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If there exist constants $c > 0$, $n_0 > 0$ such that for all $n \geqslant n_0$:

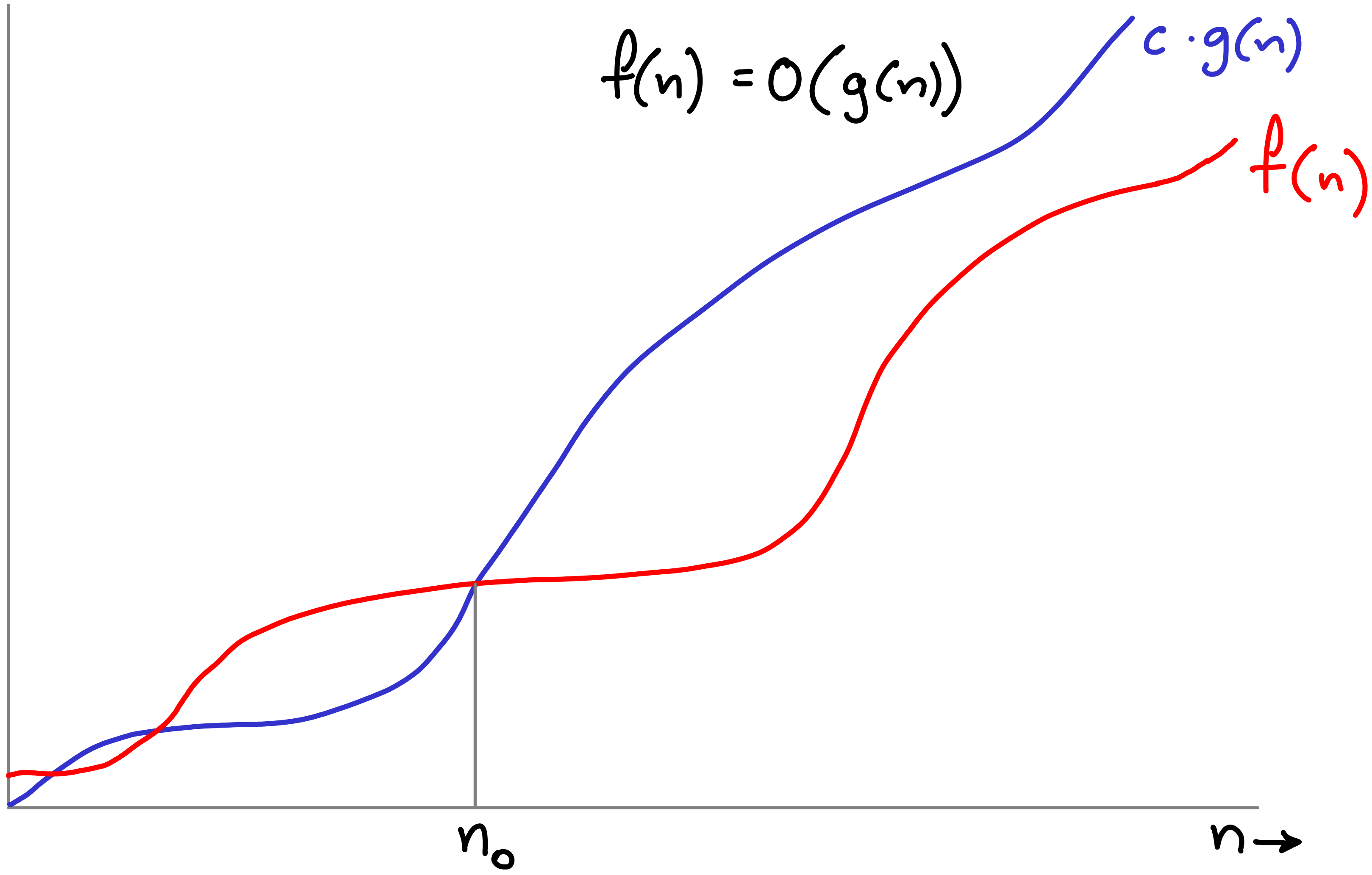
$$f(n) \leqslant c \cdot g(n)$$

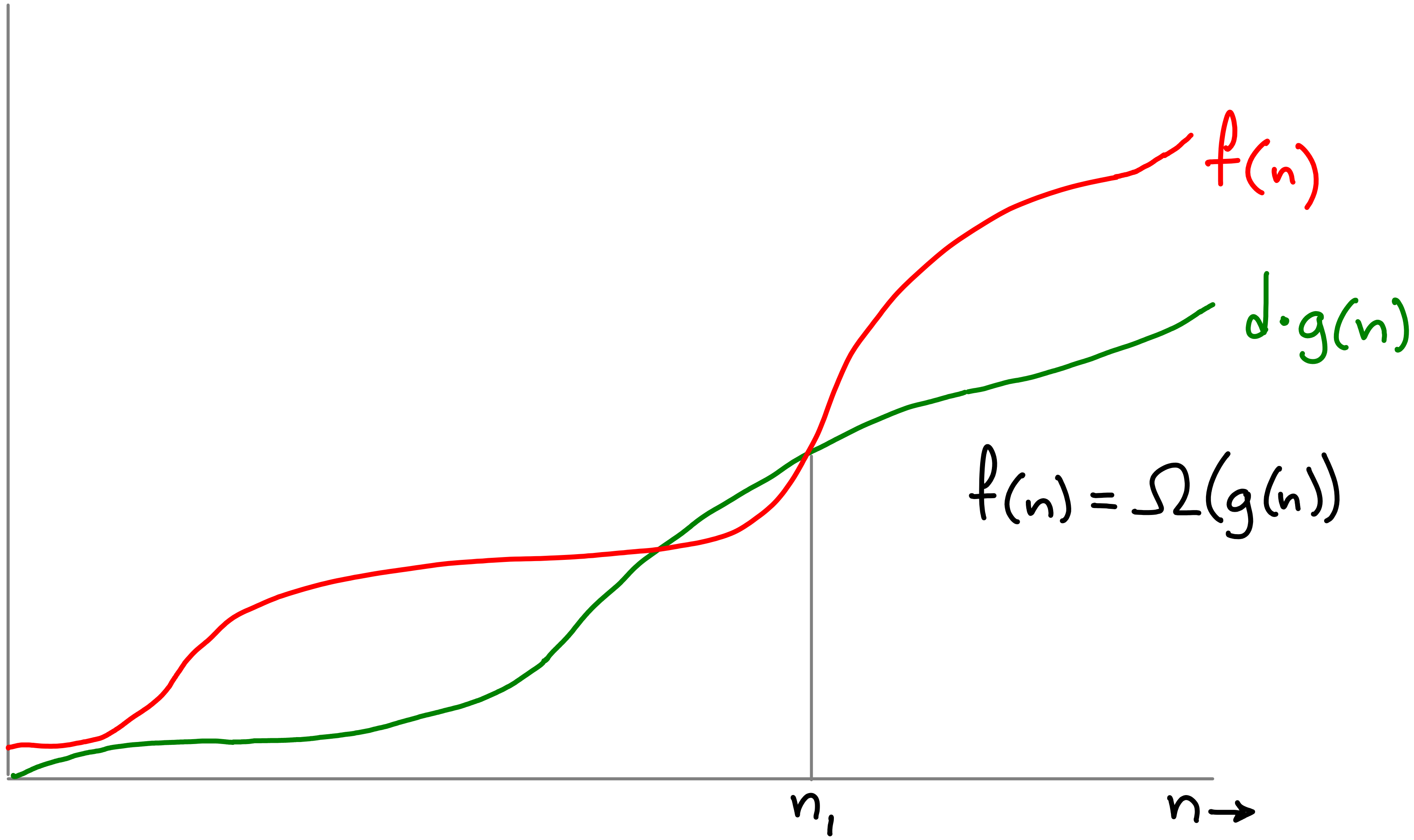
then we say $f(n) = O(g(n))$ Big-O

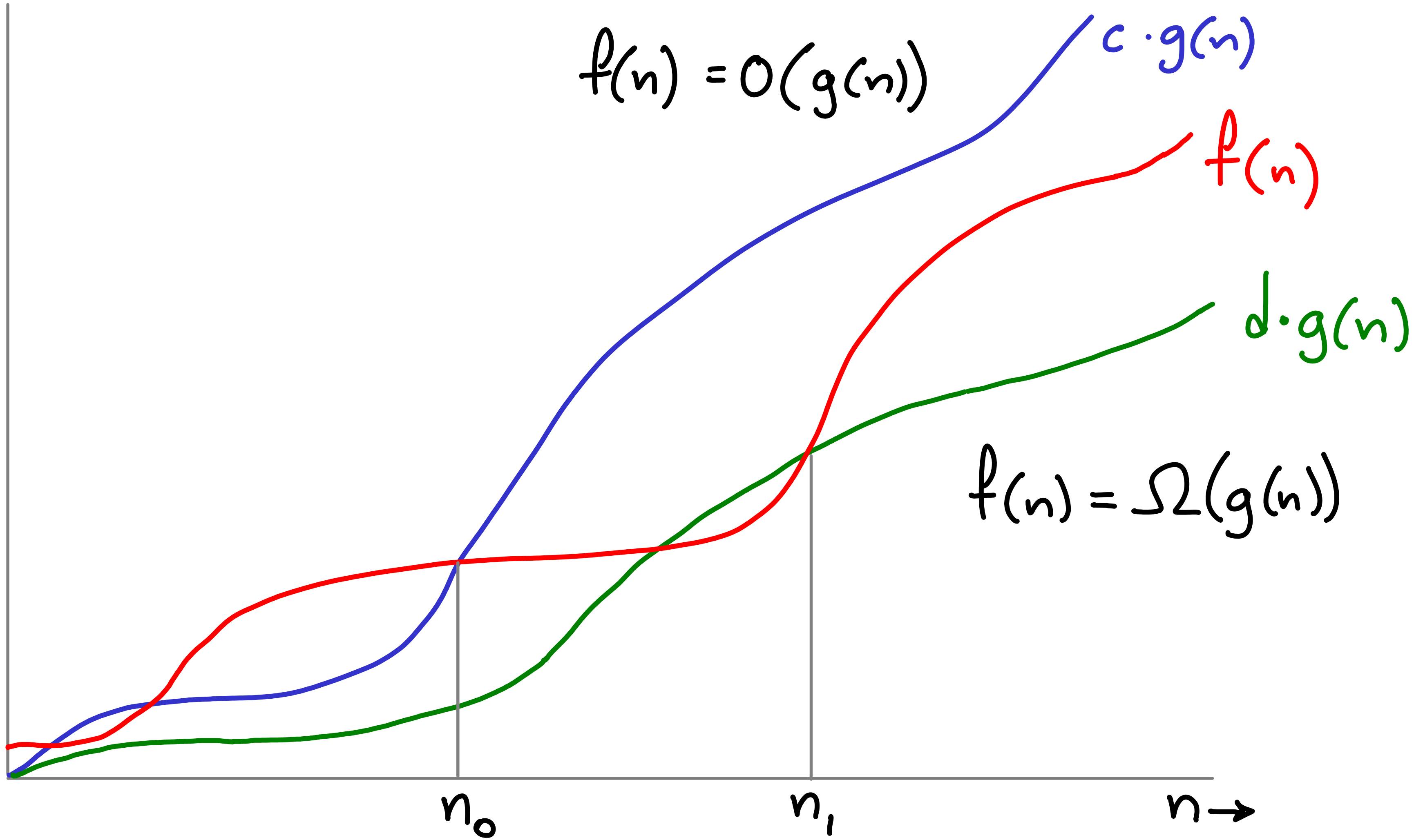
If there exist constants $d > 0$, $n_1 > 0$ such that for all $n \geqslant n_1$:

