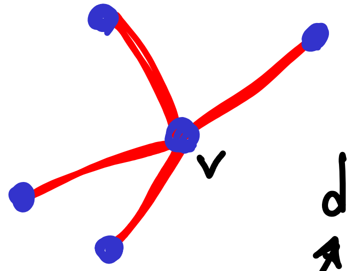
Query adjacency: Matrix $O(1)$
(is v_j my neighbor ?) List $O(|V|)$

Adjacency matrix size: $O(|V|^2)$

Adjacency list size: $O(|V|+|E|)$

directed or not

Same for "dense" graphs, i.e. $|E| \sim |V|^2$



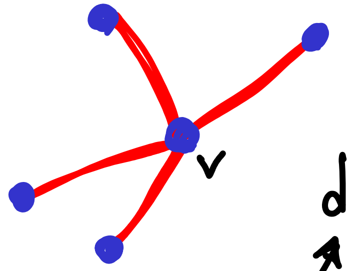
$d(v)=4$

Query adjacency : Matrix $O(1)$

(is v_j my neighbor ?) List $O(|V|)$ but really $O(\text{degree}(v))$

Adjacency matrix size: $O(|V|^2)$
Adjacency list size: $O(|V| + |E|)$ } directed or not

Same for "dense" graphs, i.e. $|E| \sim |V|^2$



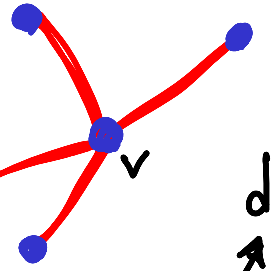
$d(v) = 4$

Query adjacency : Matrix $O(1)$
(is v_j my neighbor ?) List $O(|V|)$ but really $O(\text{degree}(v))$

Enumerate neighbors : List ?
(of one vertex) Matrix ?

$$\left.\begin{array}{ll}\text{Adjacency matrix} & \text{size: } O(|V|^2) \\ \text{Adjacency list} & \text{size: } O(|V|+|E|)\end{array}\right\}\ \text{directed or not}$$

Same for "dense" graphs, i.e. $|E| \sim |V|^2$



$d(v)=4$

Query adjacency : Matrix $O(1)$

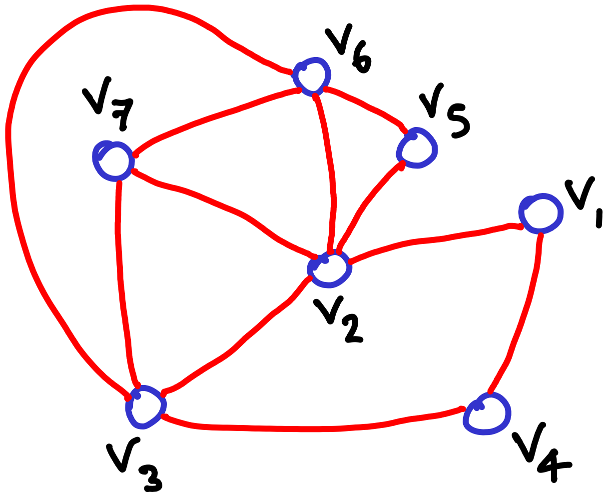
(is v_j my neighbor ?) List $O(|V|)$ but really $O(\text{degree}(v))$

Enumerate neighbors : List $O(\text{degree})$

(of one vertex) Matrix $O(|V|)$

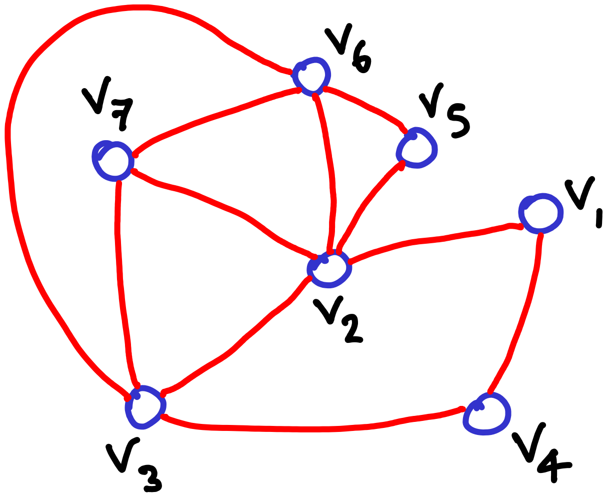
PLANE GRAPH

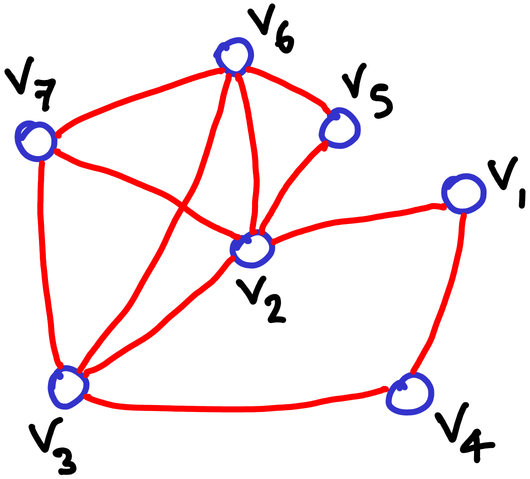
no crossings



PLANE GRAPH

no crossings



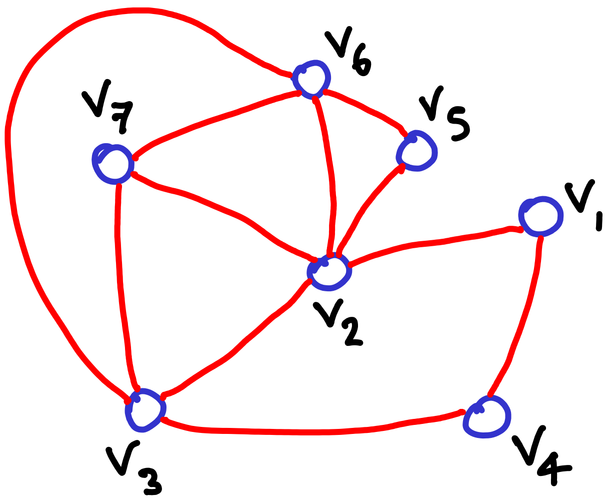


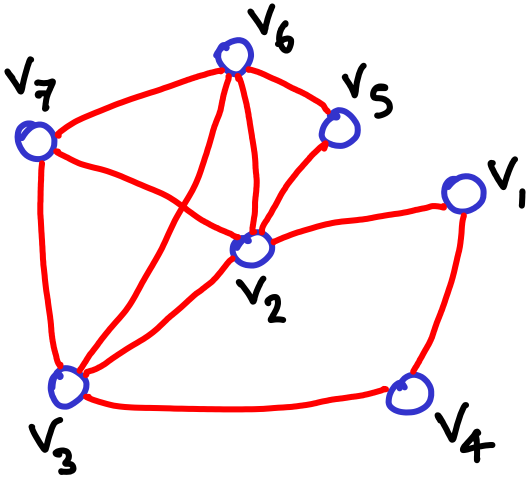
PLANAR GRAPH

can redraw without crossings

PLANE GRAPH

no crossings

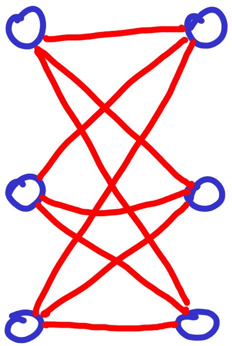




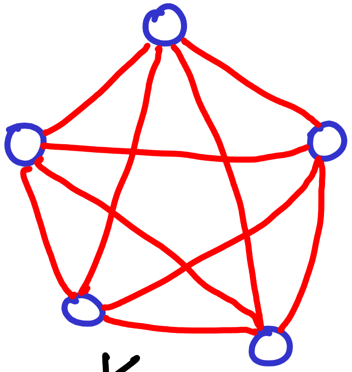
PLANAR GRAPH

can redraw without crossings

Non-planar graphs
(can't redraw)



