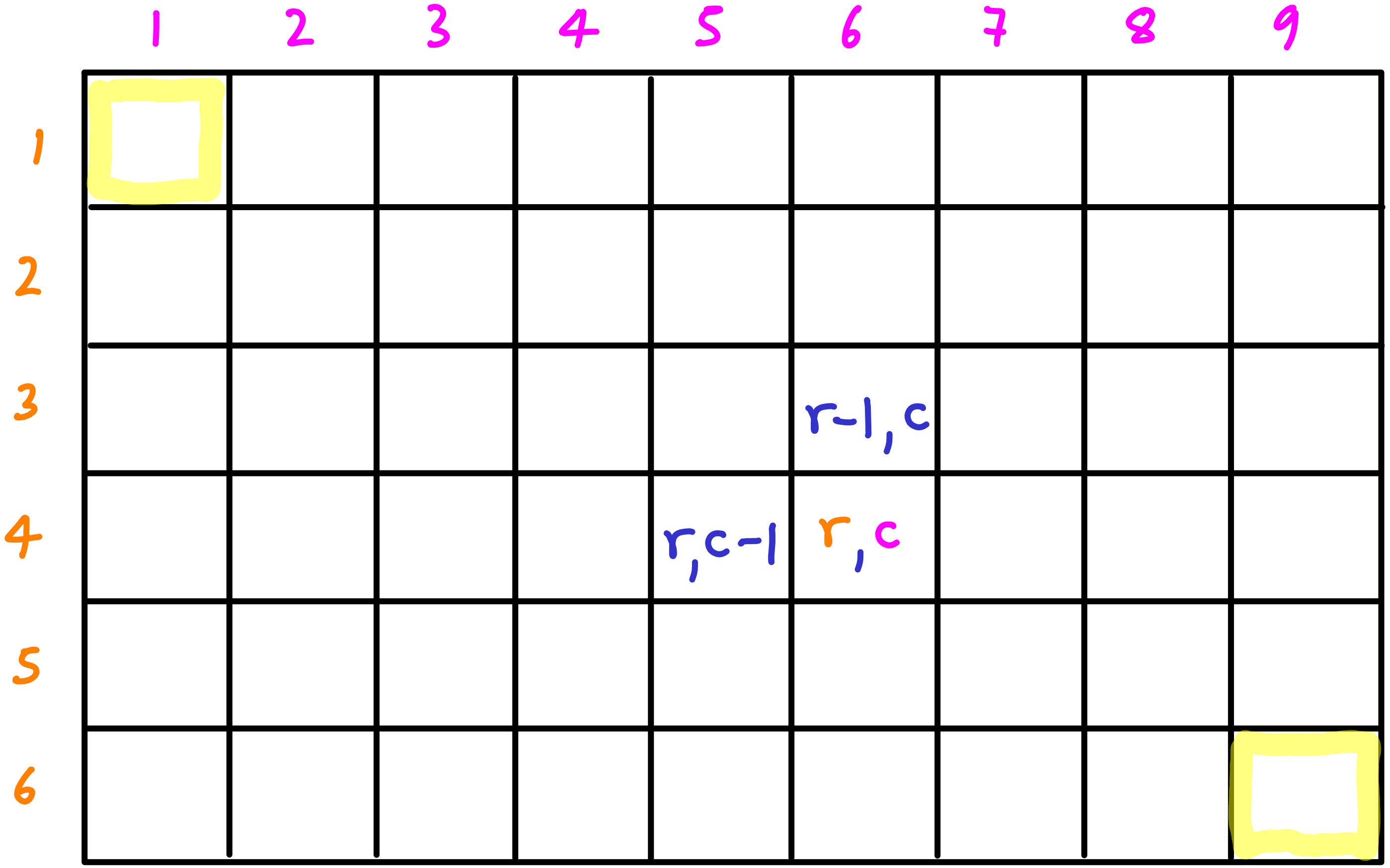
Starting at top-left of n x m grid, moving only down or right, how many ways to reach bottom-right?



