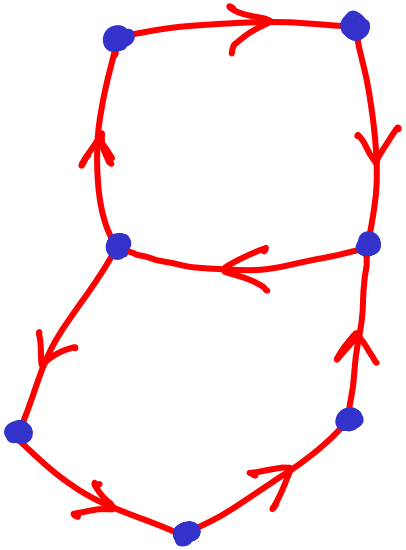
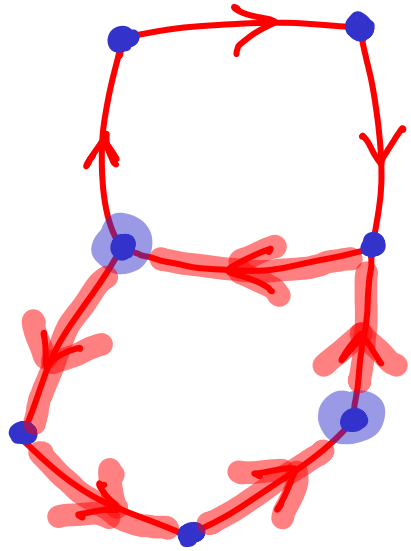


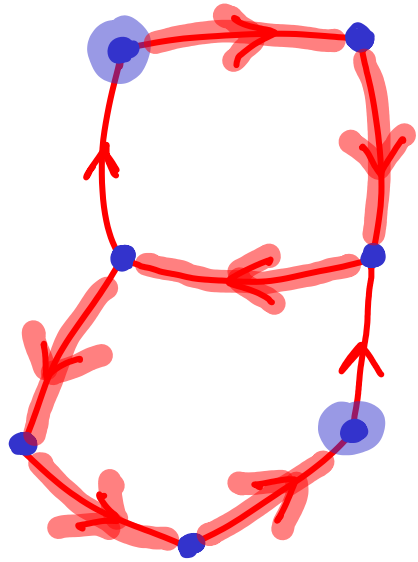
# STRONGLY CONNECTED GRAPHS

(directed) graph s.t. every vertex can be reached from every vertex.

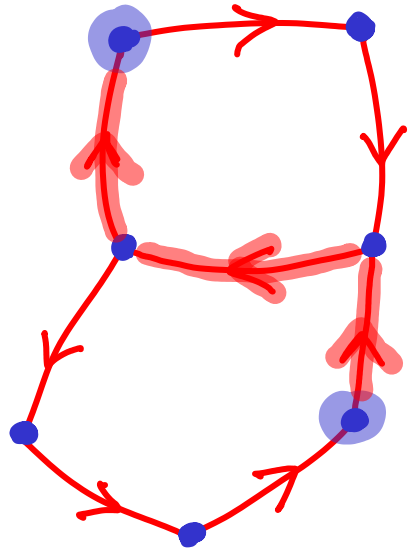




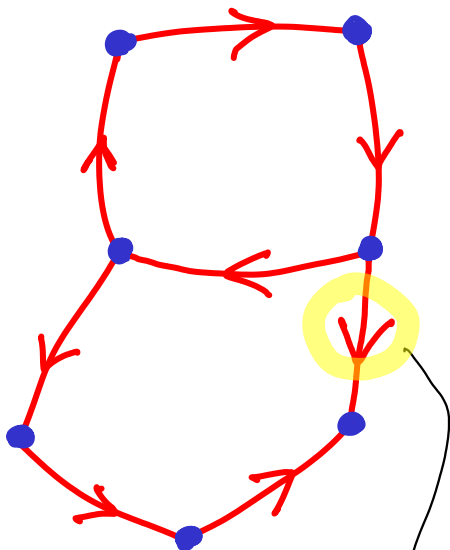
(directed) graph s.t. every vertex can be reached from every vertex.



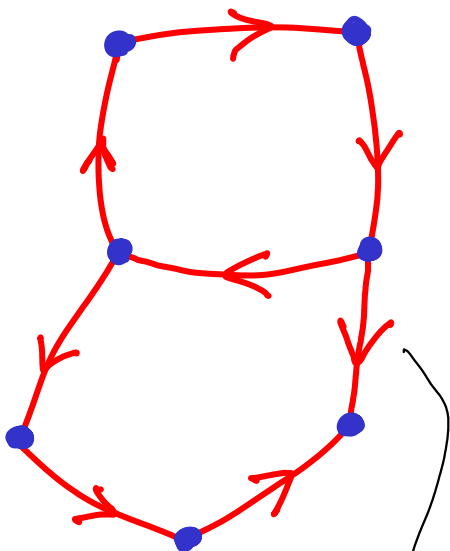
(directed) graph s.t. every vertex can be reached from every vertex.



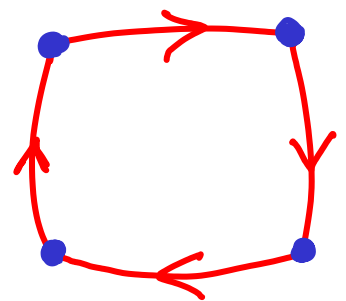
(directed) graph s.t. every vertex can be reached from every vertex.