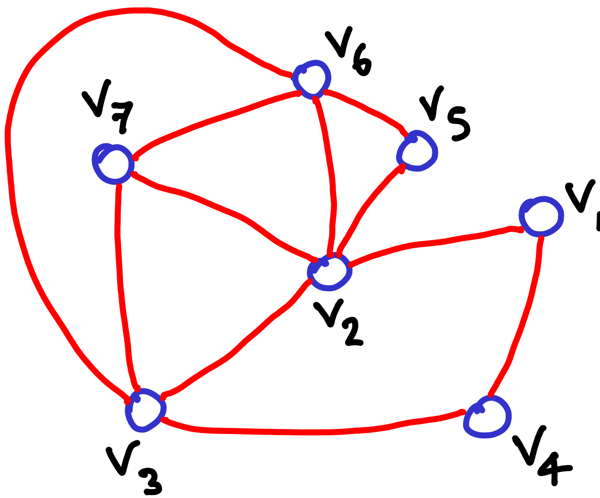
$K_{3,3}$

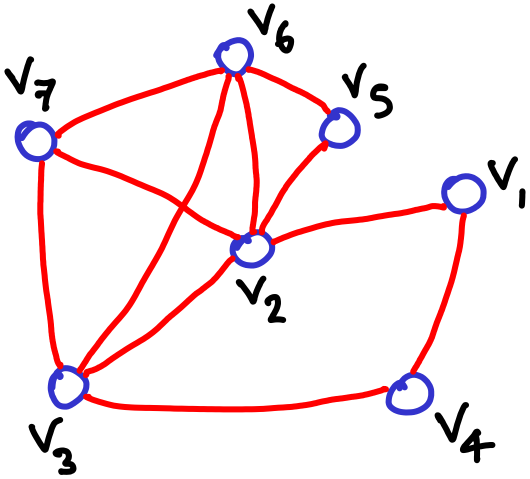


K_5

PLANE GRAPH

no crossings



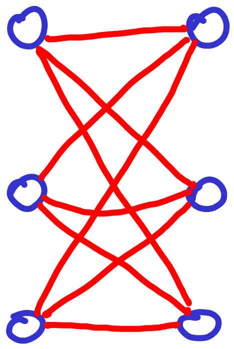


PLANAR GRAPH

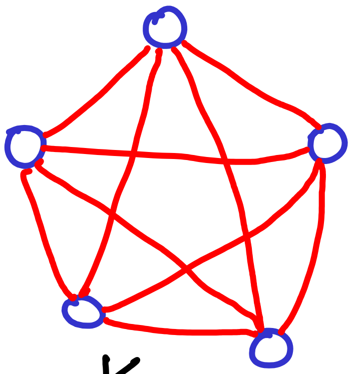
can redraw without crossings

Non-planar graphs

(can't redraw)

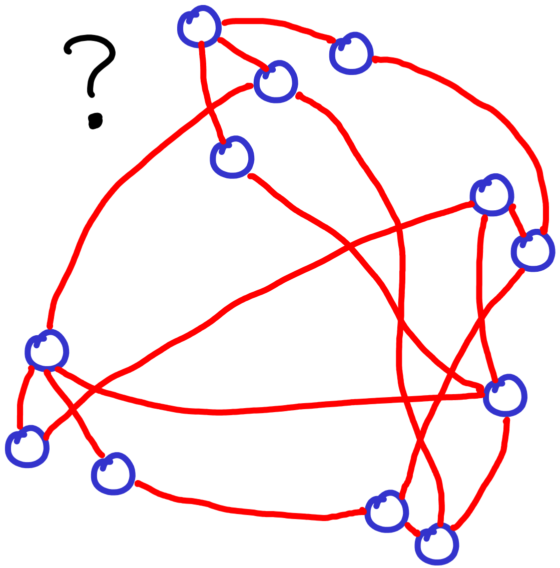


$K_{3,3}$



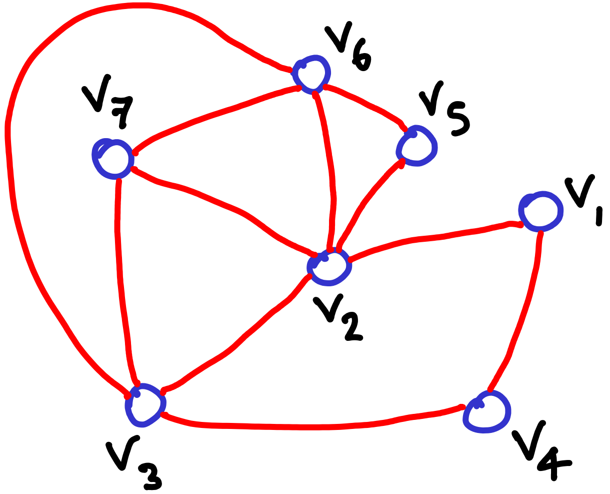
K_5

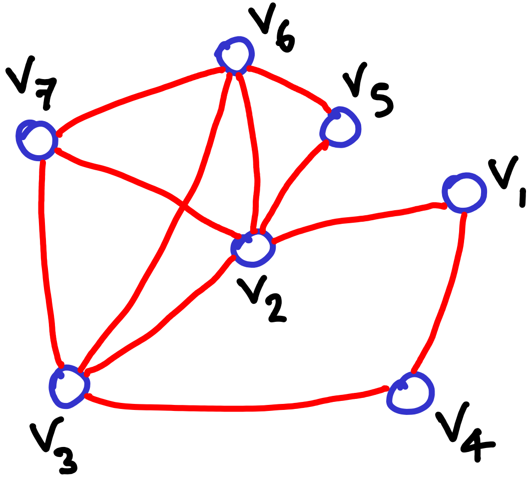
?



PLANE GRAPH

no crossings



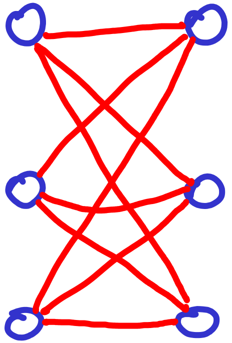


PLANAR GRAPH

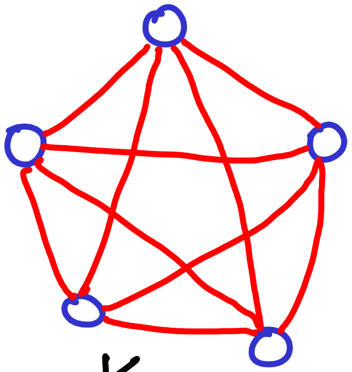
can redraw without crossings

Non-planar graphs

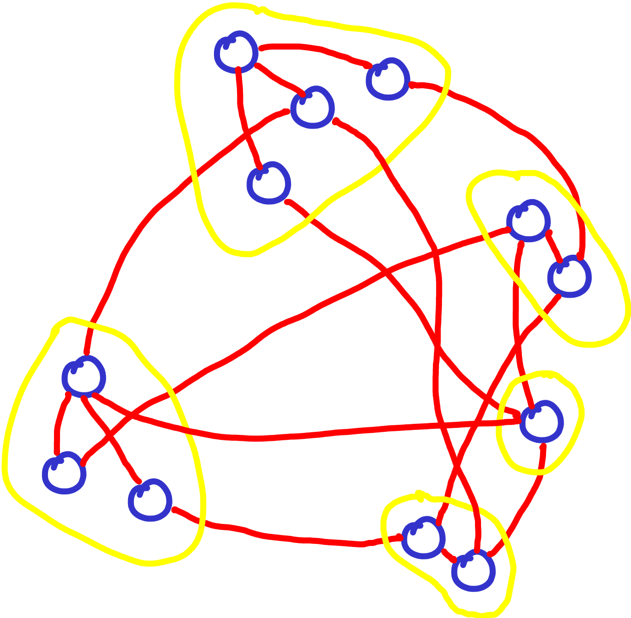
(can't redraw)