$$f(n) \geqslant d \cdot g(n)$$

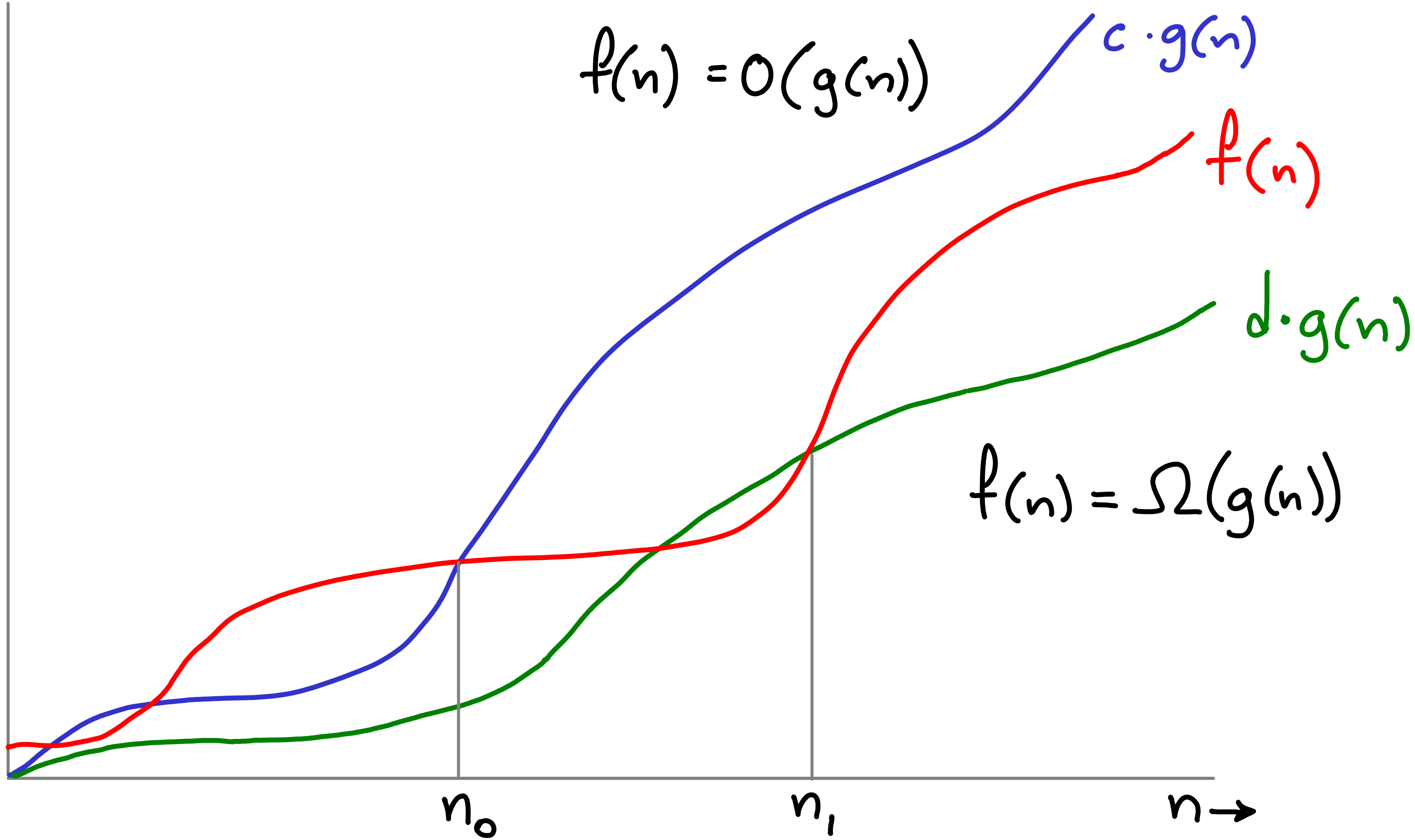
then we say $f(n) = \Omega(g(n))$ Omega

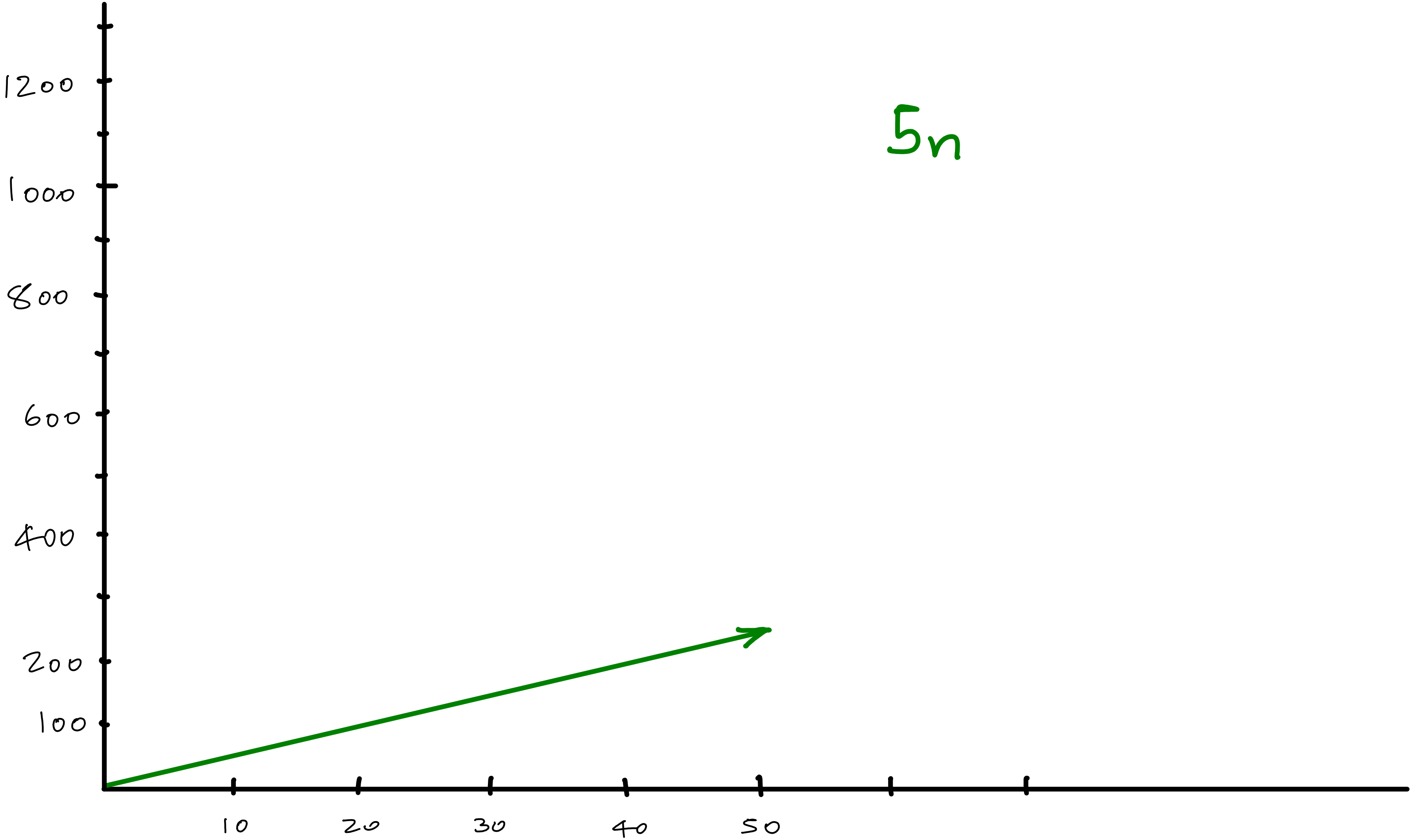

$f(n) = O(g(n))$
$c \cdot g(n)$
$f(n)$
n_0
$n \rightarrow$


$f(n)$
$d \cdot g(n)$
$f(n) = \Omega(g(n))$
n_1
$n \rightarrow$


$f(n) = O(g(n))$