$$A[r,c] = A[r-1,c] + A[r,c-1]$$

Starting at top-left of $n \times m$ grid, moving only down or right, how many ways to reach bottom-right?

repetitive subproblems

want to avoid repetition

$A[r,c]$

$A[r-1,c]$ $A[r,c-1]$

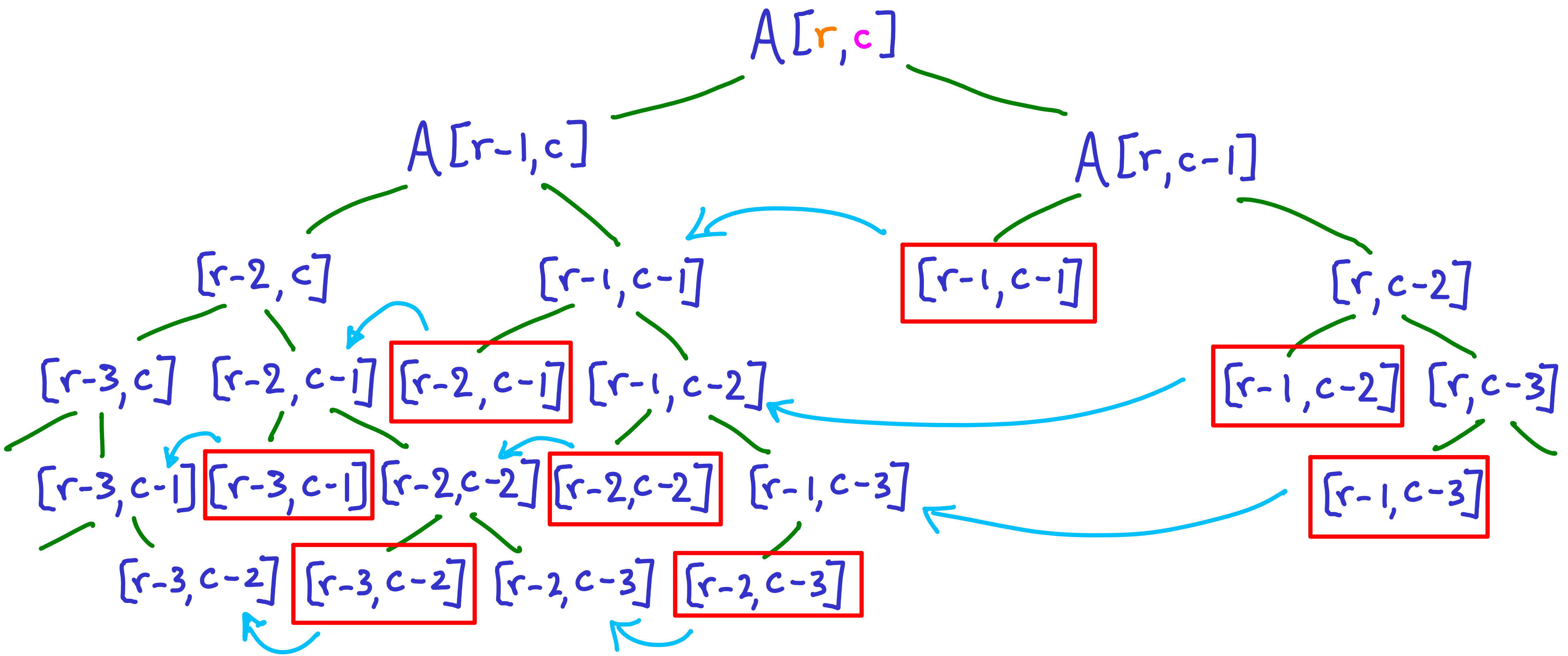
$[r-2,c]$ $[r-1,c-1]$* $\boxed{[r-1,c-1]}$ $[r,c-2]$

$[r-3,c]$ $[r-2,c-1]$* $\boxed{[r-2,c-1]}$ $[r-1,c-2]$* $\cancel{[r-2,c-1]}$ $\cancel{[r-1,c-2]}$ $\boxed{[r-1,c-2]}$ $[r,c-3]$