$K_{3,3}$

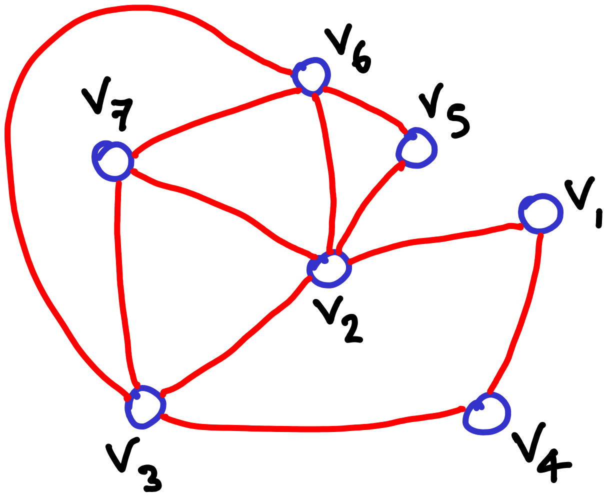


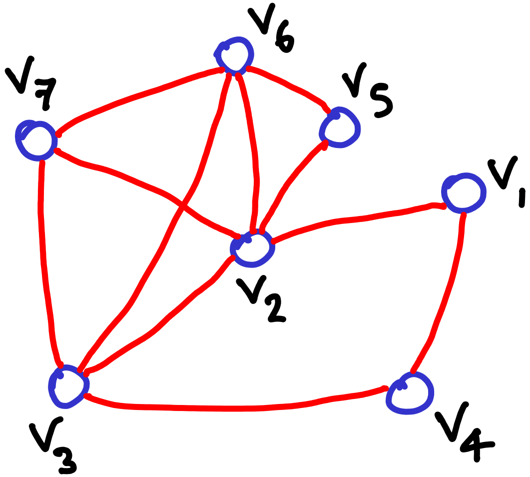
K_5



PLANE GRAPH

no crossings

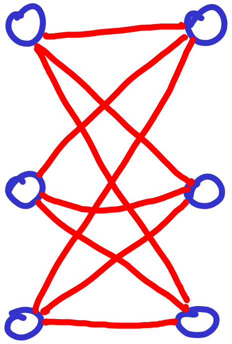




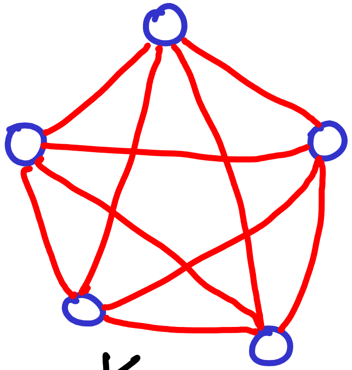
PLANAR GRAPH

can redraw without crossings

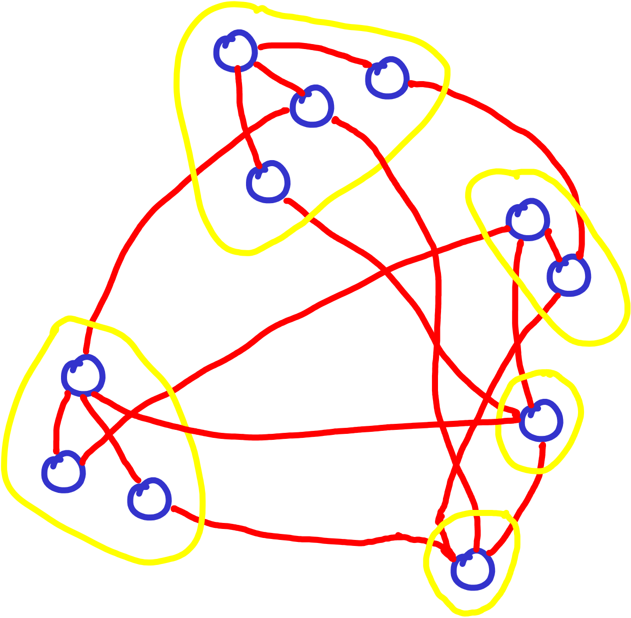
Non-planar graphs
(can't redraw)