$c \cdot g(n)$
$f(n)$
$d \cdot g(n)$
$f(n) = \Omega(g(n))$
n_0
n_1
$n \rightarrow$

If $f(n) = O(g(n))$ AND $f(n) = \Omega(g(n))$ then $f(n) = \Theta(g(n))$ [Theta]




1200
1000
800
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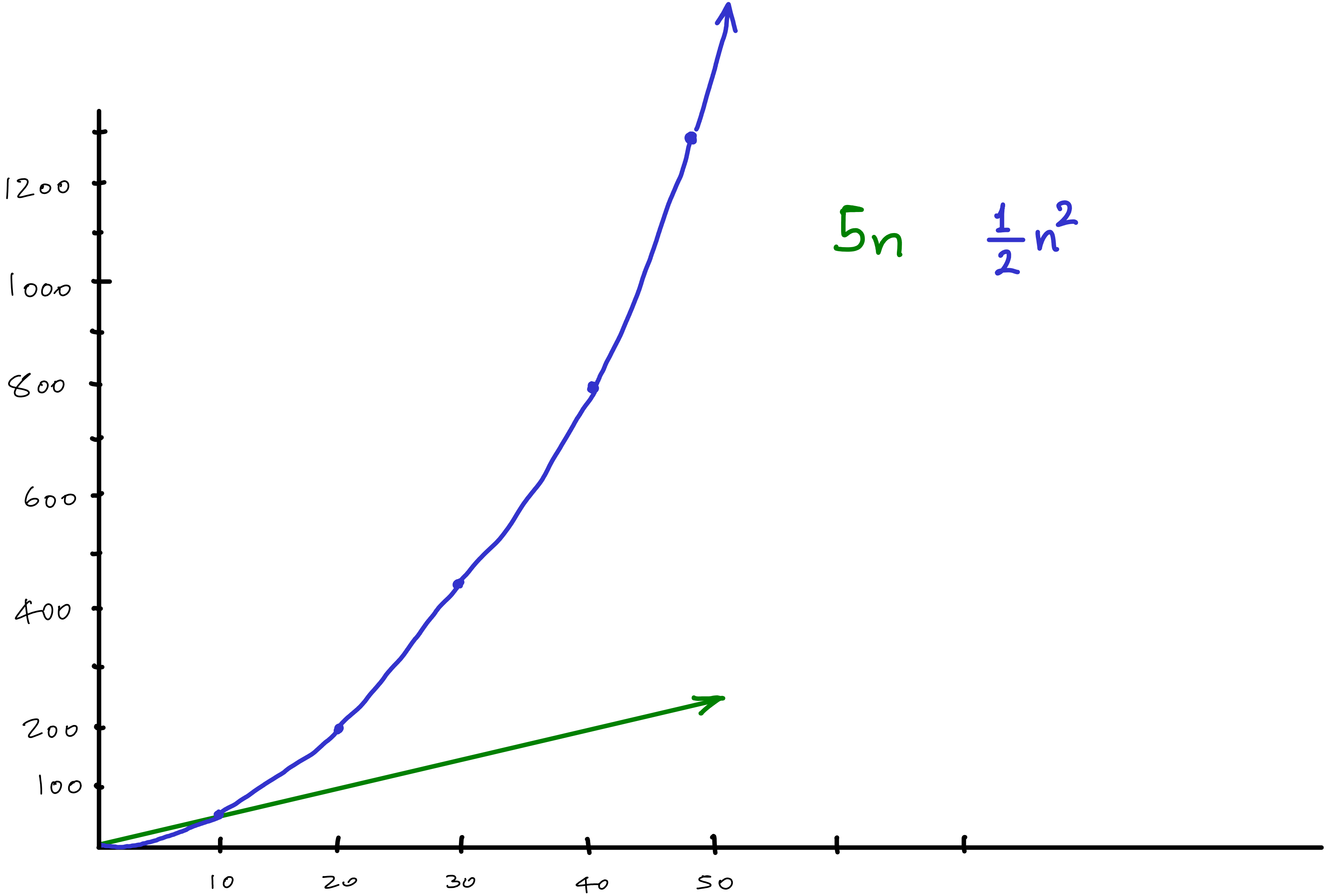
30
40
50
5n