no longer  
strongly  
connected

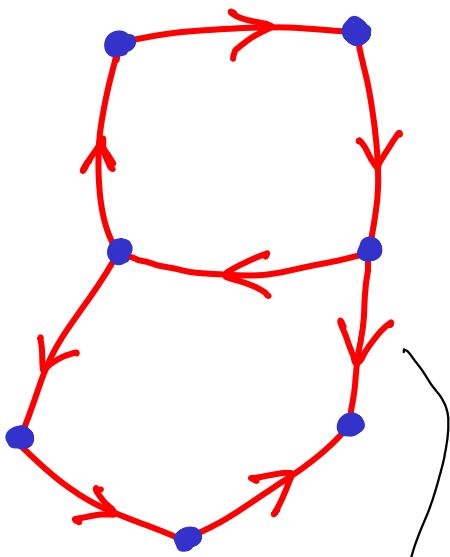


no longer  
strongly  
connected

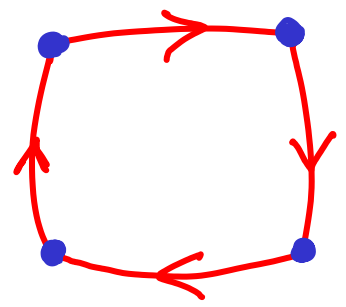


chain reaction:  
several vertices  
no longer  
mutually reachable

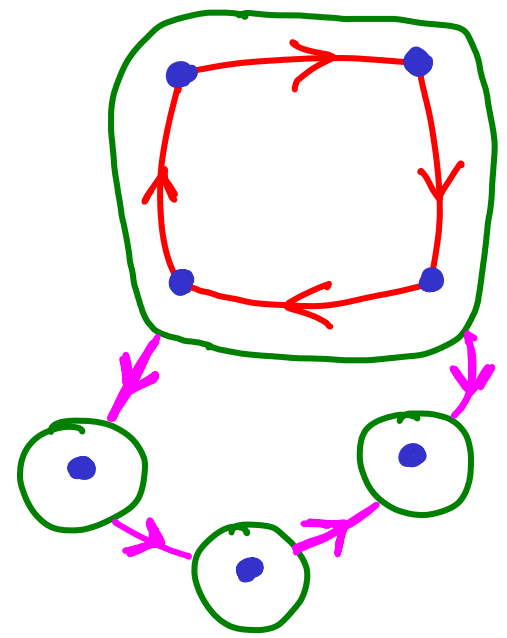
# STRONGLY CONNECTED COMPONENTS



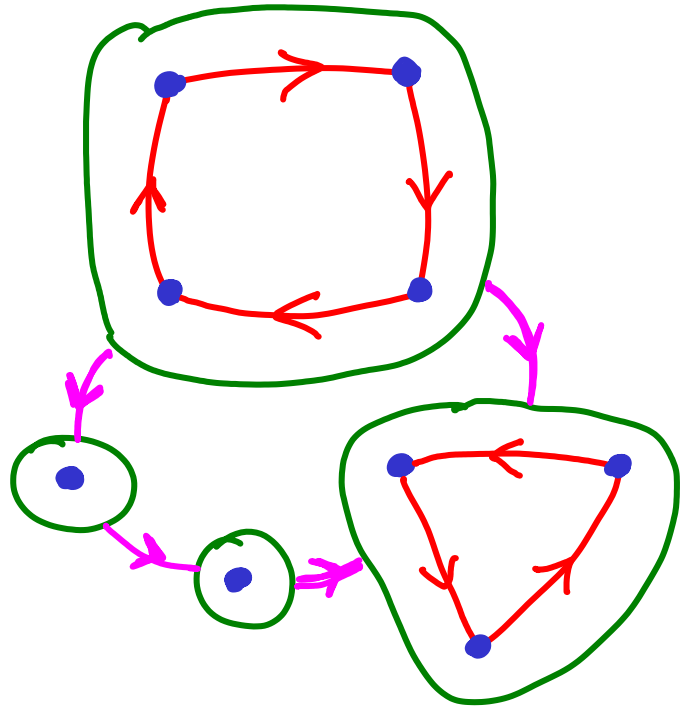
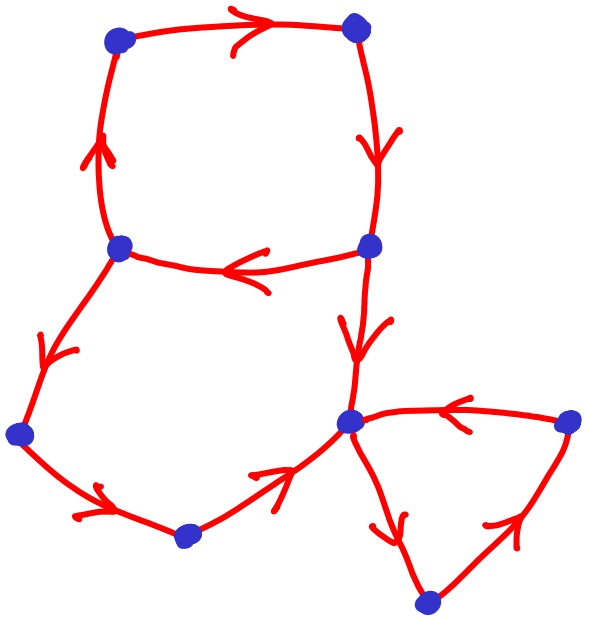
no longer strongly connected



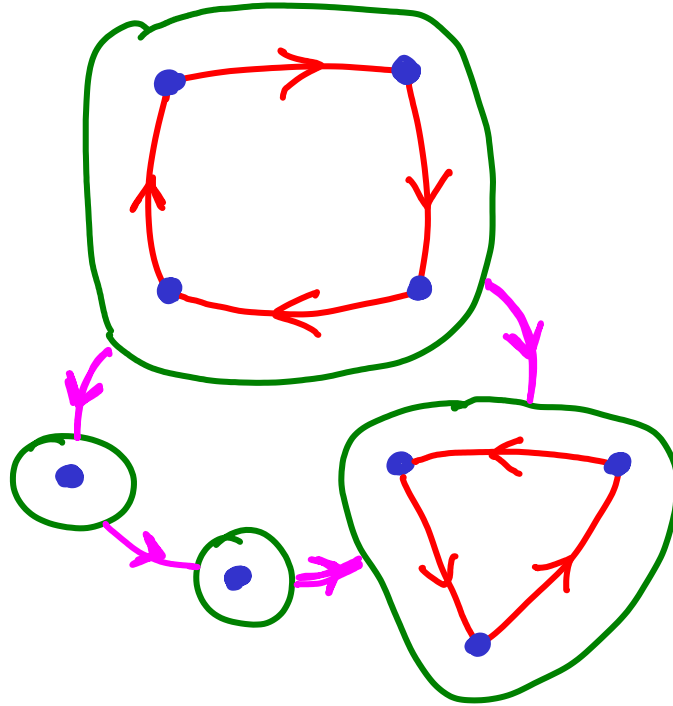
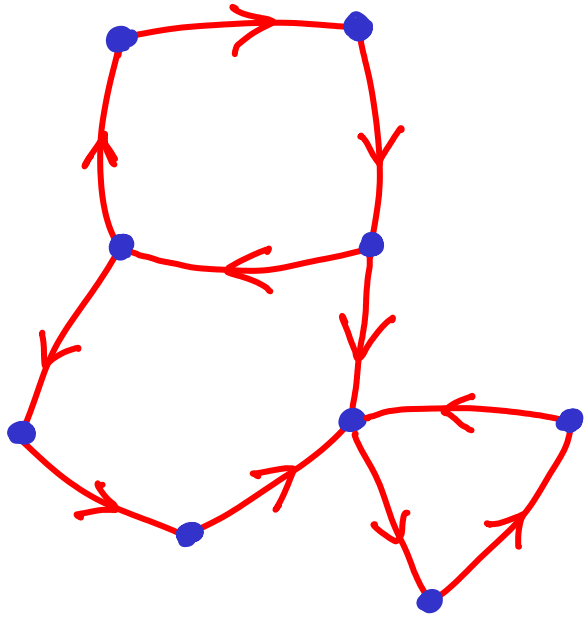
chain reaction:  
several vertices  
no longer mutually reachable



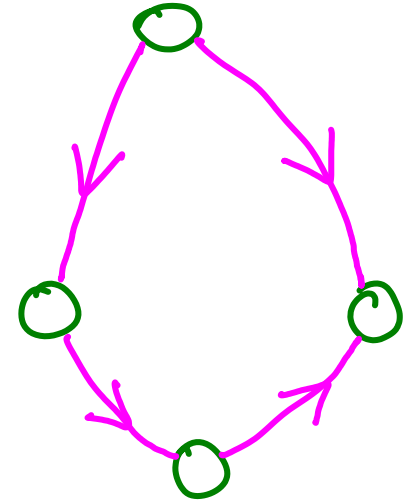
we get groups (components)  
each of which is strongly connected





