

Dynamic arrays: dealing with $\Phi = -\text{size}_i$ assuming $\Phi_0 = 0$

Dynamic arrays: dealing with $\phi = -\text{size}_i$ assuming $\phi_0 = 0$

The problem: $\phi_n - \phi_0 < 0$

Dynamic arrays: dealing with $\phi = -\text{size}_i$ assuming $\phi_0 = 0$

The problem: $\phi_n - \phi_0 < 0$

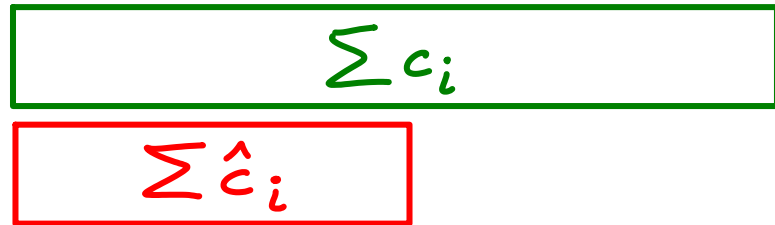
$$\sum \hat{c}_i = \sum c_i + \phi_n - \phi_0$$

Dynamic arrays: dealing with $\phi = -\text{size}_i$ assuming $\phi_0 = 0$

The problem: $\phi_n - \phi_0 < 0$
 $\sum \hat{c}_i = \sum c_i + \phi_n - \phi_0$ } $\sum \hat{c}_i < \sum c_i$

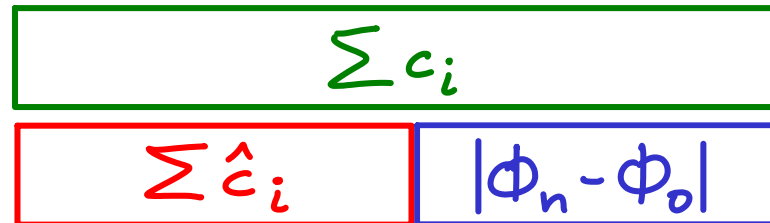
Dynamic arrays: dealing with $\phi = -\text{size}_i$ assuming $\phi_0 = 0$

The problem: $\phi_n - \phi_0 < 0$
 $\sum \hat{c}_i = \sum c_i + \phi_n - \phi_0$ } $\underline{\sum \hat{c}_i} < \underline{\sum c_i}$



Dynamic arrays: dealing with $\phi = -\text{size}_i$ assuming $\phi_0 = 0$

The problem: $\phi_n - \phi_0 < 0$
 $\sum \hat{c}_i = \sum c_i + \underline{\phi_n - \phi_0}$ } $\sum \hat{c}_i < \sum c_i$

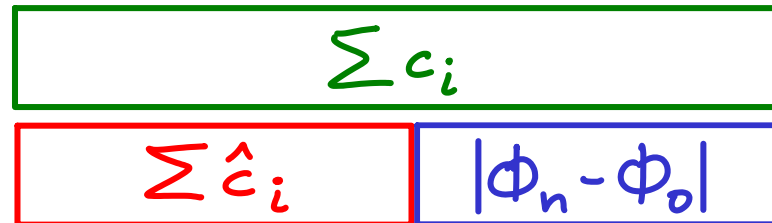


Dynamic arrays: dealing with $\phi = -\text{size}_i$ assuming $\phi_0 = 0$

The problem: $\phi_n - \phi_0 < 0$
 $\left. \begin{array}{l} \phi_n - \phi_0 < 0 \\ \sum \hat{c}_i = \sum c_i + \phi_n - \phi_0 \end{array} \right\} \sum \hat{c}_i < \sum c_i$

operation type:

• cheap: $c_i = 1, \Delta\phi_i = 0 \rightarrow \hat{c}_i = 1$

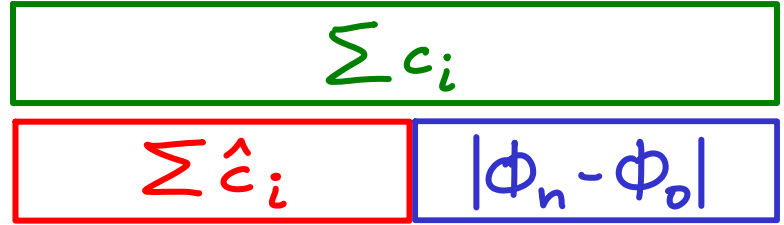


Dynamic arrays: dealing with $\phi = -\text{size}_i$ assuming $\phi_0 = 0$

The problem: $\phi_n - \phi_0 < 0$
 $\left. \begin{array}{l} \phi_n - \phi_0 < 0 \\ \sum \hat{c}_i = \sum c_i + \phi_n - \phi_0 \end{array} \right\} \sum \hat{c}_i < \sum c_i$

operation type:

- cheap: $c_i = 1, \Delta\phi_i = 0 \rightarrow \hat{c}_i = 1$
- expensive: $c_i = i, \Delta\phi_i = ?$



Dynamic arrays: dealing with $\Phi = -\text{size}_i$ assuming $\Phi_0 = 0$

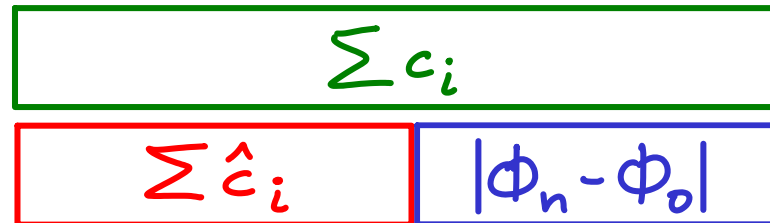
The problem: $\Phi_n - \Phi_0 < 0$
 $\left. \begin{aligned} \sum \hat{c}_i &= \sum c_i + \underbrace{\Phi_n - \Phi_0} \end{aligned} \right\} \sum \hat{c}_i < \sum c_i$

operation type:

• cheap: $c_i = 1, \Delta\Phi_i = 0 \rightarrow \hat{c}_i = 1$

• expensive: $c_i = i, \Delta\Phi_i = \underline{-(i-1)}$

array size: $i-1 \rightarrow 2(i-1)$



Dynamic arrays: dealing with $\Phi = -\text{size}_i$ assuming $\Phi_0 = 0$

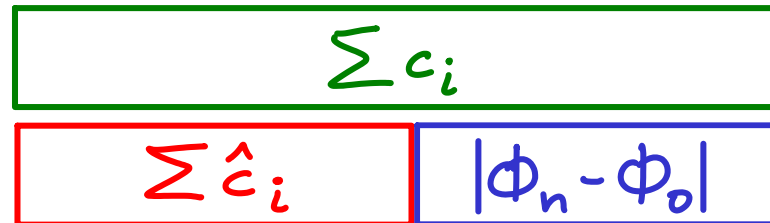
The problem: $\Phi_n - \Phi_0 < 0$
 $\left. \begin{array}{l} \sum \hat{c}_i = \sum c_i + \underbrace{\Phi_n - \Phi_0} \end{array} \right\} \sum \hat{c}_i < \sum c_i$

operation type:

• cheap: $c_i = 1, \Delta\Phi_i = 0 \rightarrow \hat{c}_i = 1$

• expensive: $c_i = i, \Delta\Phi_i = -(i-1) = \underline{1 - c_i}$

array size: $i-1 \rightarrow 2(i-1)$



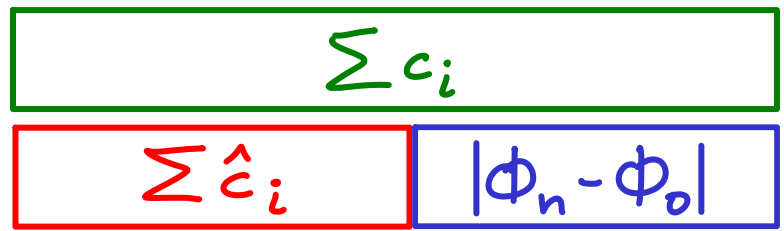
Dynamic arrays: dealing with $\Phi = -\text{size}_i$ assuming $\Phi_0 = 0$

The problem: $\Phi_n - \Phi_0 < 0$
 $\left. \begin{array}{l} \Phi_n - \Phi_0 < 0 \\ \sum \hat{c}_i = \sum c_i + \Phi_n - \Phi_0 \end{array} \right\} \sum \hat{c}_i < \sum c_i$

operation type:

- cheap: $c_i = 1, \Delta\Phi_i = 0 \rightarrow \hat{c}_i = 1$
 - expensive: $c_i = i, \Delta\Phi_i = -(i-1) = 1 - c_i \rightarrow \hat{c}_i = 1$
- $\left. \begin{array}{l} \text{cheap} \\ \text{expensive} \end{array} \right\} \sum \hat{c}_i ?$

array size: $i-1 \rightarrow 2(i-1)$



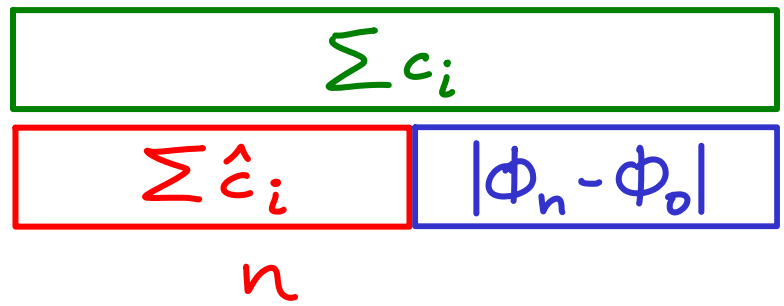
Dynamic arrays: dealing with $\Phi = -\text{size}_i$ assuming $\Phi_0 = 0$

The problem: $\Phi_n - \Phi_0 < 0$
 $\left. \begin{array}{l} \Phi_n - \Phi_0 < 0 \\ \sum \hat{c}_i = \sum c_i + \Phi_n - \Phi_0 \end{array} \right\} \sum \hat{c}_i < \sum c_i$

operation type:

- cheap: $c_i = 1, \Delta\Phi_i = 0 \rightarrow \hat{c}_i = 1$
 - expensive: $c_i = i, \Delta\Phi_i = -(i-1) = 1 - c_i \rightarrow \hat{c}_i = 1$
- $\left. \begin{array}{l} \text{cheap} \\ \text{expensive} \end{array} \right\} \sum \hat{c}_i = n$

array size: $i-1 \rightarrow 2(i-1)$



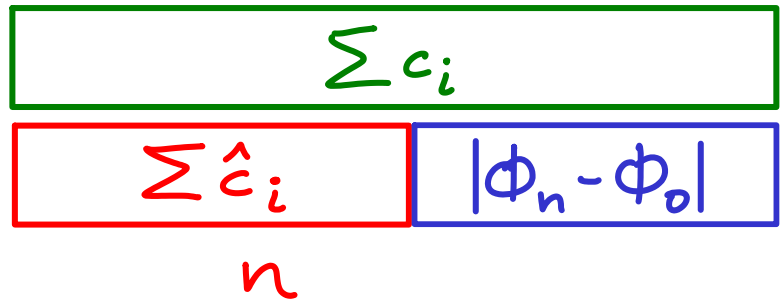
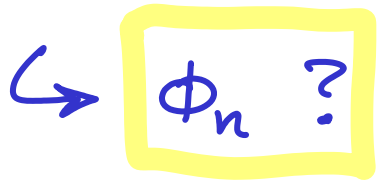
Dynamic arrays: dealing with $\Phi = -\text{size}_i$ assuming $\Phi_0 = 0$

The problem: $\Phi_n - \Phi_0 < 0$
 $\left. \begin{array}{l} \Phi_n - \Phi_0 < 0 \\ \sum \hat{c}_i = \sum c_i + \Phi_n - \Phi_0 \end{array} \right\} \sum \hat{c}_i < \sum c_i$

operation type:

- cheap: $c_i = 1, \Delta\Phi_i = 0 \rightarrow \hat{c}_i = 1$
 - expensive: $c_i = i, \Delta\Phi_i = -(i-1) = 1 - c_i \rightarrow \hat{c}_i = 1$
- $\left. \begin{array}{l} \text{cheap} \\ \text{expensive} \end{array} \right\} \sum \hat{c}_i = n$

array size: $i-1 \rightarrow 2(i-1)$



Dynamic arrays: dealing with $\Phi = -\text{size}_i$ assuming $\Phi_0 = 0$

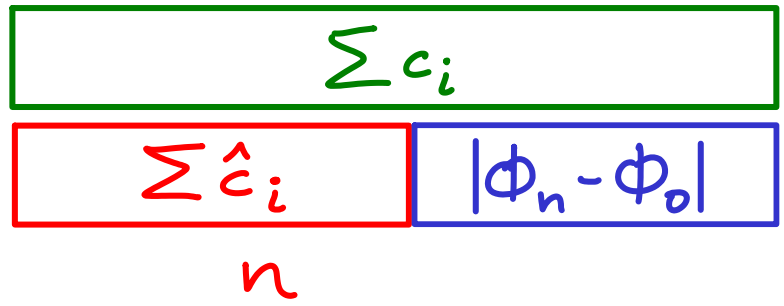
The problem: $\Phi_n - \Phi_0 < 0$
 $\left. \begin{array}{l} \sum \hat{c}_i = \sum c_i + \Phi_n - \Phi_0 \end{array} \right\} \sum \hat{c}_i < \sum c_i$

operation type:

- cheap: $c_i = 1, \Delta\Phi_i = 0 \rightarrow \hat{c}_i = 1$
 - expensive: $c_i = i, \Delta\Phi_i = -(i-1) = 1 - c_i \rightarrow \hat{c}_i = 1$
- $\left. \right\} \sum \hat{c}_i = n$

array size: $i-1 \rightarrow 2(i-1)$

$\hookrightarrow \Phi_n \geq -2(n-1)$



Dynamic arrays: dealing with $\Phi = -\text{size}_i$ assuming $\Phi_0 = 0$

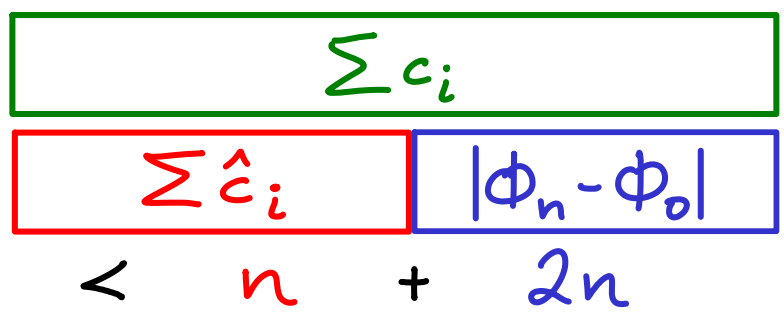
The problem: $\Phi_n - \Phi_0 < 0$
 $\left. \begin{array}{l} \sum \hat{c}_i = \sum c_i + \Phi_n - \Phi_0 \\ \sum \hat{c}_i < \sum c_i \end{array} \right\}$

operation type:

- cheap: $c_i = 1, \Delta\Phi_i = 0 \rightarrow \hat{c}_i = 1$
 - expensive: $c_i = i, \Delta\Phi_i = -(i-1) = 1 - c_i \rightarrow \hat{c}_i = 1$
- $\left. \right\} \sum \hat{c}_i = n$

array size: $i-1 \rightarrow 2(i-1)$

$\hookrightarrow \Phi_n \geq -2(n-1)$
 $|\Phi_n - \Phi_0| < 2n$



Dynamic arrays: dealing with $\Phi = -\text{size}_i$ assuming $\Phi_0 = 0$

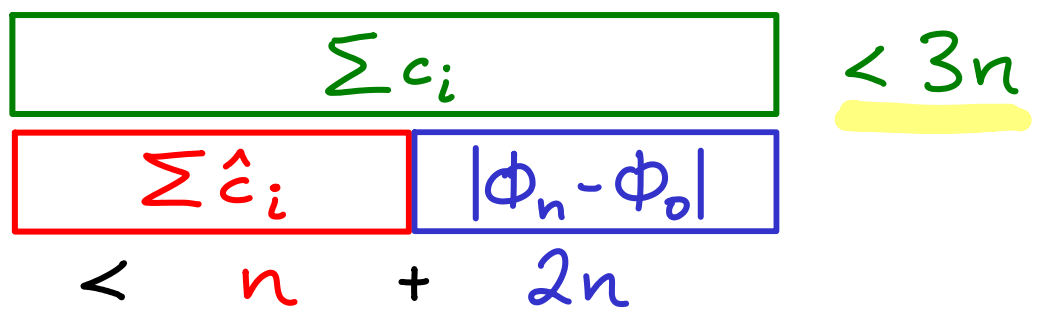
The problem: $\Phi_n - \Phi_0 < 0$
 $\left. \begin{aligned} \sum \hat{c}_i &= \sum c_i + \Phi_n - \Phi_0 \\ \sum \hat{c}_i &< \sum c_i \end{aligned} \right\}$

operation type:

- cheap: $c_i = 1, \Delta\Phi_i = 0 \rightarrow \hat{c}_i = 1$
 - expensive: $c_i = i, \Delta\Phi_i = -(i-1) = 1 - c_i \rightarrow \hat{c}_i = 1$
- $\left. \right\} \sum \hat{c}_i = n$

array size: $i-1 \rightarrow 2(i-1)$

$\hookrightarrow \Phi_n \geq -2(n-1)$
 $|\Phi_n - \Phi_0| < 2n$



For any amortized analysis:

if $\phi_n - \phi_0 < 0$

calculate the maximum possible $|\phi_n - \phi_0|$

& add to your upper bound for $\sum \hat{c}_i$

$$\sum c_i$$

$$\sum \hat{c}_i \quad |\phi_n - \phi_0|$$