1200
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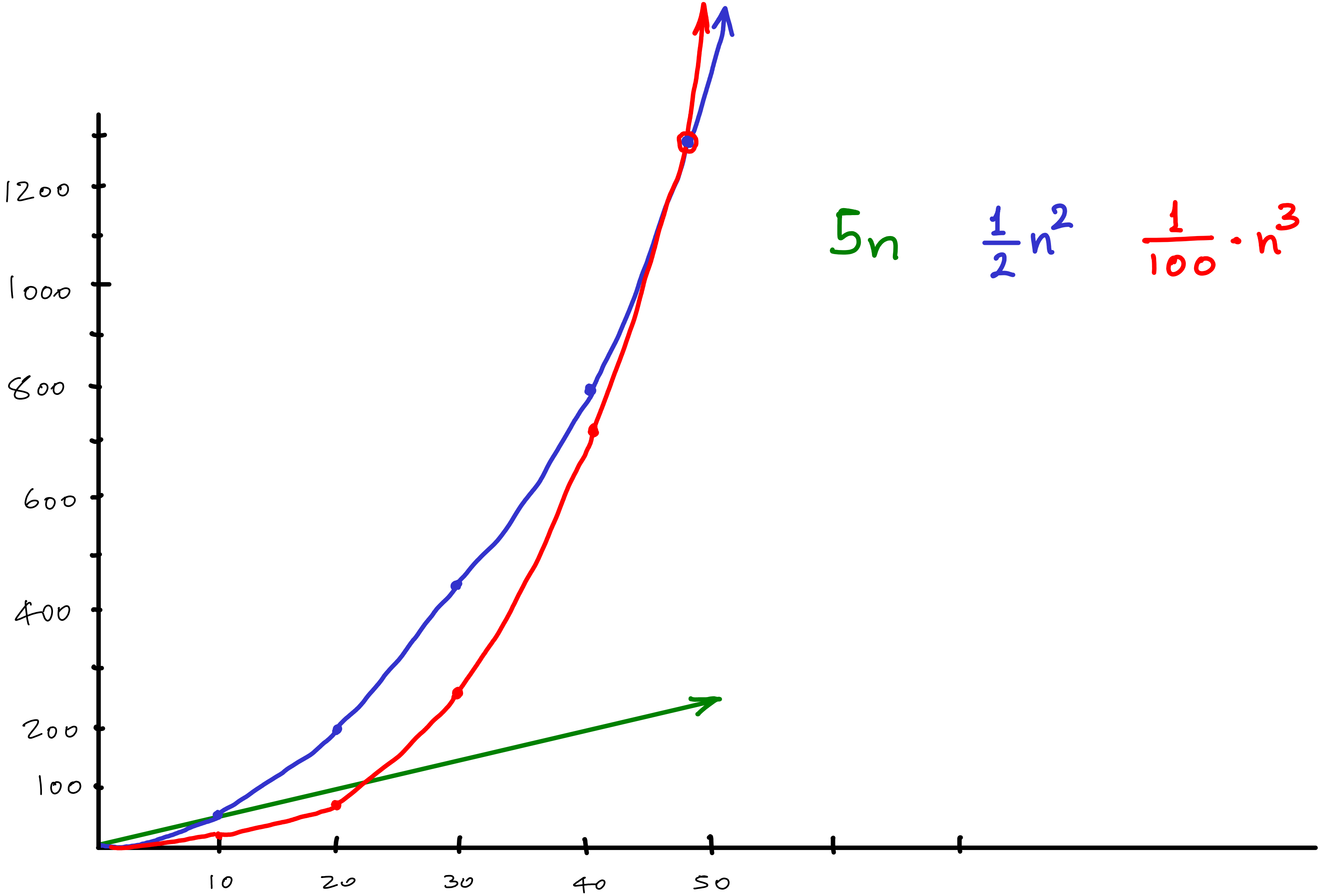
$\frac{1}{2}n^2$