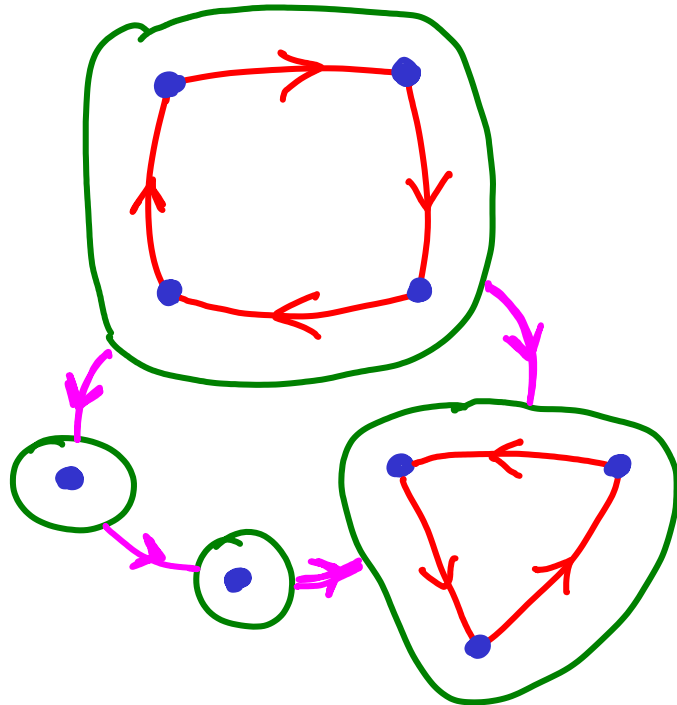
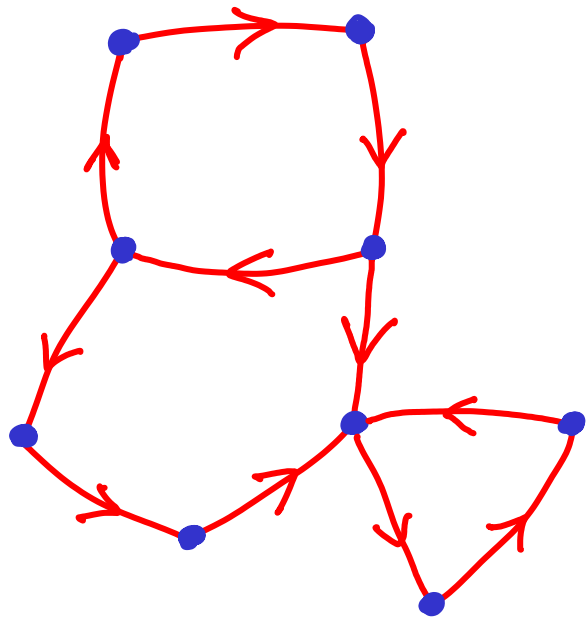


new graph:

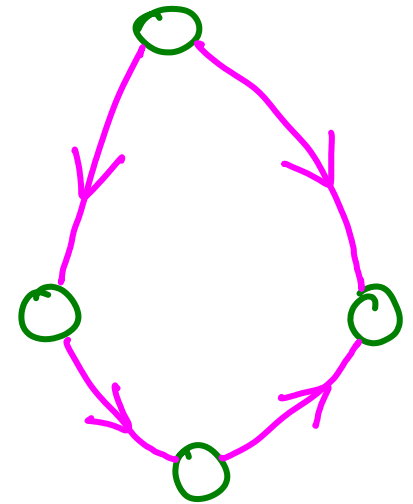


each vertex  
represents a S.C.C.

What type of graph  
is this?



new graph:

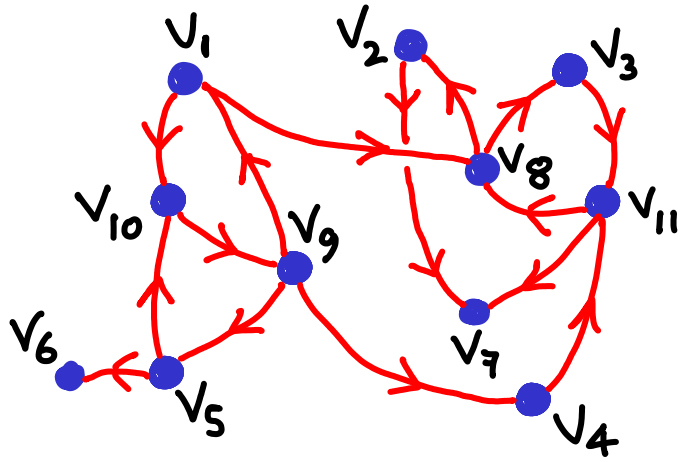


each vertex  
represents a S.C.C.

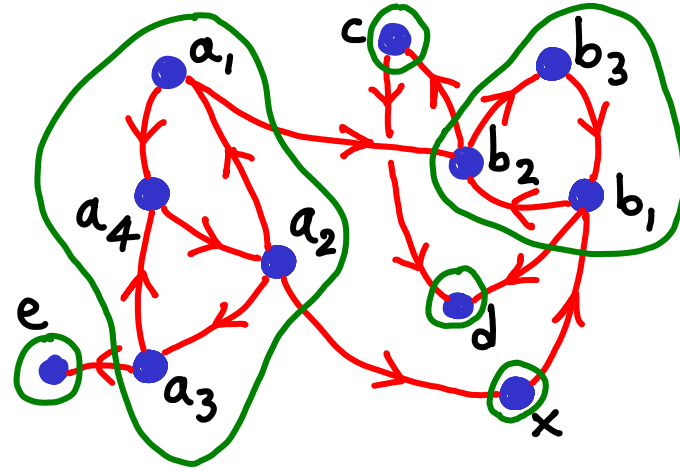
The new graph must be a DAG

↳ (any cycle would merge components)

# FINDING ALL STRONGLY CONNECTED COMPONENTS

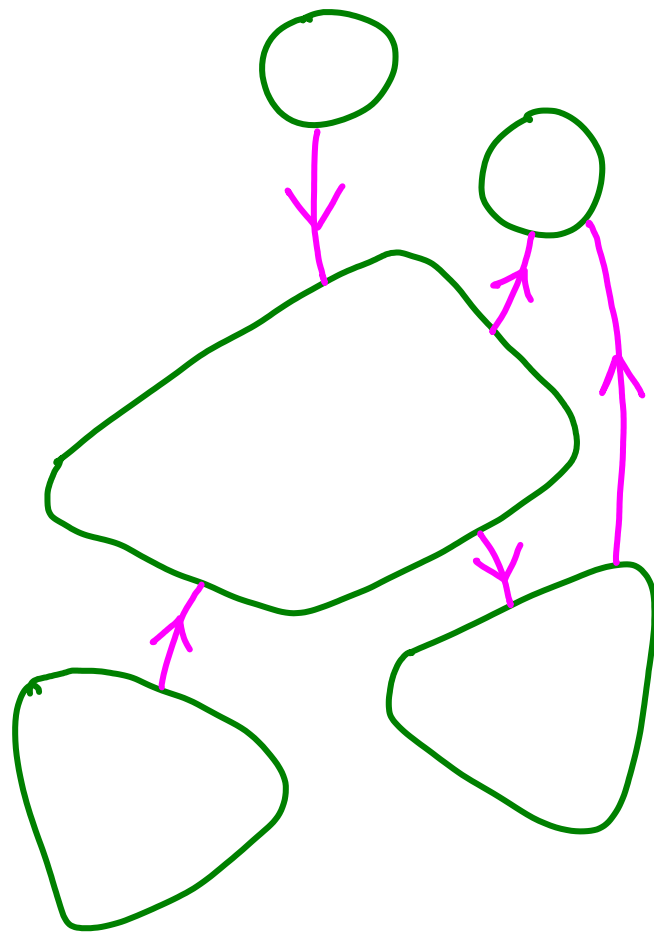
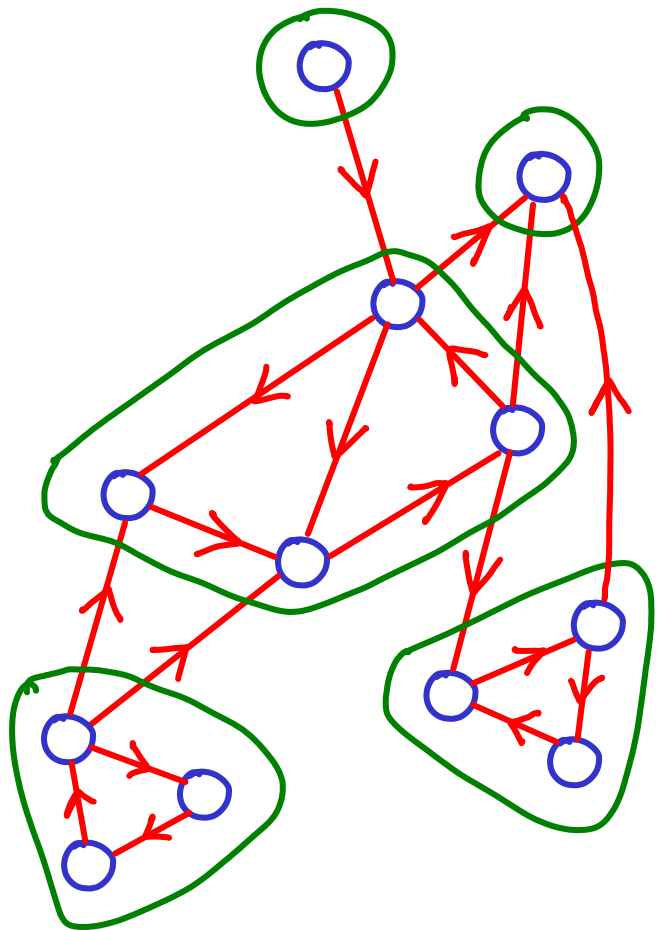