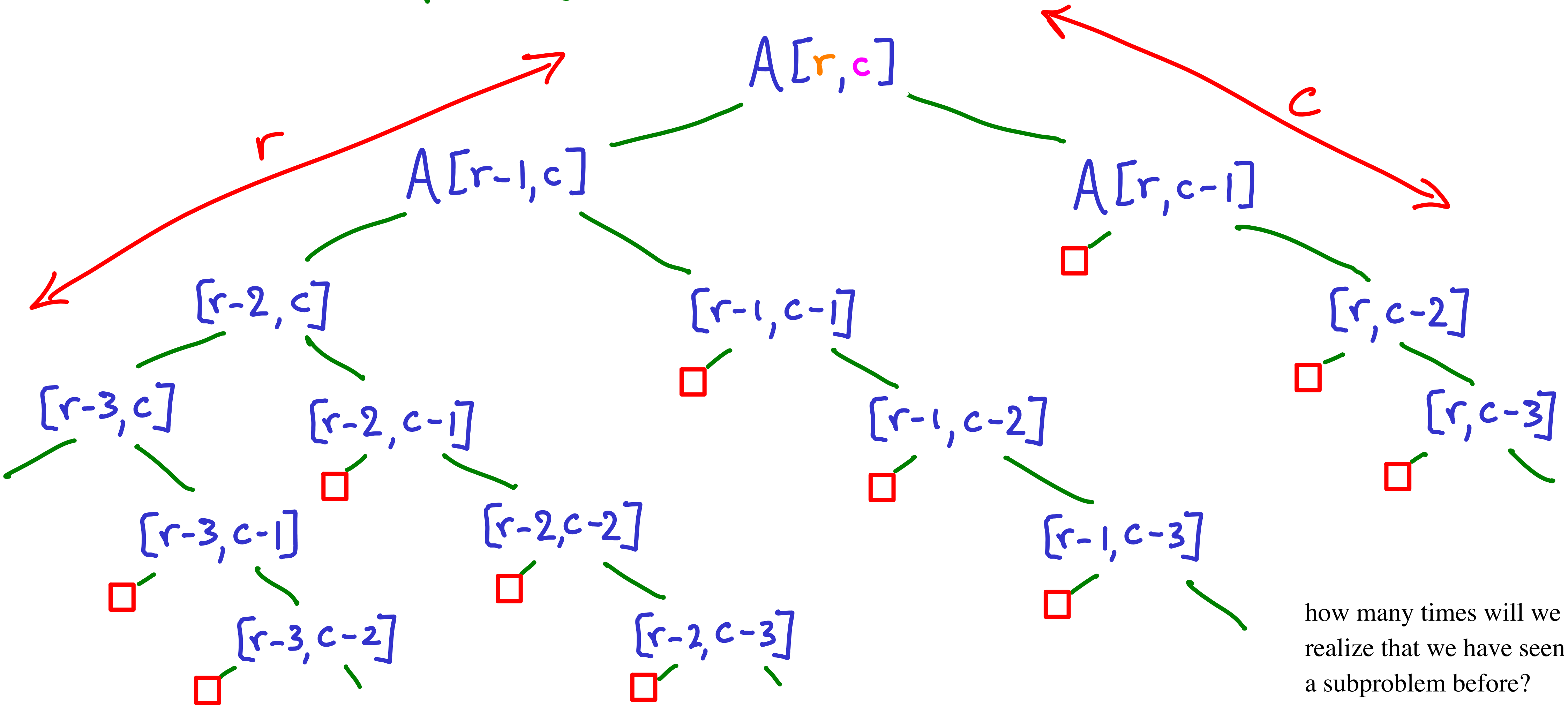
$[1,c]$ $[r,1]$

$\min\{r,c\}$ full levels $\Omega(2^n)$ for $n \times n$

Starting at top-left of $n \times m$ grid, moving only down or right, how many ways to reach bottom-right?