1200
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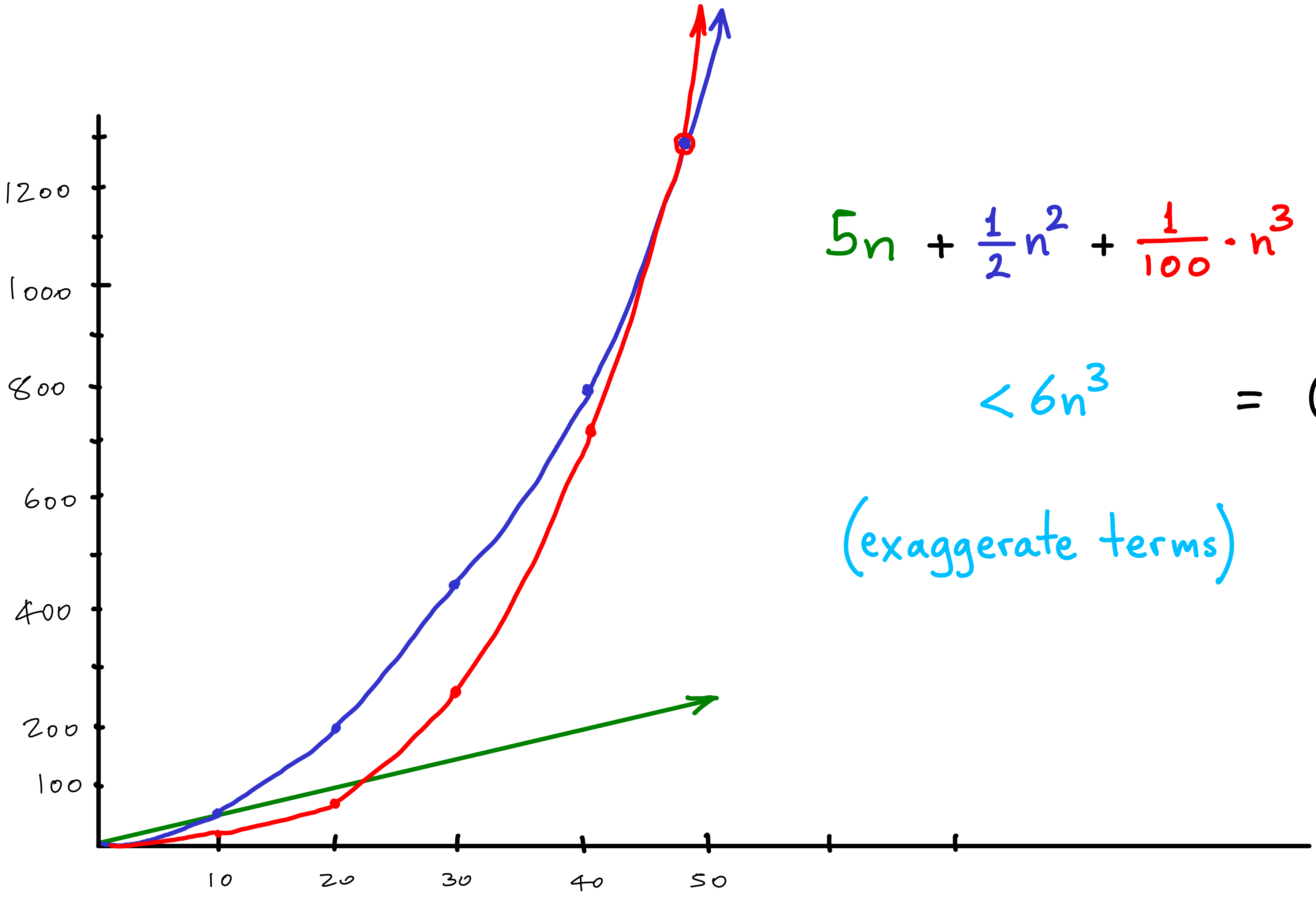
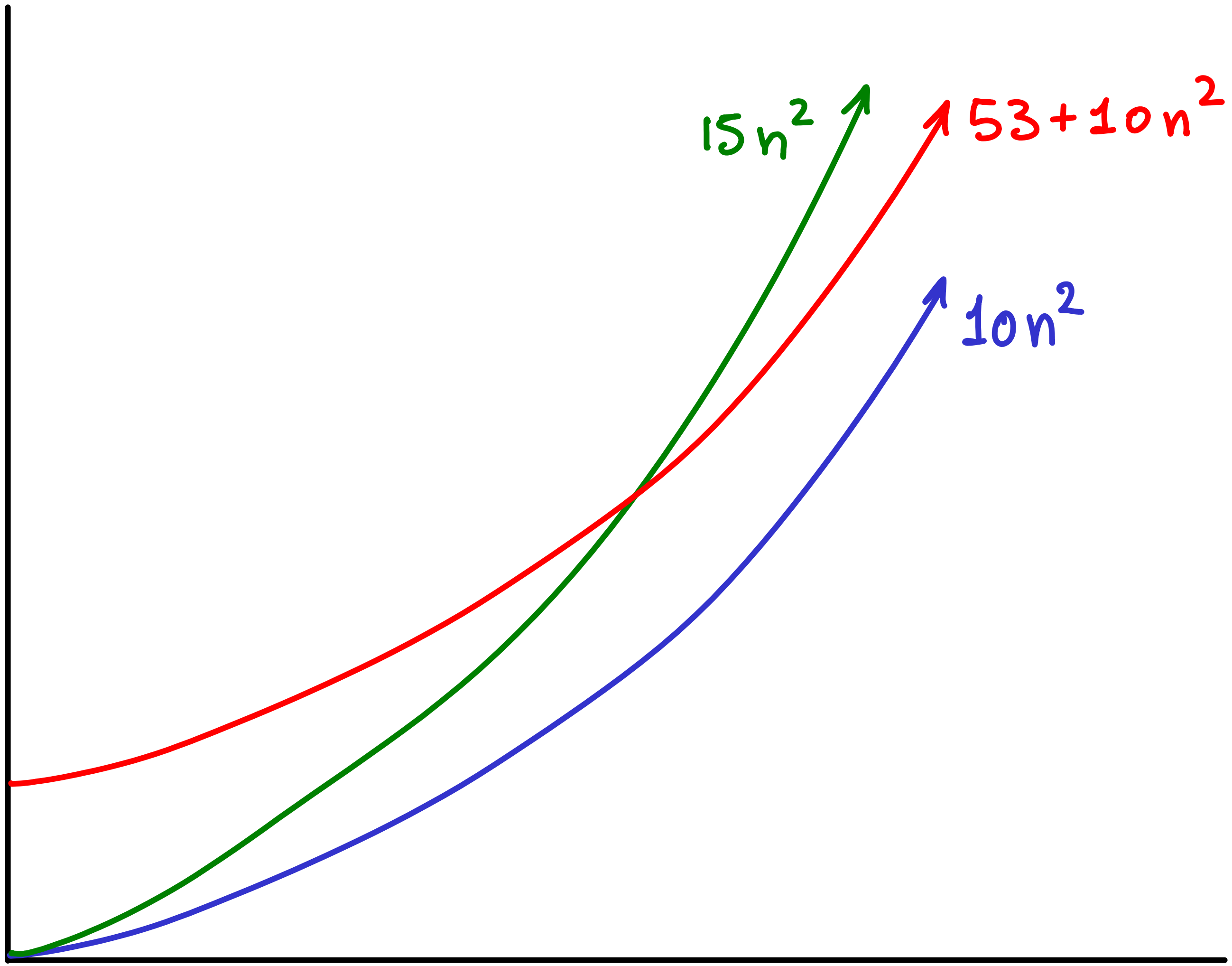


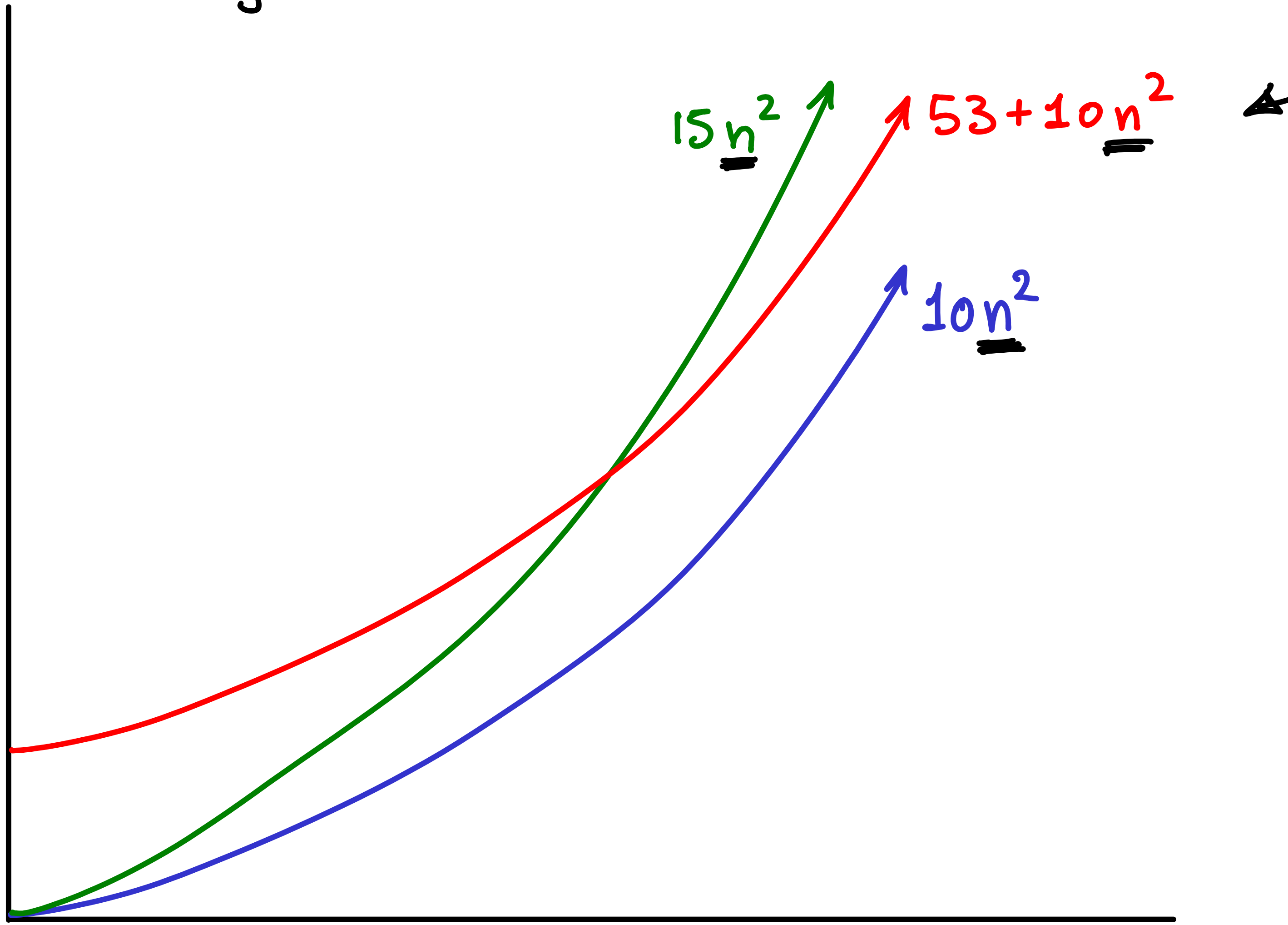
$$5n + \frac{1}{2}n^2 + \frac{1}{100}\cdot n^3$$