$K_{3,3}$

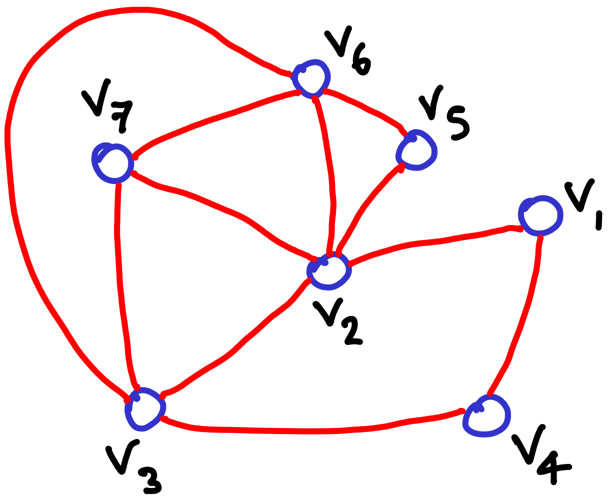


K_5



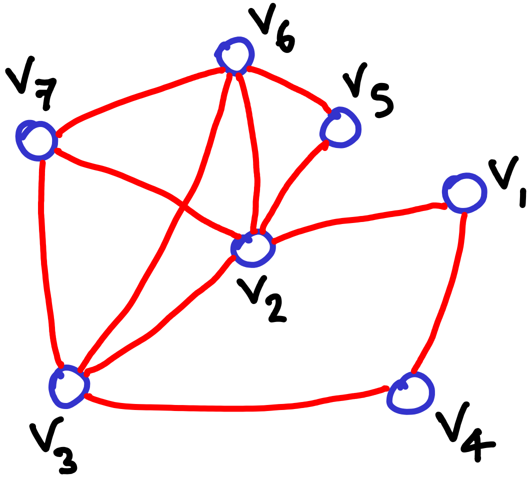
PLANE GRAPH

no crossings



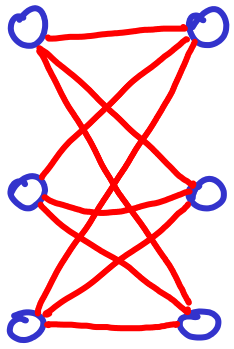
PLANAR GRAPH

can redraw without crossings

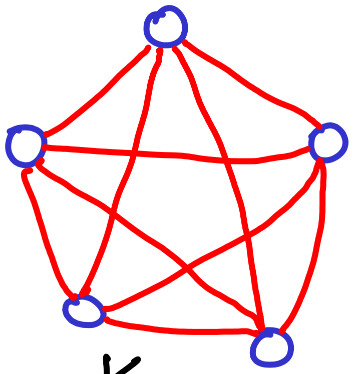


Non-planar graphs

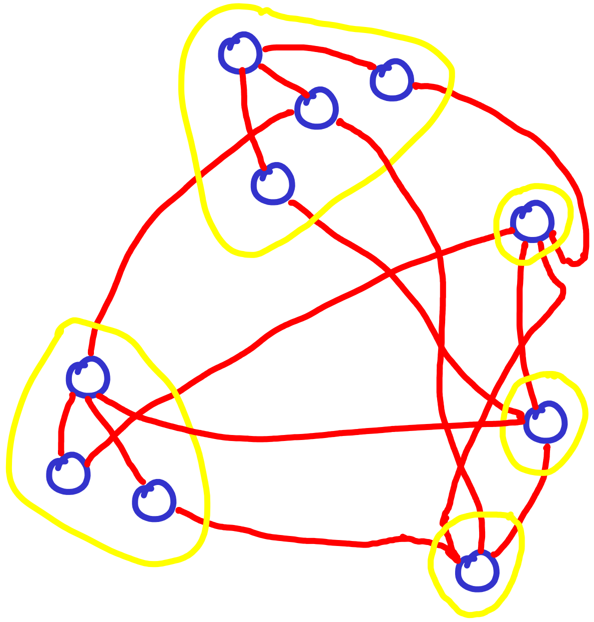
(can't redraw)



$K_{3,3}$

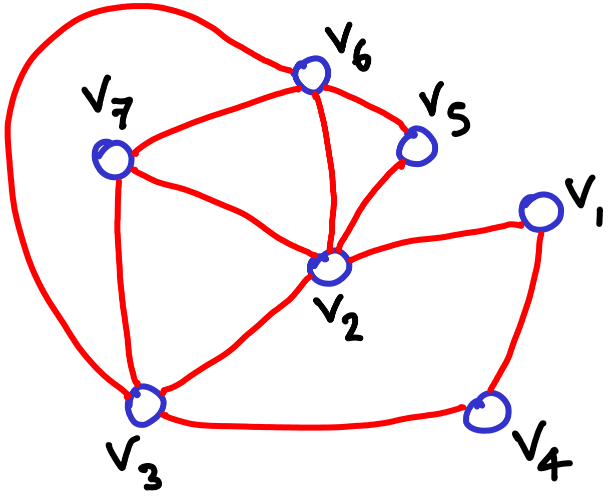


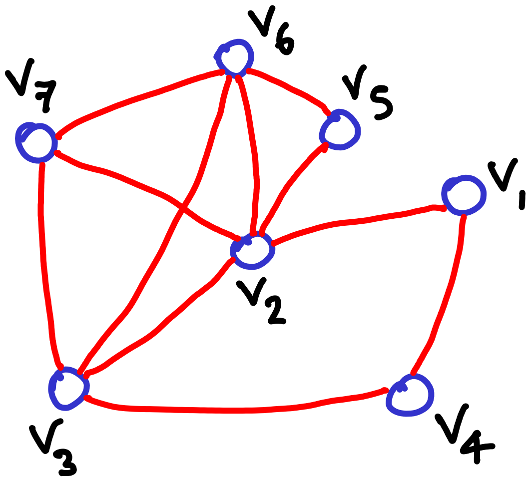
K_5



PLANE GRAPH

no crossings



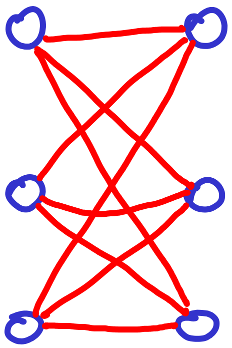


PLANAR GRAPH

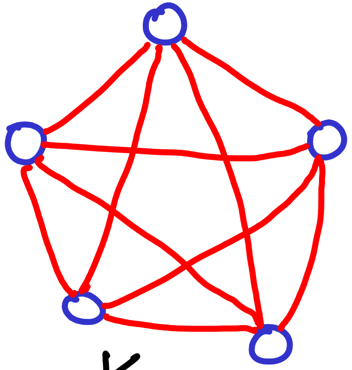
can redraw without crossings

Non-planar graphs

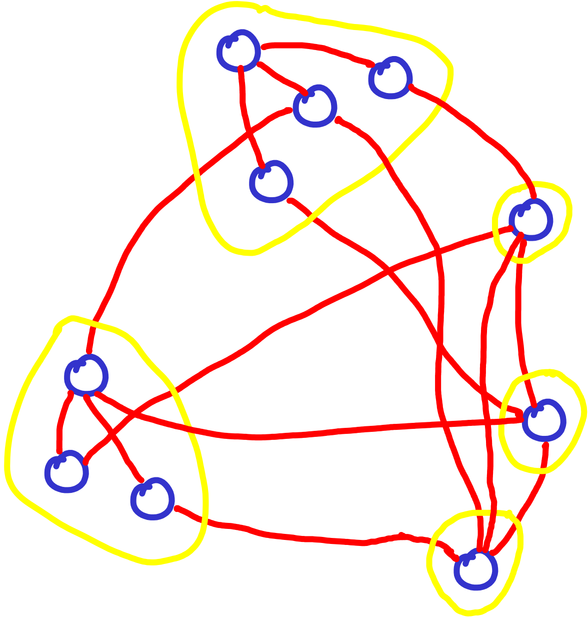
(can't redraw)