How many times will we recurse in a unique way? → $r \cdot c$ distinct subproblems

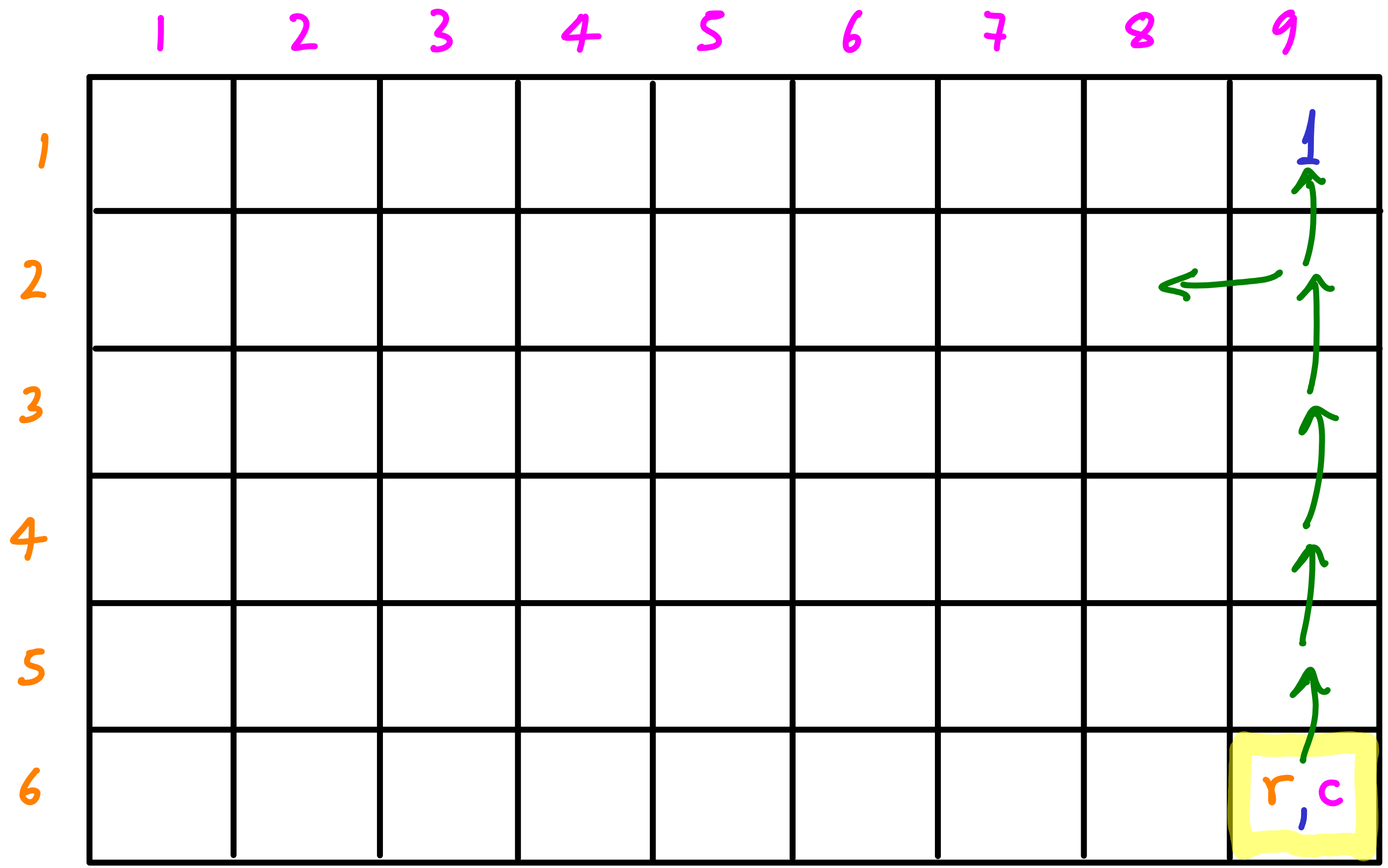


how many times will we realize that we have seen a subproblem before?