$$< 6n^3 \qquad = O(n^3)$$

(exaggerate terms)


$15n^2$
$53+10n^2$
$10n^2$

For large n these are within a constant multiplicative factor



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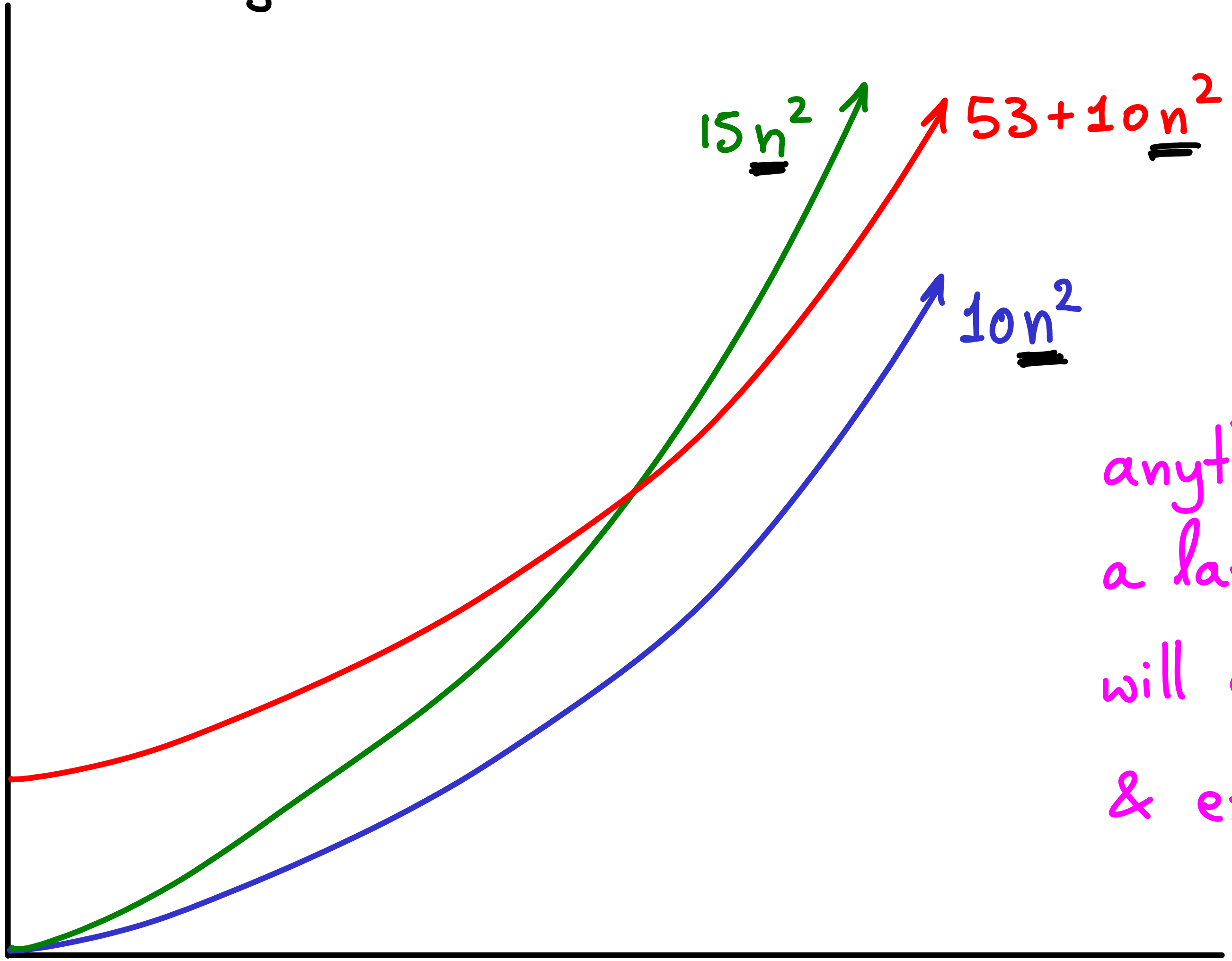
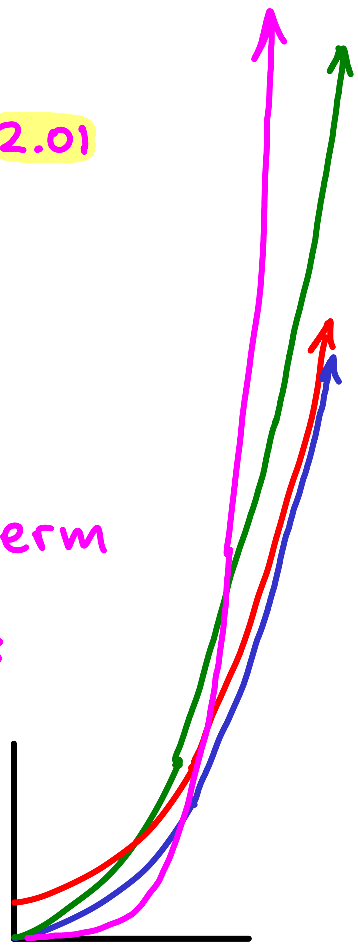


$0.00001 \cdot n^{2.01}$

anything with
a larger dominating term
will eventually surpass
& exceed by a lot.