$K_{3,3}$

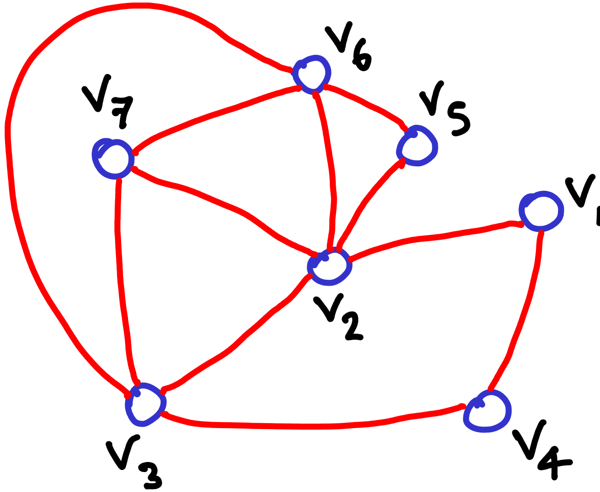


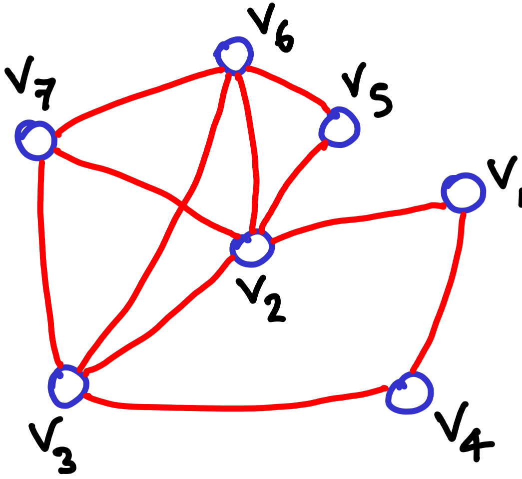
K_5



PLANE GRAPH

no crossings



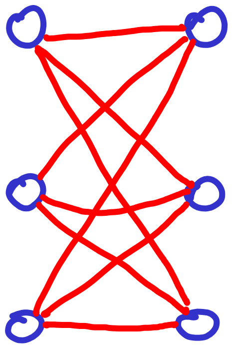


PLANAR GRAPH

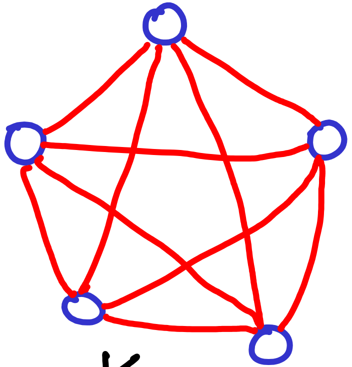
can redraw without crossings

Non-planar graphs

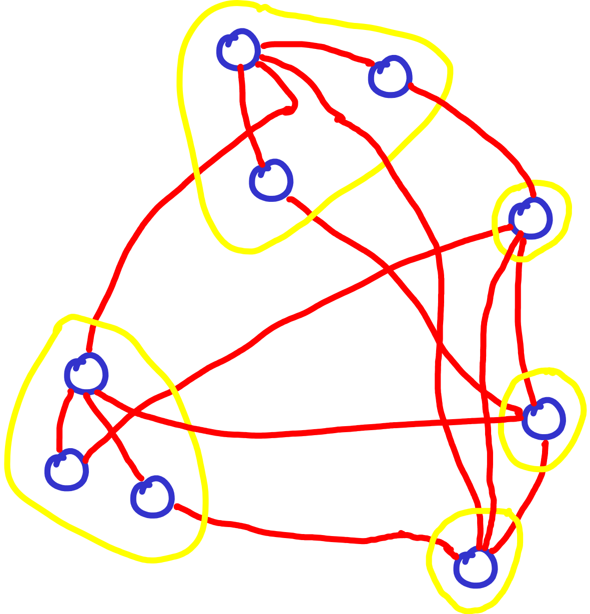
(can't redraw)



$K_{3,3}$

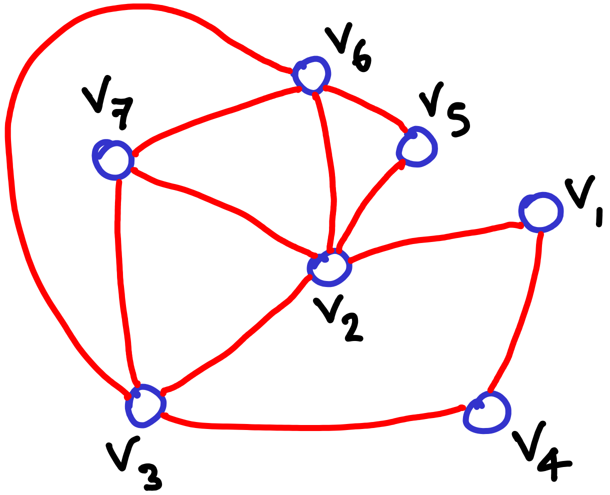


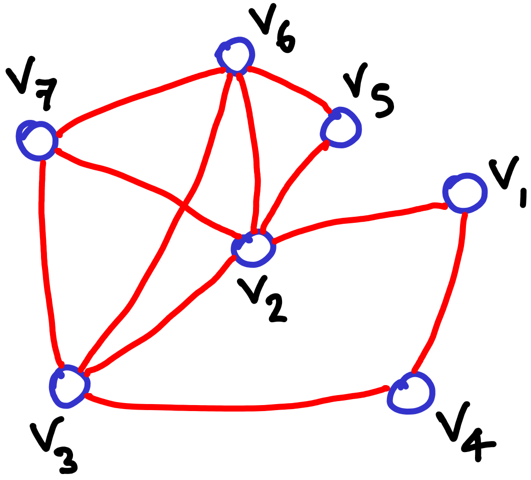
K_5



PLANE GRAPH

no crossings



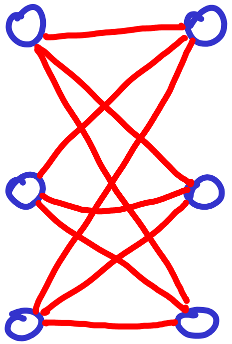


PLANAR GRAPH

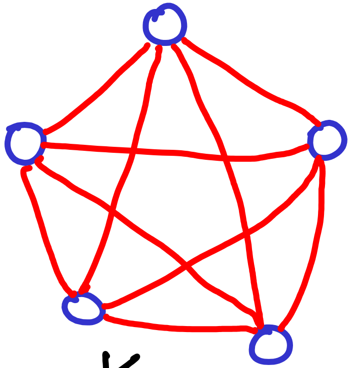
can redraw without crossings

Non-planar graphs

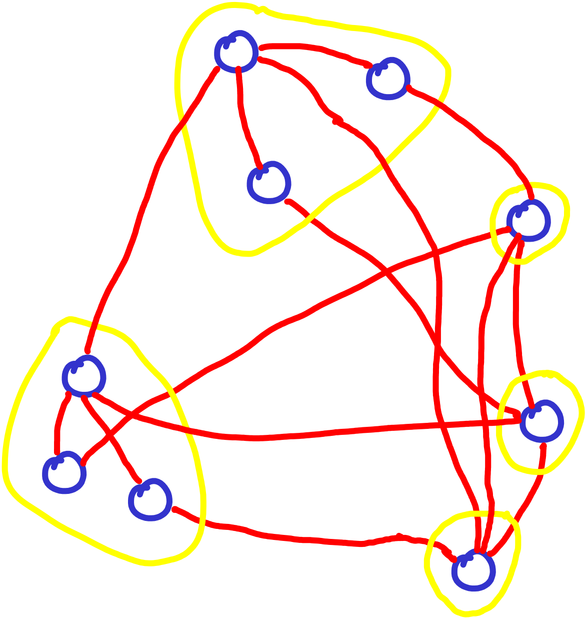
(can't redraw)



