Dynamic arrays: dealing with $\Phi = -\text{size}_i$ assuming $\Phi_0 = 0$

The problem: $\Phi_n - \Phi_0 < 0$

$$\sum \hat{c}_i = \sum c_i + \Phi_n - \Phi_0$$

$$\sum \hat{c}_i < \sum c_i$$

operation type:

- cheap: $c_i = 1$, $\Delta\Phi_i = 0 \rightarrow \hat{c}_i = 1$
- expensive: $c_i = i$, $\Delta\Phi_i = -(i-1) = 1 - c_i \rightarrow \hat{c}_i = 1$

$$\sum \hat{c}_i = n$$

array size: $i-1 \rightarrow 2(i-1)$

$\hookrightarrow \Phi_n \geq -2(n-1)$

$$|\Phi_n - \Phi_0| < 2n$$

$$\sum c_i < 3n$$

$$\sum \hat{c}_i + |\Phi_n - \Phi_0| < n + 2n$$

For any amortized analysis:

if $\Phi_n - \Phi_0 < 0$

calculate the maximum possible $|\Phi_n - \Phi_0|$

& add to your upper bound for $\sum \hat{c}_i$

$$\boxed{\sum c_i}$$

$$\boxed{\sum \hat{c}_i}\boxed{|\Phi_n - \Phi_0|}$$