Polynomials: $a + bn + cn^2 + dn^3 \cdots + zn^k$

$a, b, c, d, \ldots, z$: constants

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Also assuming one of each term (compare to: $a_1 n + a_2 n + \dots + a_n n$)

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Logarithms: $50 \cdot \log n^3 + \log^{20} n + \underline{n}^{0.1} = O(\underline{n}^{0.1})$

"weaker" than polynomial

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Exponential: $100 \cdot n^{50} + \underline{3}^n + 40 \cdot 2^n = O(\underline{3}^n)$

"stronger" than polynomial

Ordering some common functions

Within each row, subdivide

	n^n	
	$n!$	
exponential:	k^n $(k>1)$	1.1^n, 2^n, 3^n etc
polynomial:	n^k	$n^{0.1}$, $\sqrt{n}$, n, $n^{1.1}$, n^2, n^3, etc
powers of logs:	$\log^k n = (\log n)^k$	$\log n$, $\log^2 n$, $\log^3 n$, etc ↰ base doesn't matter!
constants:	1, 50, 2^{100} $= O(1)$	

$$\left(50 \cdot \log n + 10 \log^5 n + n^{0.1}\right) \cdot \left(100 \cdot n^{50} + 3^n + 40 \cdot 2^n\right) = ?$$

$$\left(50 \cdot \log n + 10 \log^5 n + n^{0.1}\right) \cdot \left(100 \cdot n^{50} + 3^n + 40 \cdot 2^n\right) =$$

$$O(n^{0.1}) \cdot O(3^n) \quad = \quad ?$$

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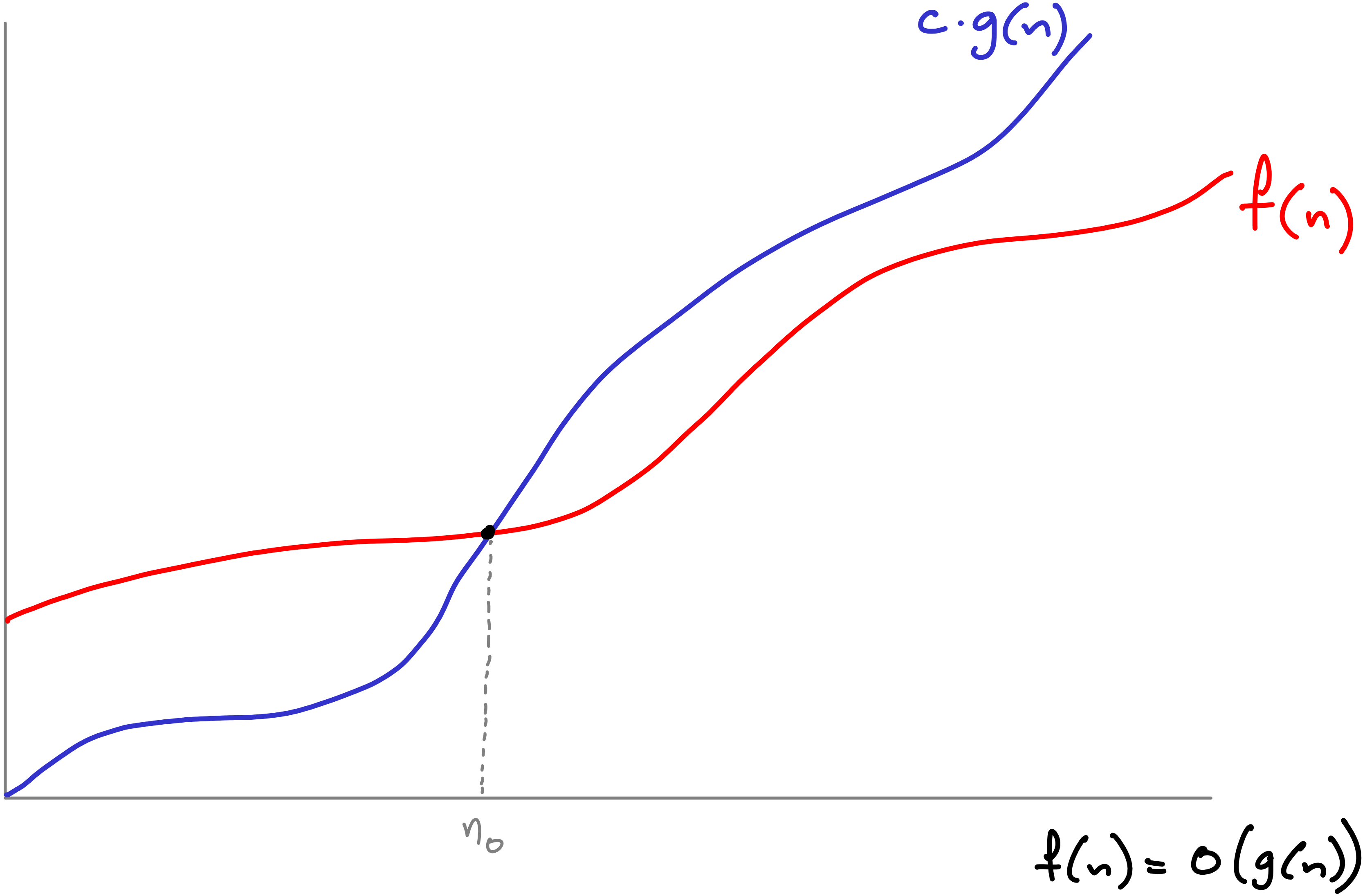
- is $n^3 = O(5n^2)$? No.

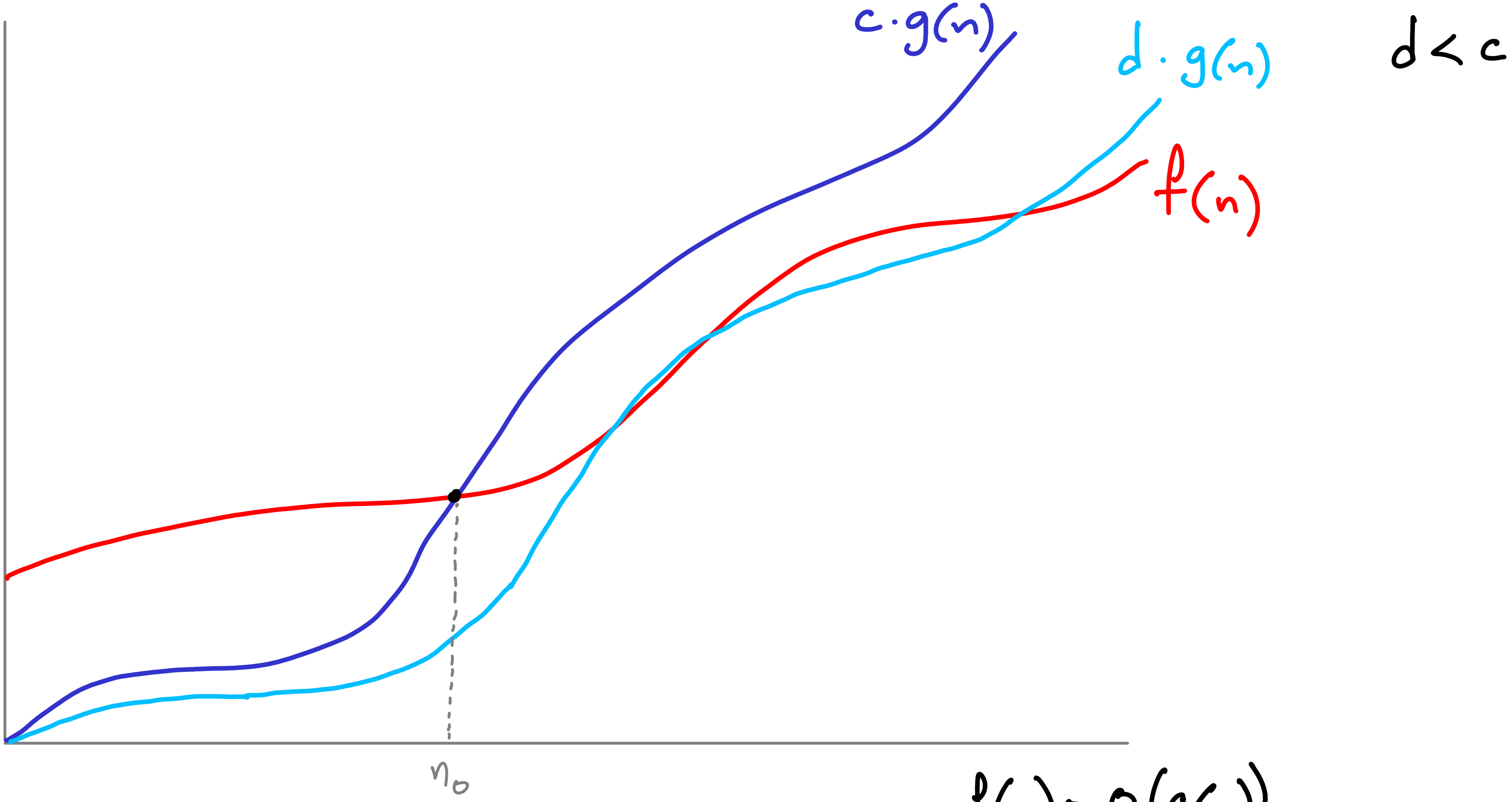
There is no c such that $n^3 \leq c \cdot 5n^2$