MEMOIZATION (making memos)

For this problem, m×n table

$$A[r,c] = A[r-1,c] + A[r,c-1]$$



Recursion:

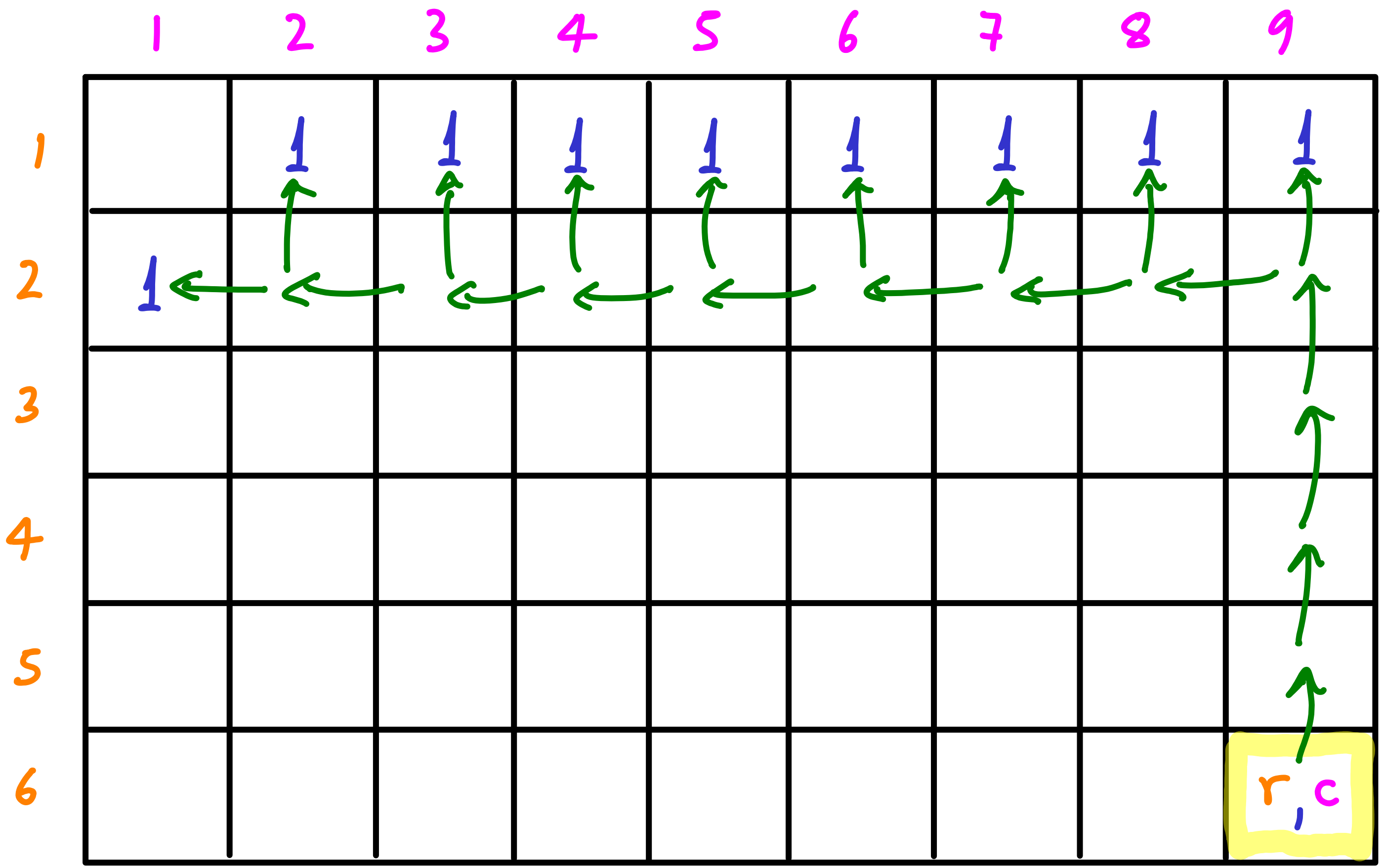
first find $A[r-1,c]$ ↑

then find $A[r,c-1]$ ←

MEMOIZATION (making memos)