$K_{3,3}$

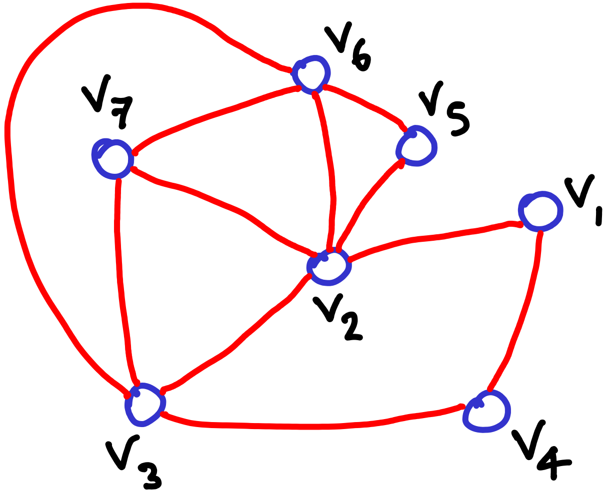


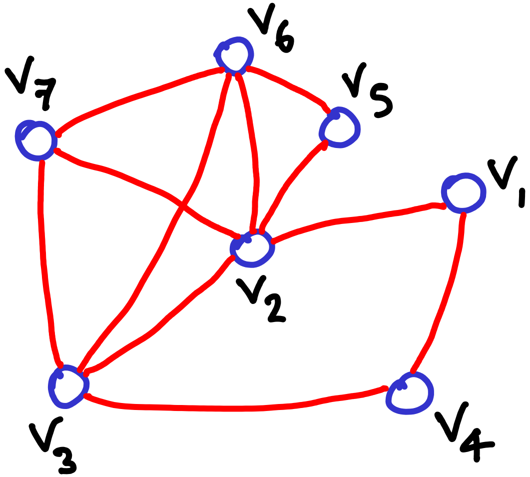
K_5



PLANE GRAPH

no crossings



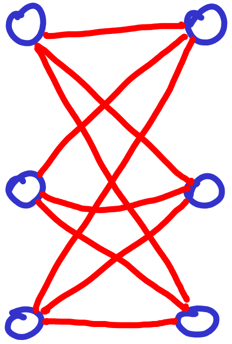


PLANAR GRAPH

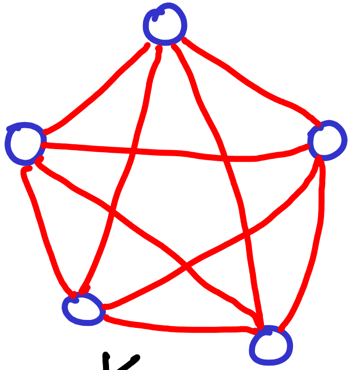
can redraw without crossings

Non-planar graphs

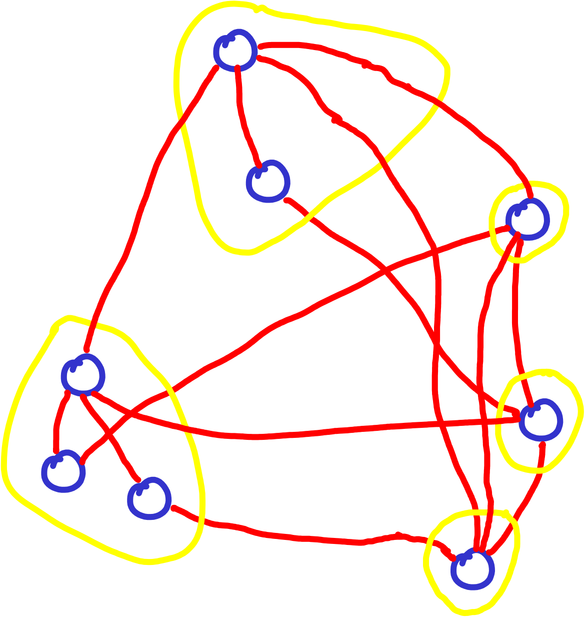
(can't redraw)



$K_{3,3}$

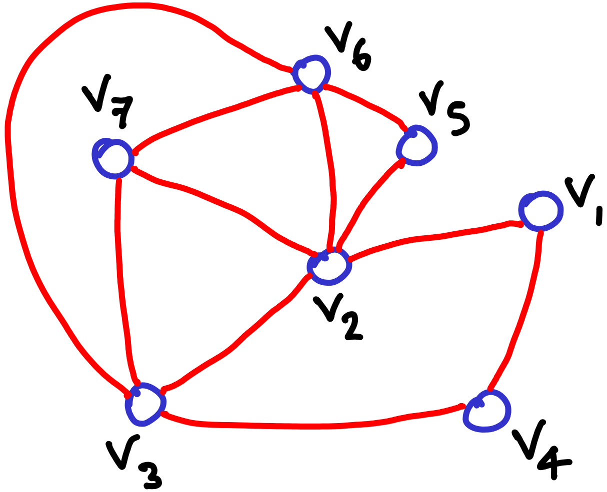


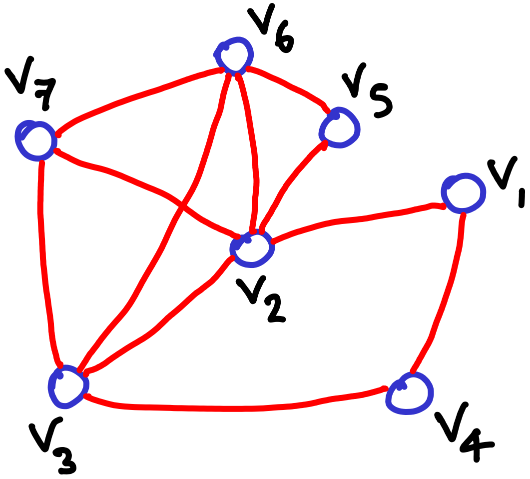
K_5



PLANE GRAPH

no crossings



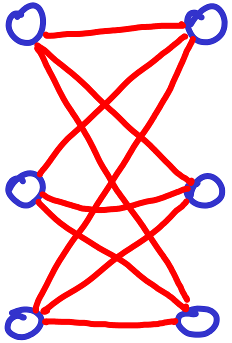


PLANAR GRAPH

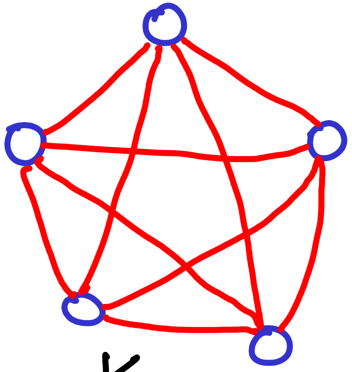
can redraw without crossings

Non-planar graphs

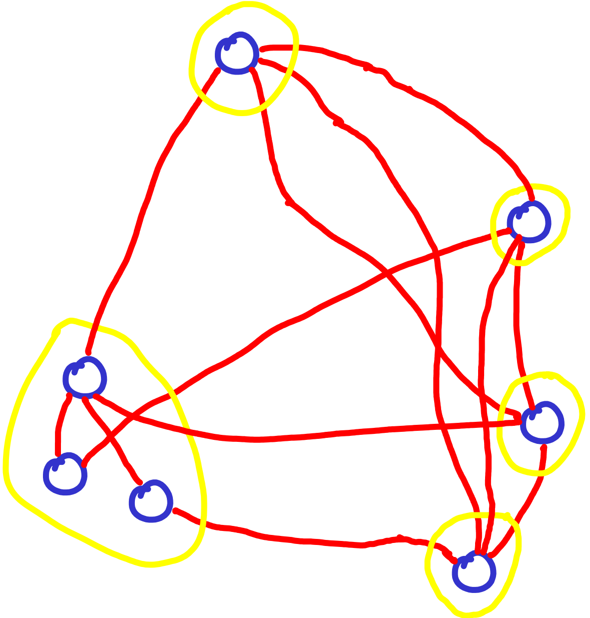
(can't redraw)