(for all large n)

Also, the 5 doesn't belong in $O(5n^2)$

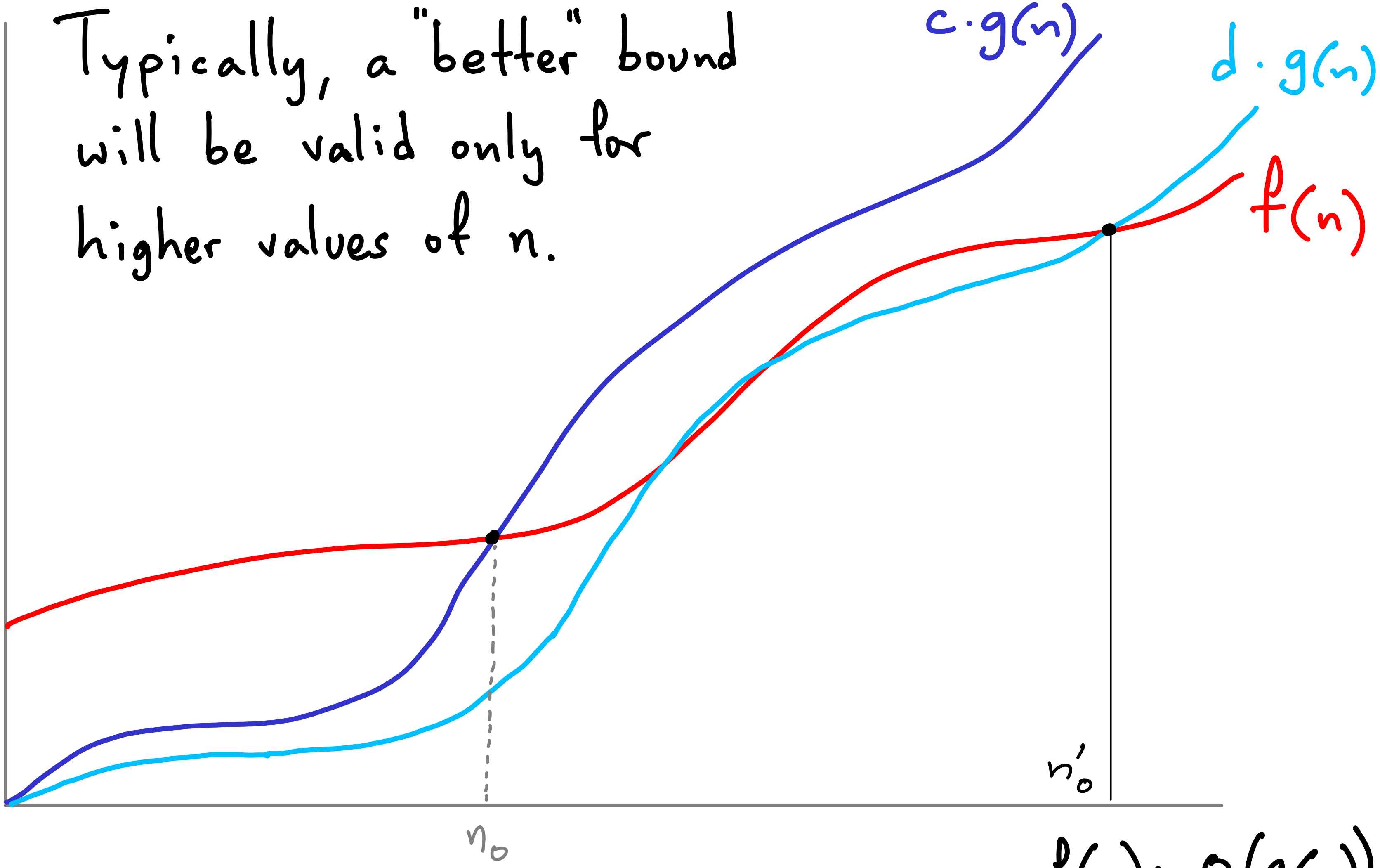


$f(n) = O(g(n))$



$f(n) = o(g(n))$

Typically, a "better" bound will be valid only for higher values of n.



$d < c$

↓

better

$f(n) = O(g(n))$

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↳ prove $= O(n^2)$

↳ prove $= \Omega(n^2)$

Prove $\frac{1}{2}n^2+3n-10 = \Theta(n^2)$

↳ prove $=O(n^2)$ → find c_1 & n_1 s.t. $\frac{1}{2}n^2+3n-10 \leq c_1 \cdot n^2$ for $n>n_1$

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(exaggerate & simplify)

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(underestimate & simplify) $> \frac{4}{10}n^2$ for $n \ge 10$

$c_2 = 0.4$ & $n_2 = 10$ work

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$$\frac{\frac{1}{2}n^2}{n^2} + \frac{3n}{n^2} - \frac{10}{n^2} \le \frac{cn^2}{n^2}$$

(this step is ok)

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  $$\frac{\frac{1}{2}\cancel{n^2}}{\cancel{n^2}} + \boxed{\xcancel{\frac{3n}{n^2} - \frac{10}{n^2}}} \leq \frac{c\cancel{n^2}}{\cancel{n^2}}$$

  so as $n \to \infty$, $\frac{1}{2} \leq c$ [NOT OK]

Prove $6n^3 \neq \Theta(n^2)$

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if it were true, then $c_1 n^2 \leq 6n^3 \leq c_2 n^2$ for $n \geqslant n_0$

Prove $6n^3 \neq \Theta(n^2)$

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$\boxed{\text{NO}}$

$6n \leq c_2$?

$$\left. n \leq \frac{c_2}{6} \right\} \text{NO.}$$

Whatever constant c_2 we choose, n will eventually surpass it.

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e.g., $n^3 = \omega(n^2)$ but $5n^2 \neq \omega(n^2)$

Recap of rules and model of computation used in this course.

(unless mentioned otherwise)

- Any number occupies $O(1)$ storage ...and can be read in $O(1)$ time

  Even irrationals.

- We can do simple arithmetic in $O(1)$ time. (on $O(1)$ elements)

  ($+, -, *, \div$, but also $\sqrt{\ }$, $\lceil\ \rceil$, $\lfloor\ \rfloor$, powers, etc)

- We care only about what happens for unimaginably large input size n

- We focus on worst-case time/space complexity