what we see



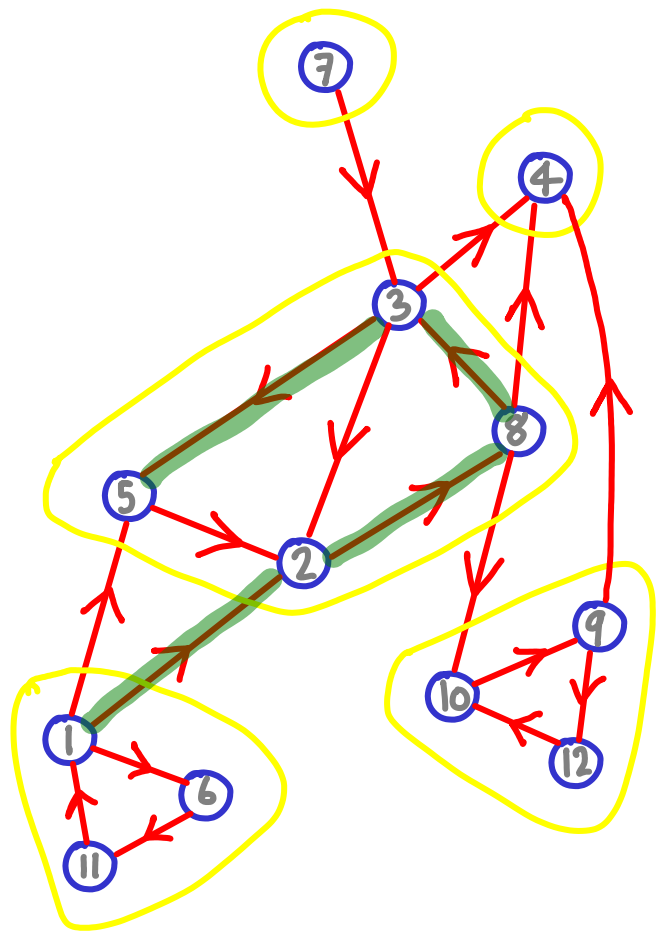
what we want

# FINDING STRONGLY CONNECTED COMPONENTS



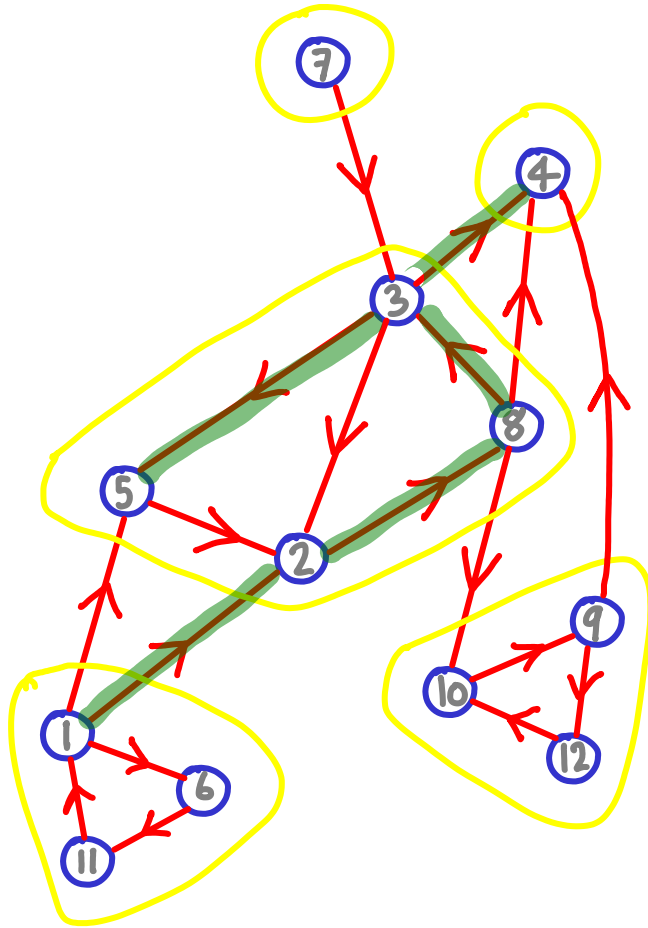
5  
finished

DFS 1 → 2 → 8 → 3 → 5



DFS from arbitrary vertex

4 5  
finished

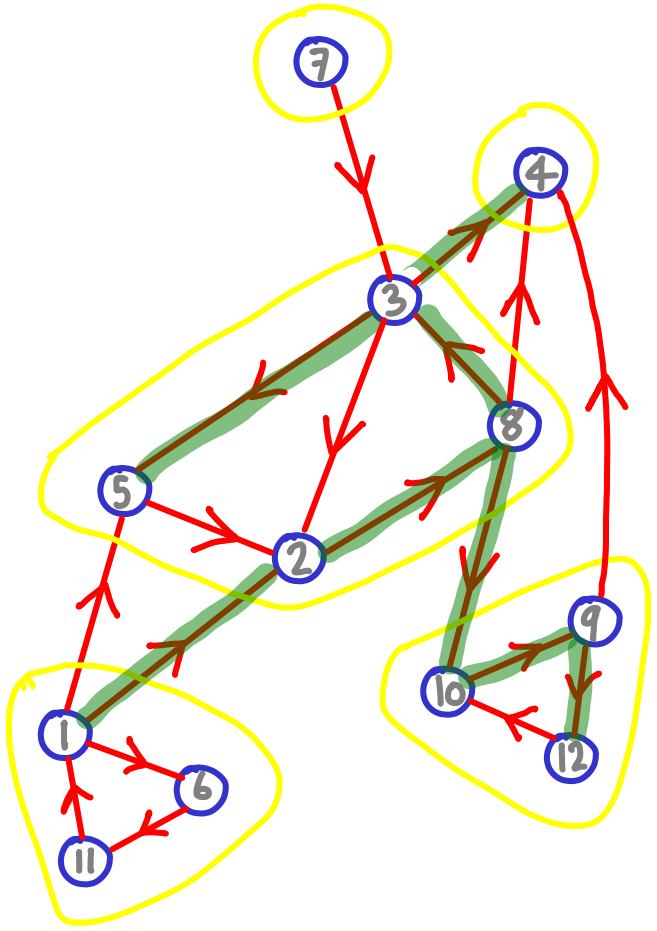


DFS 1 → 2 → 8 → 3 → 5  
↙  
3 → 4