$K_{3,3}$

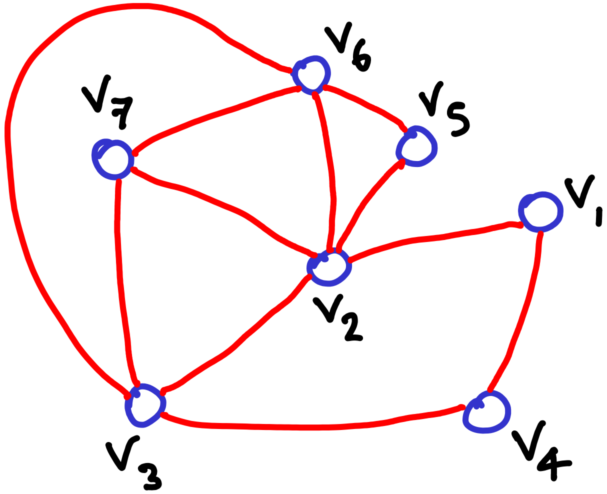


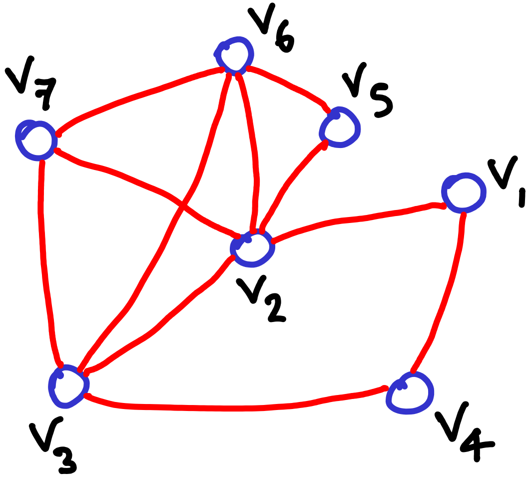
K_5



PLANE GRAPH

no crossings

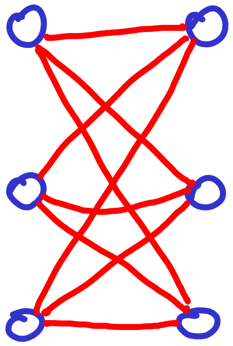




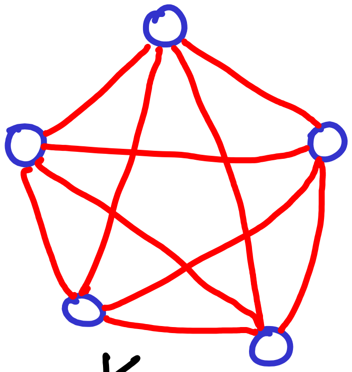
PLANAR GRAPH

can redraw without crossings

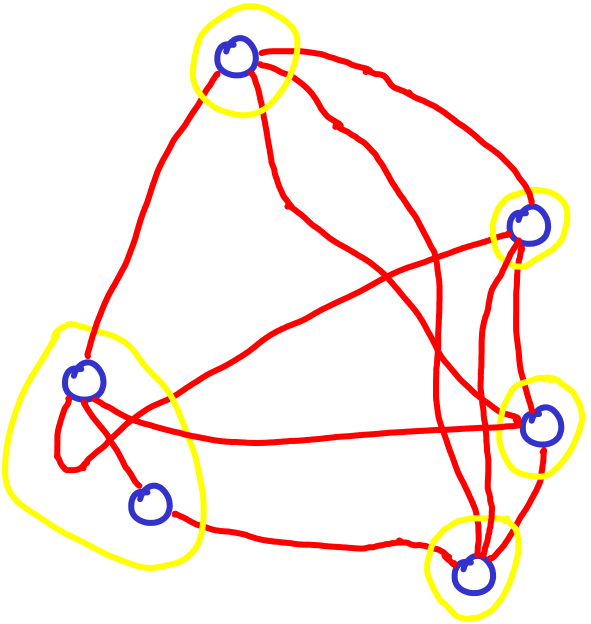
Non-planar graphs (can't redraw)



$K_{3,3}$

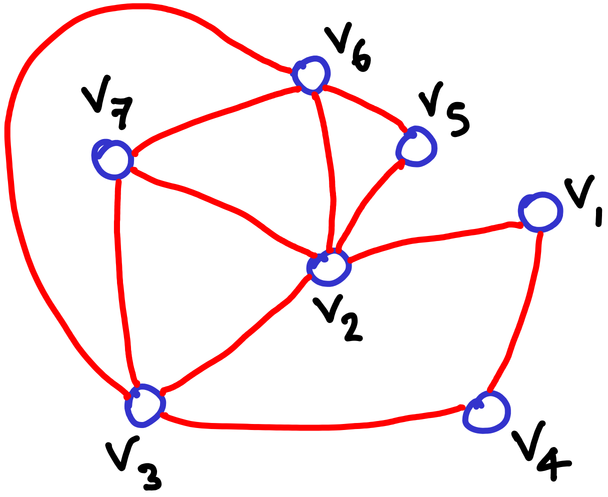


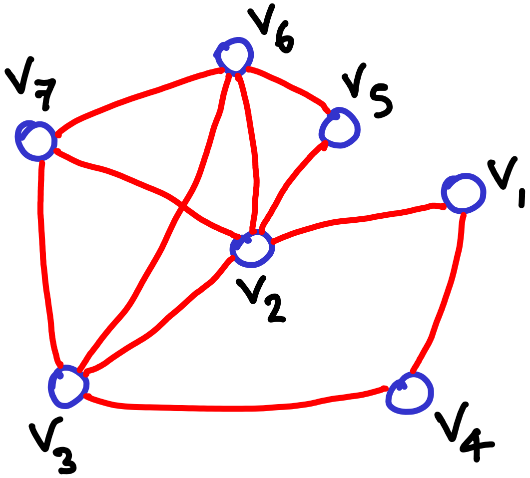
K_5



PLANE GRAPH

no crossings



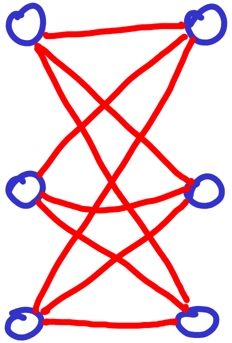


PLANAR GRAPH

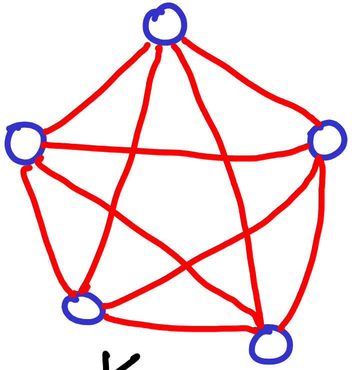
can redraw without crossings

Non-planar graphs

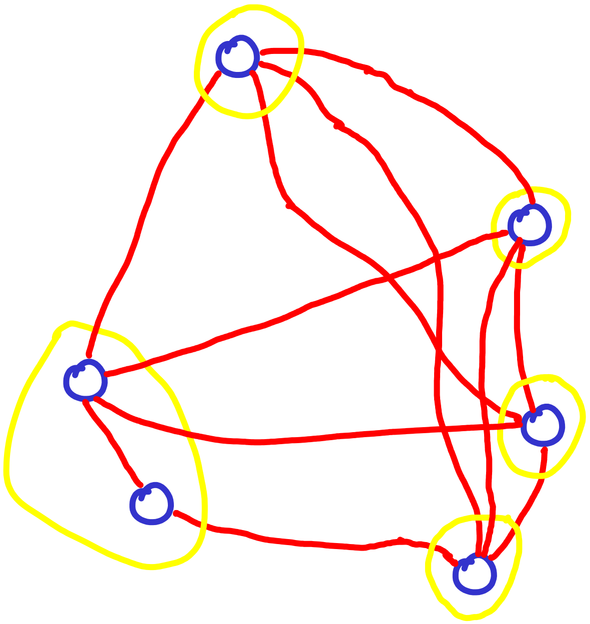
(can't redraw)