For this problem, $m \times n$ table

$$A[r,c] = A[r-1,c] + A[r,c-1]$$



Recursion:

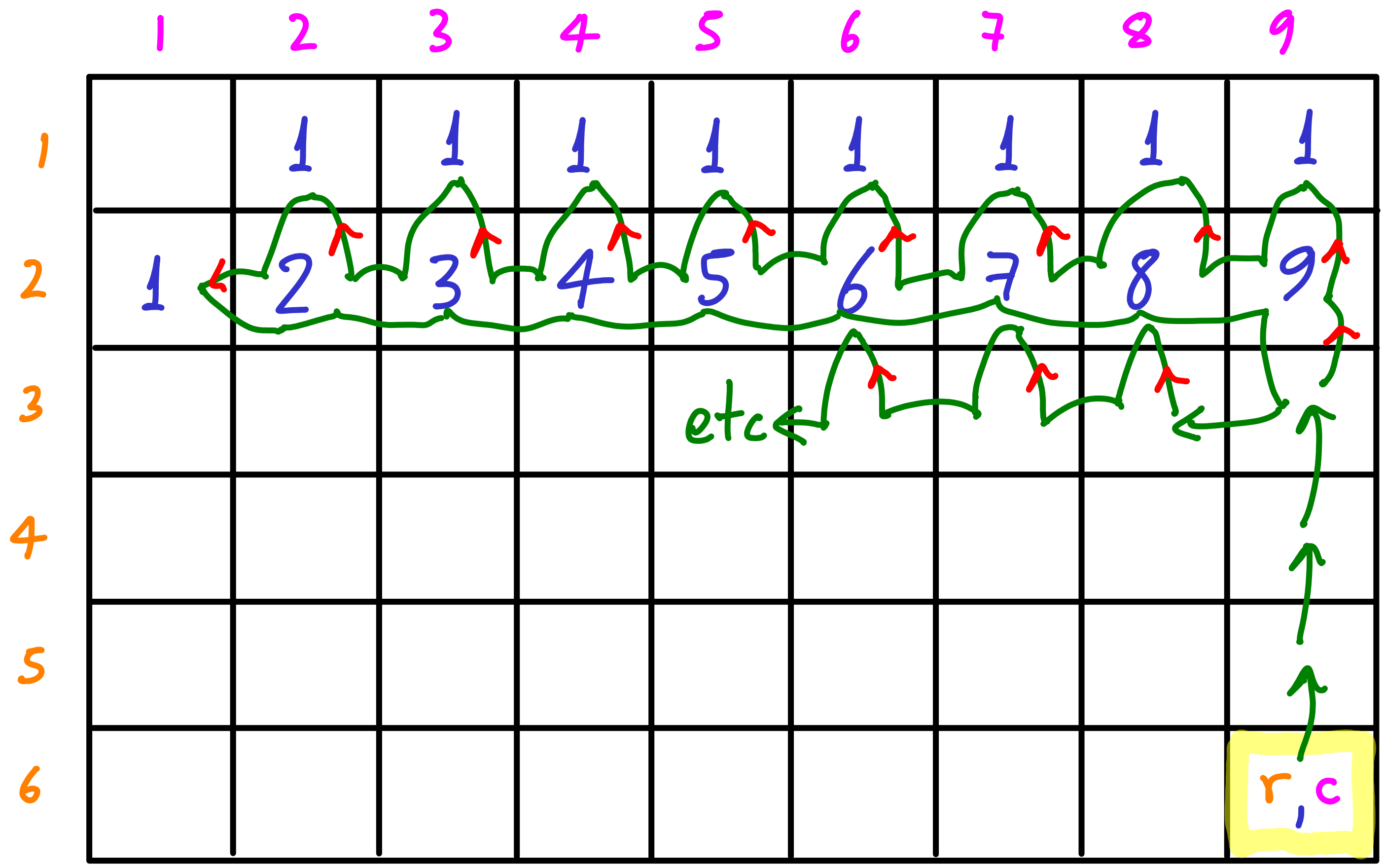
first find $A[r-1,c]$ ↑

then find $A[r,c-1]$ ←

MEMOIZATION (making memos)