DFS from arbitrary vertex



2 8 10 9 12 3 4 5  
finished



DFS

1 → 2 → 8 → 3 → 5

3 → 4

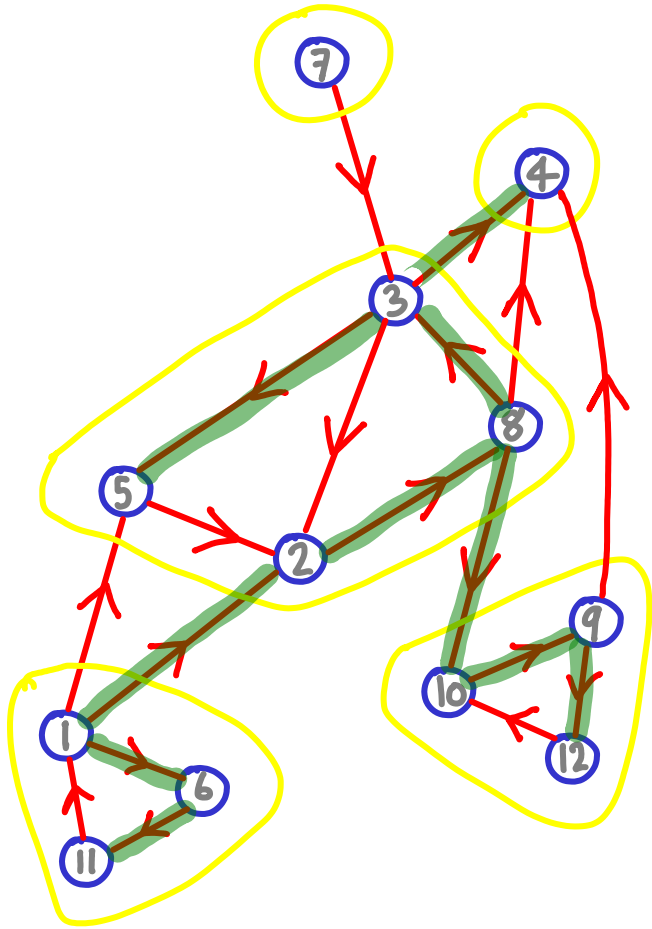
8 → 10 → 9 → 12

1  
2

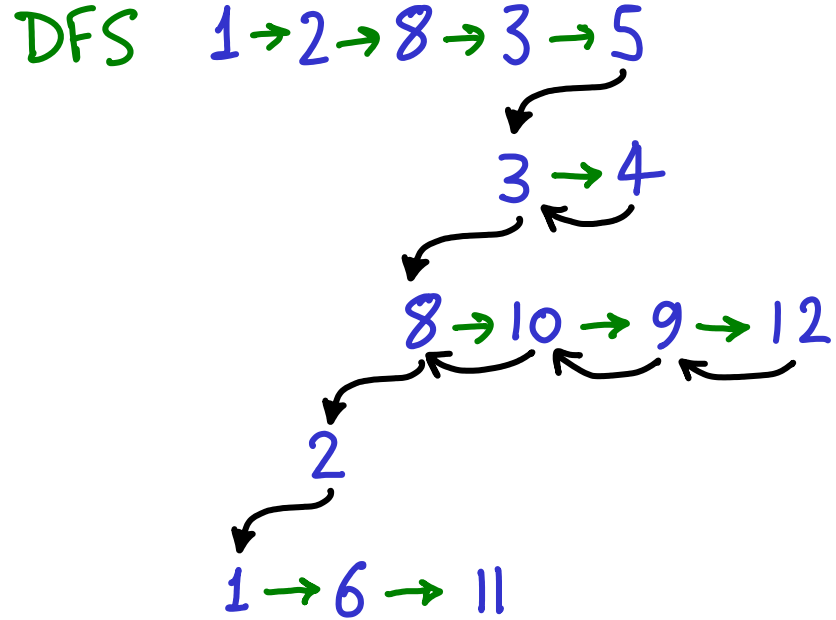
DFS from arbitrary vertex



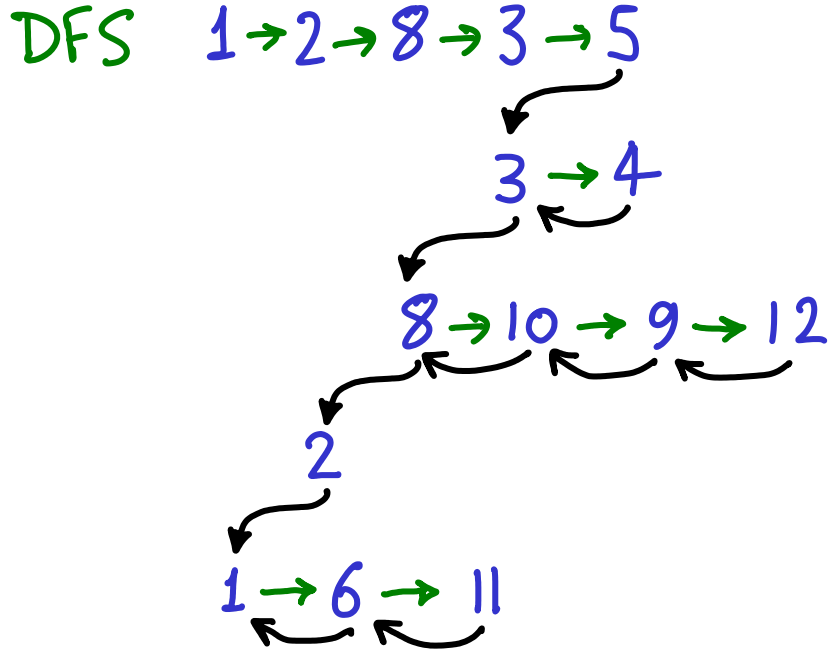
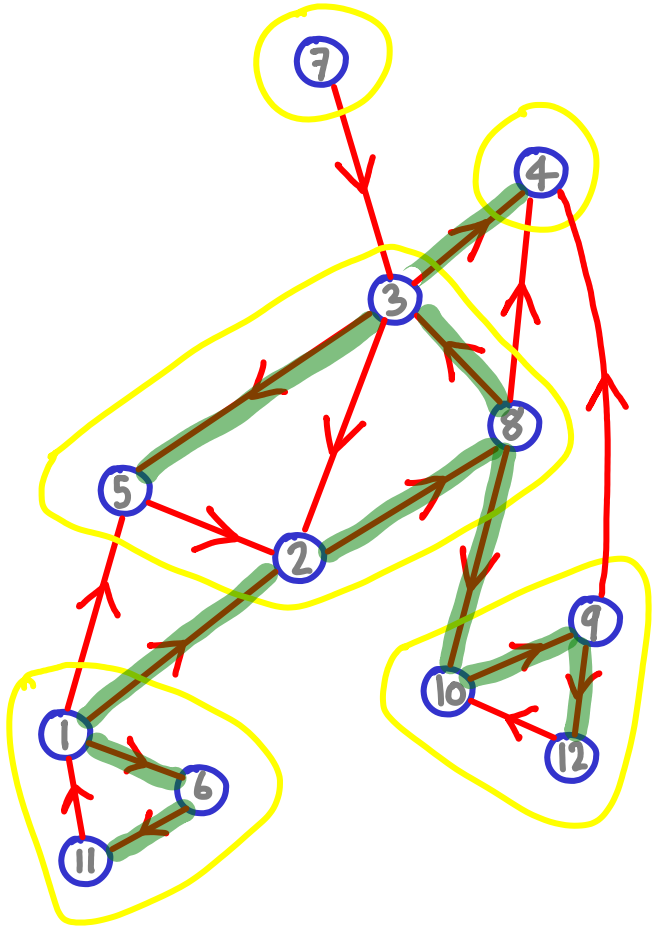
2 8 10 9 12 3 4 5  
finished