$K_{3,3}$

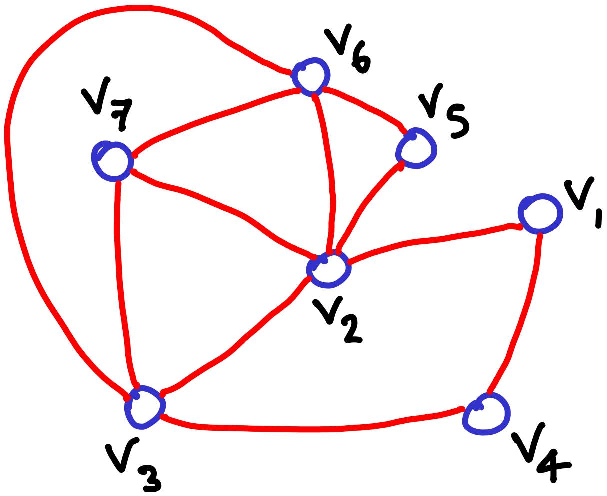


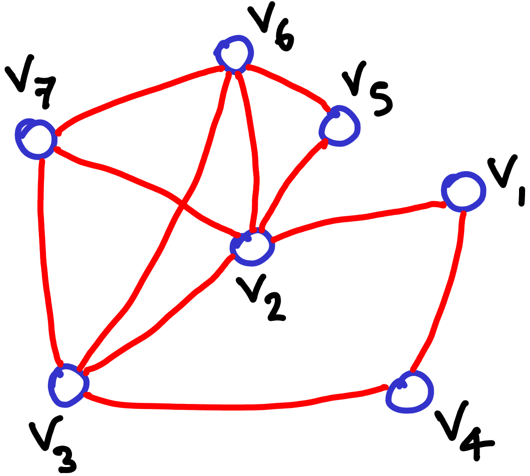
K_5



PLANE GRAPH

no crossings

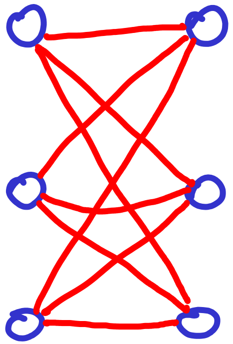




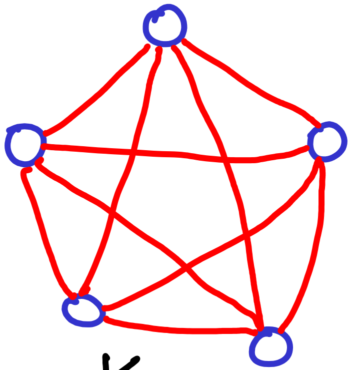
PLANAR GRAPH

can redraw without crossings

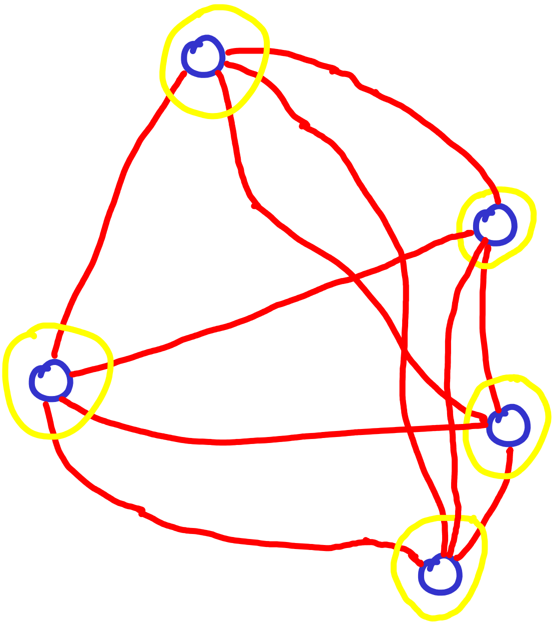
Non-planar graphs (can't redraw)



$K_{3,3}$

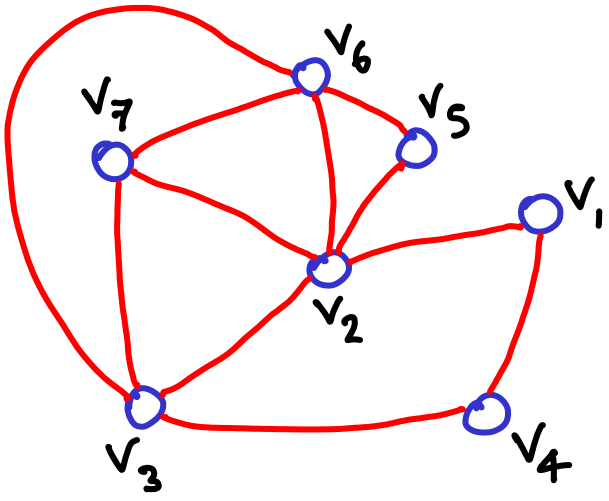


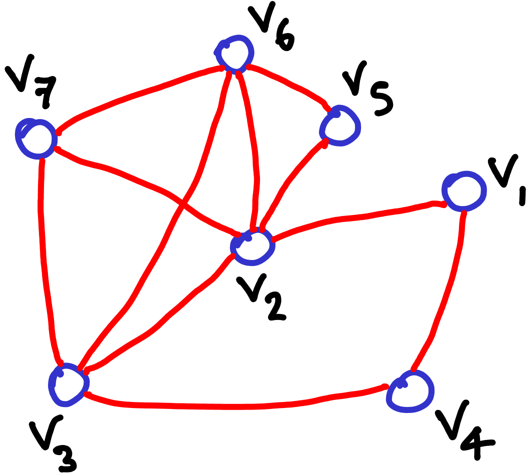
K_5



PLANE GRAPH

no crossings



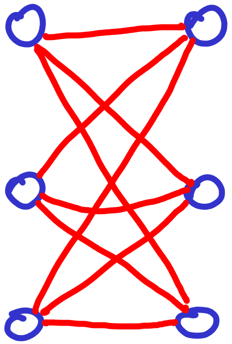


PLANAR GRAPH

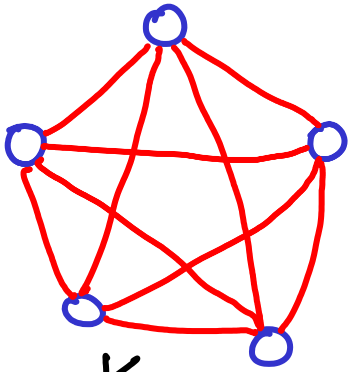
can redraw without crossings

obtained by successive contractions

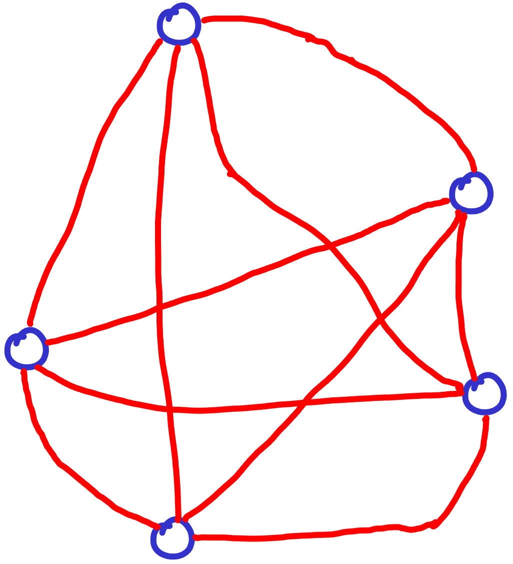
Non-planar graphs (can't redraw)



$K_{3,3}$



K_5



A graph is non-planar if and only if it "contains" a K3,3 or K5