For this problem, $m \times n$ table

$$A[r,c] = A[r-1,c] + A[r,c-1]$$



Recursion:

first find $A[r-1,c]$ $\uparrow$

then find $A[r,c-1]$ $\leftarrow$

$\Theta(n \cdot m)$

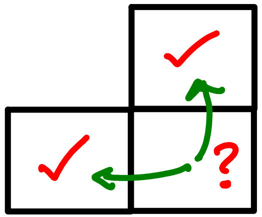
time & space

Starting at top-left of $n \times m$ grid, moving only down or right, how many ways to reach bottom-right?

DYNAMIC PROGRAMMING (bottom-up : base cases first)

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
2	1								
3	1								
4	1								
5	1								
6	1								

$$A[r,c] = \underbrace{A[r-1,c] + A[r,c-1]}$$



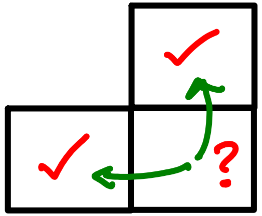
fill any cell as long as what it depends on is full

Starting at top-left of $n \times m$ grid, moving only down or right, how many ways to reach bottom-right?

DYNAMIC PROGRAMMING (bottom-up : base cases first)

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
2	1	2							
3	1								
4	1								
5	1								
6	1								

$$A[r,c] = \underbrace{A[r-1,c] + A[r,c-1]}$$



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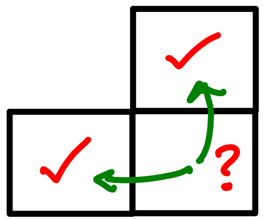
Starting at top-left of $n \times m$ grid, moving only down or right, how many ways to reach bottom-right?

DYNAMIC PROGRAMMING (bottom-up : base cases first)

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
2	1	2	3	4	5	6	7	8	
3	1	3	6						
4	1	4	10						
5	1	5							
6	1	6							

$$A[r,c] = A[r-1,c] + A[r,c-1]$$



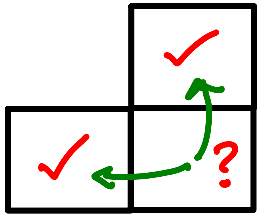
fill any cell as long as what it depends on is full

Starting at top-left of $n \times m$ grid, moving only down or right, how many ways to reach bottom-right?

DYNAMIC PROGRAMMING (bottom-up: base cases first)

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	1	1	1	1
2	1	2	3	4	5	6	7	8	9
3	1	3	6	10	15	21	28	36	45
4	1	4	10	20	35	56	84	120	165
5	1	5	15	35	70	126	210	330	495
6	1	6	21	56	126	252	462	792	1287

$$A[r,c] = \underbrace{A[r-1,c] + A[r,c-1]}$$



fill any cell as long as what it depends on is full

Starting at top-left of $n \times m$ grid, moving only down or right, how many ways to reach bottom-right? ... with obstacles

	1	2	3	4	5	6	7	8	9
1						0			
2			0						
3			0		0				
4			0				0		
5			0			0			
6									

Starting at top-left of $n \times m$ grid, moving only down or right, how many ways to reach bottom-right? ... with obstacles

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1		0						
3	1		0		0				
4	1		0				0		
5	1		0			0			
6	1								

Starting at top-left of n x m grid, moving only down or right, how many ways to reach bottom-right? ... with obstacles

	1	2	3	4	5	6	7	8	9
1	1	1	1	1	1	0	0	0	0
2	1	2	0	1	2	2	2	2	2
3	1	3	0	1	0	2	4	6	8
4	1	4	0	1	1	3	0	6	14
5	1	5	0	1	2	0	0	6	20
6	1	6	6	7	9	9	9	15	35