DFS from arbitrary vertex

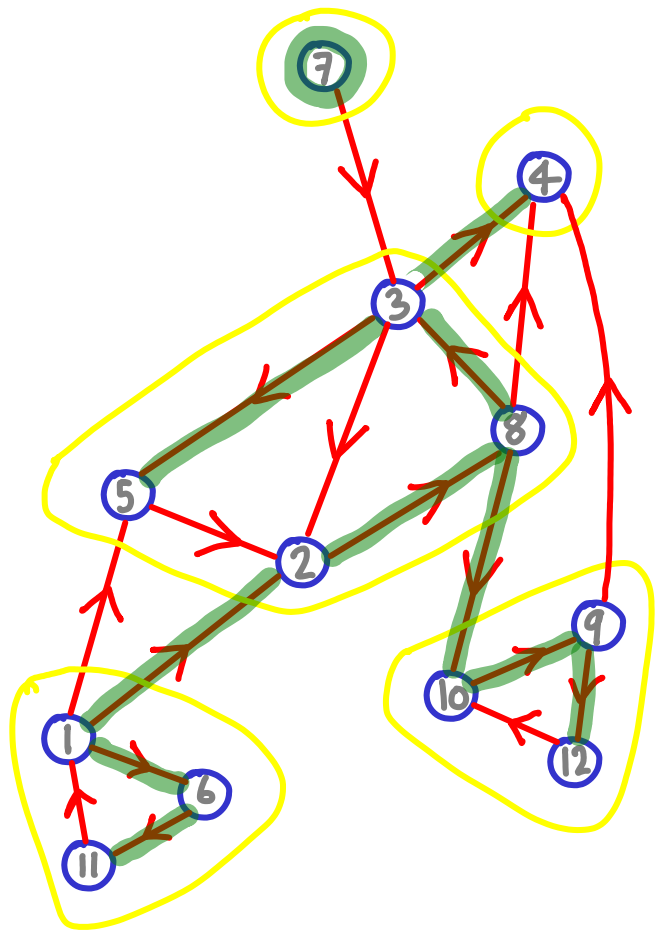


1 6 11 2 8 10 9 12 3 4 5  
 finished

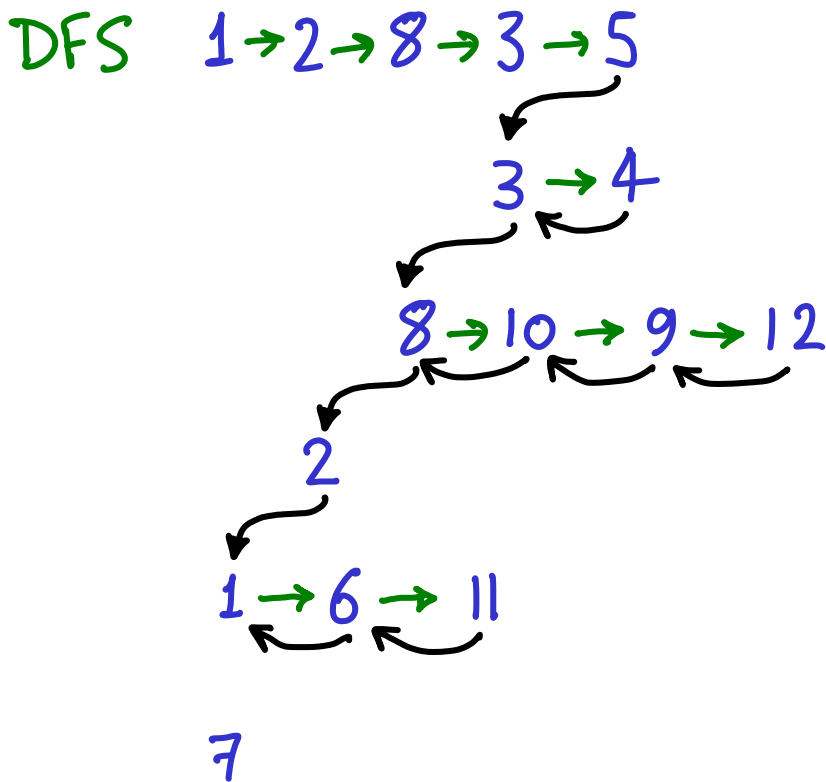


DFS from arbitrary vertex

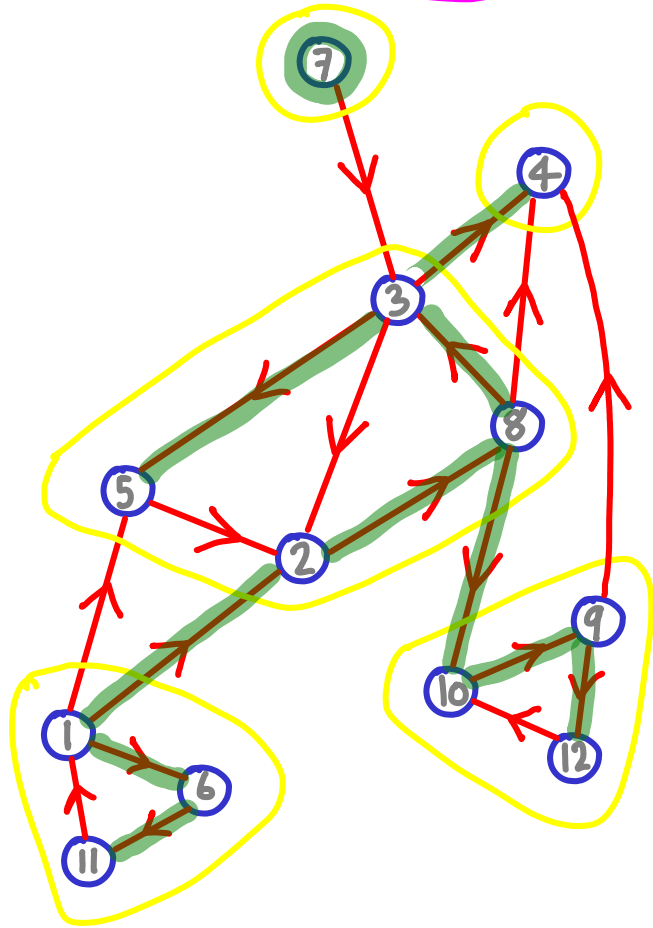
7 1 6 11 2 8 10 9 12 3 4 5  
finished



DFS from arbitrary vertex



Finishing times: 7 1 6 11 2 8 10 9 12 3 4 5  
finished first

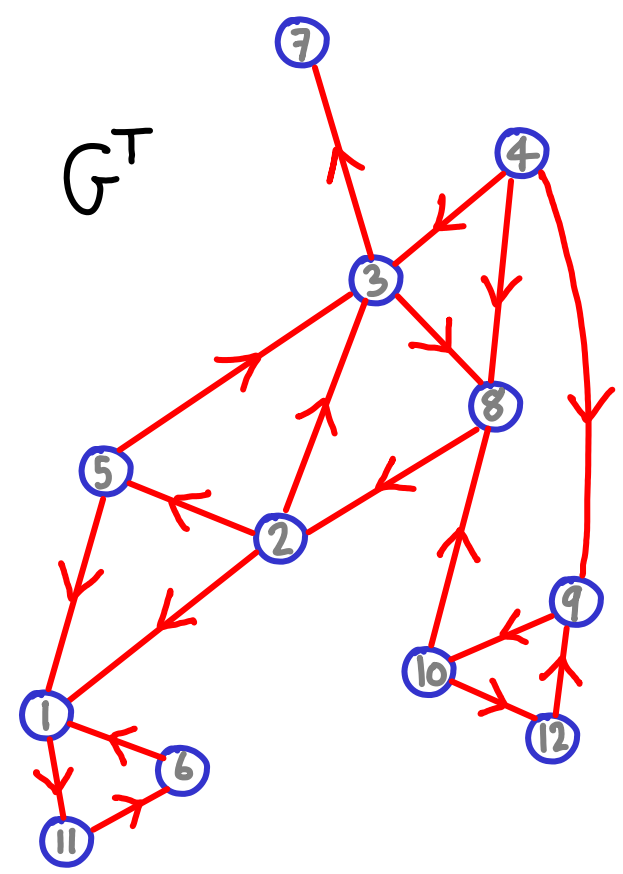
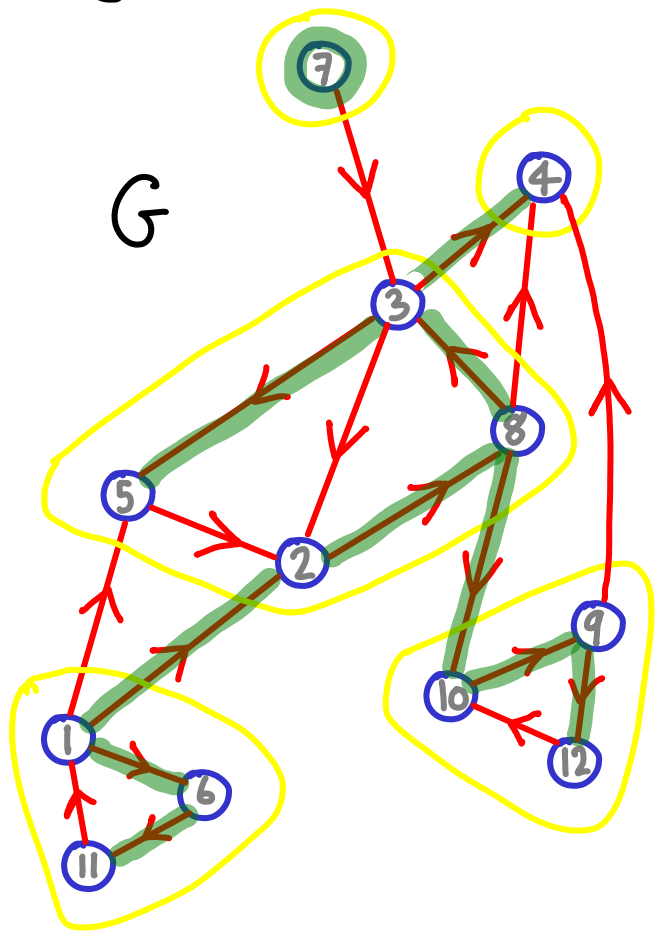


Finishing times don't give us the SCC but they help a lot.

DFS from arbitrary vertex

Finishing times: 7 1 6 11 2 8 10 9 12 3 4 5

finished first

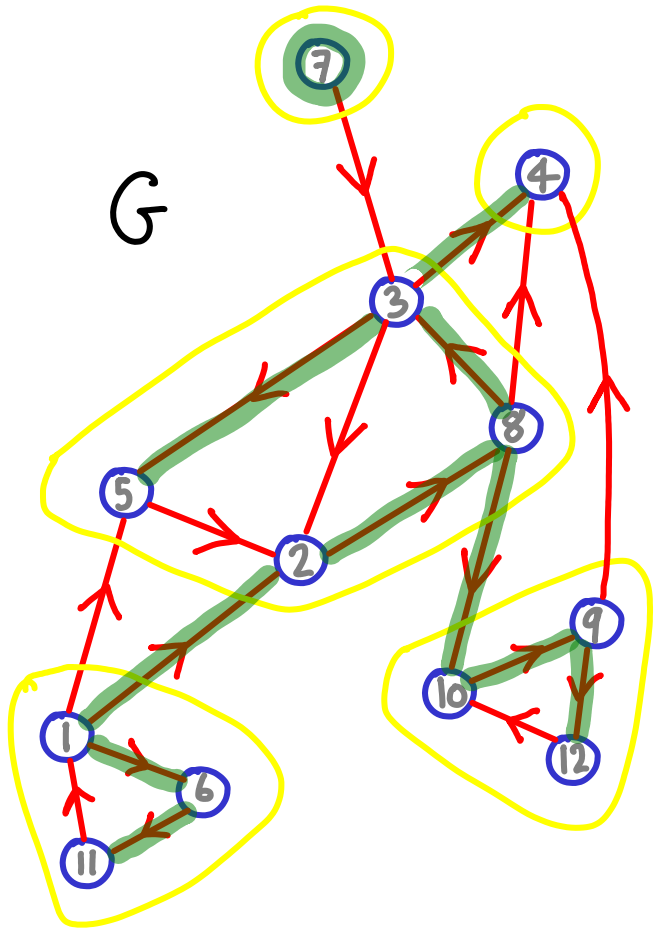


DFS from arbitrary vertex

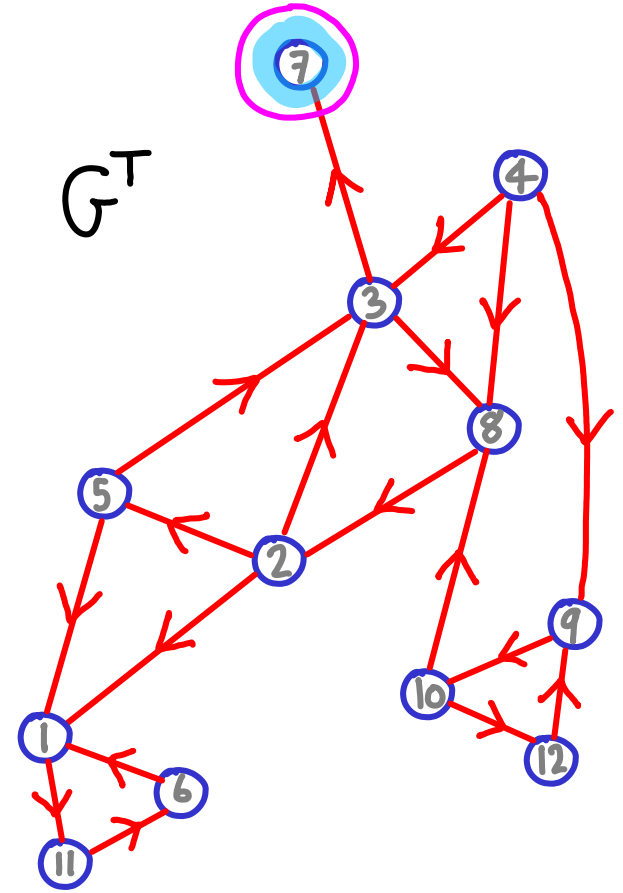
Finishing times: 7 1 6 11 2 8 10 9 12 3 4 5

finished first

DFS( $G^T$ ) in this order



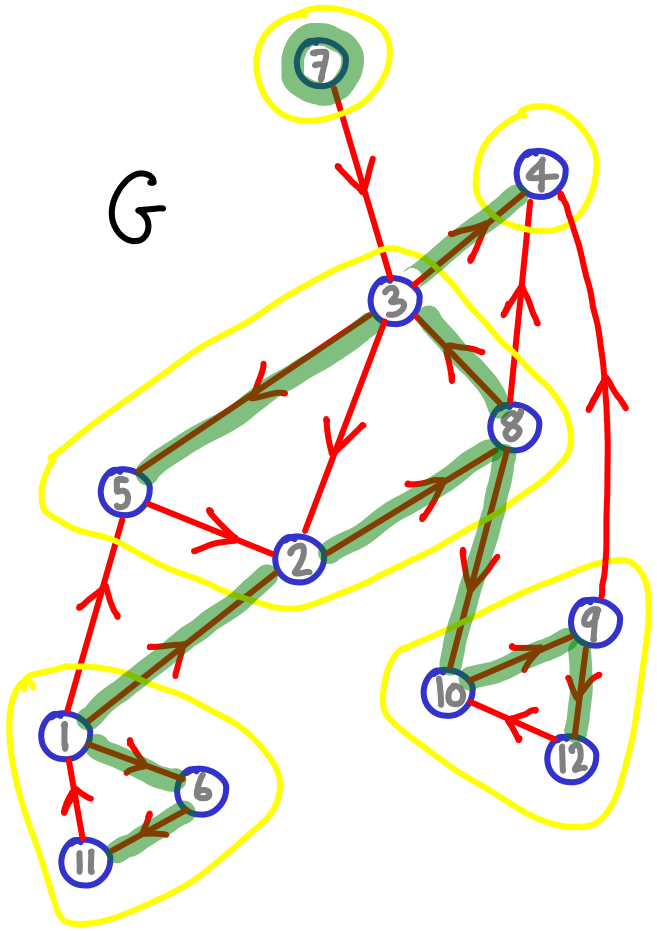
DFS from arbitrary vertex



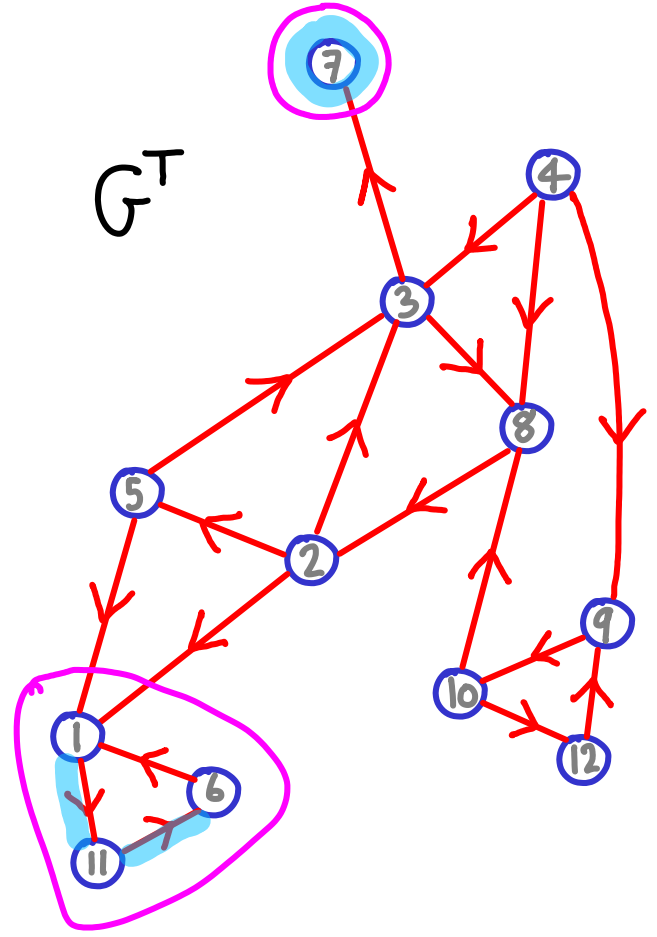
Finishing times: 7 1 6 11 2 8 10 9 12 3 4 5

finished first

DFS( $G^T$ ) in this order

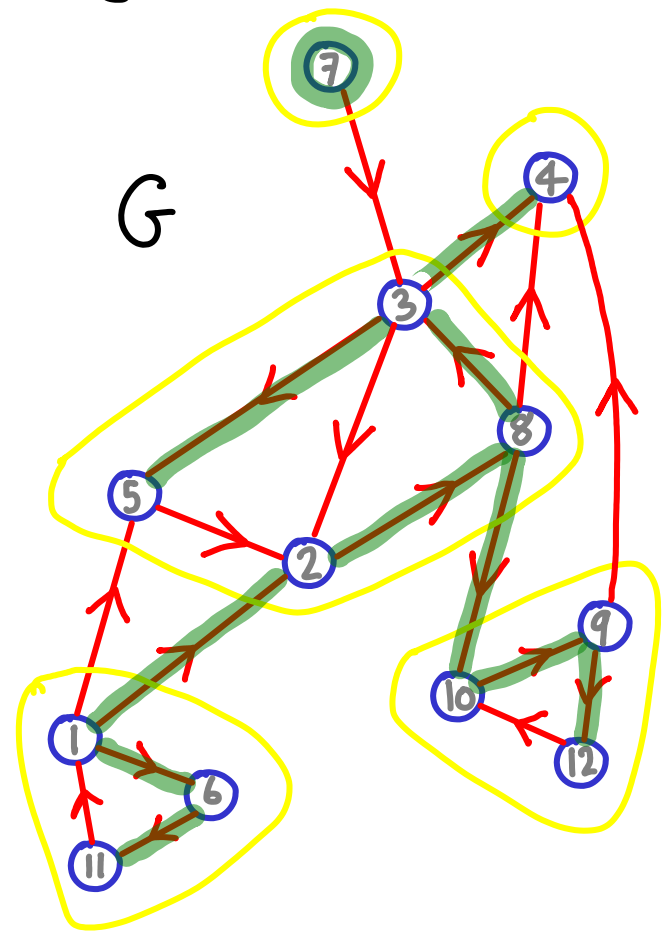


DFS from arbitrary vertex

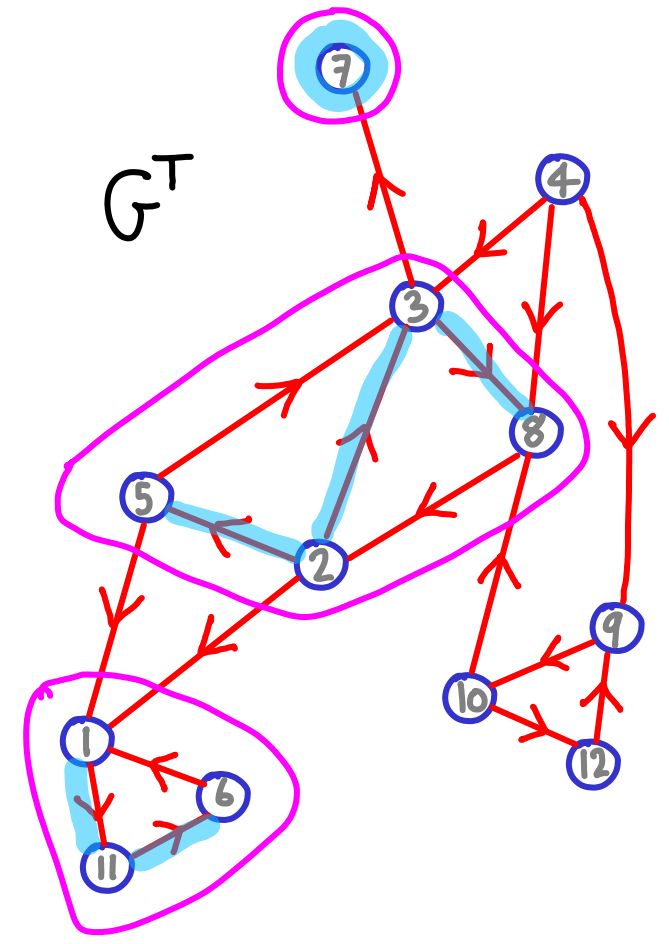


Finishing times:  $\boxed{7}$   $\boxed{1\ 6\ 11}$   $\boxed{2\ 8\ 10\ 9\ 12}$   $\boxed{3}$   $\boxed{4}$   $\boxed{5}$  finished first

DFS( $G^T$ ) in this order



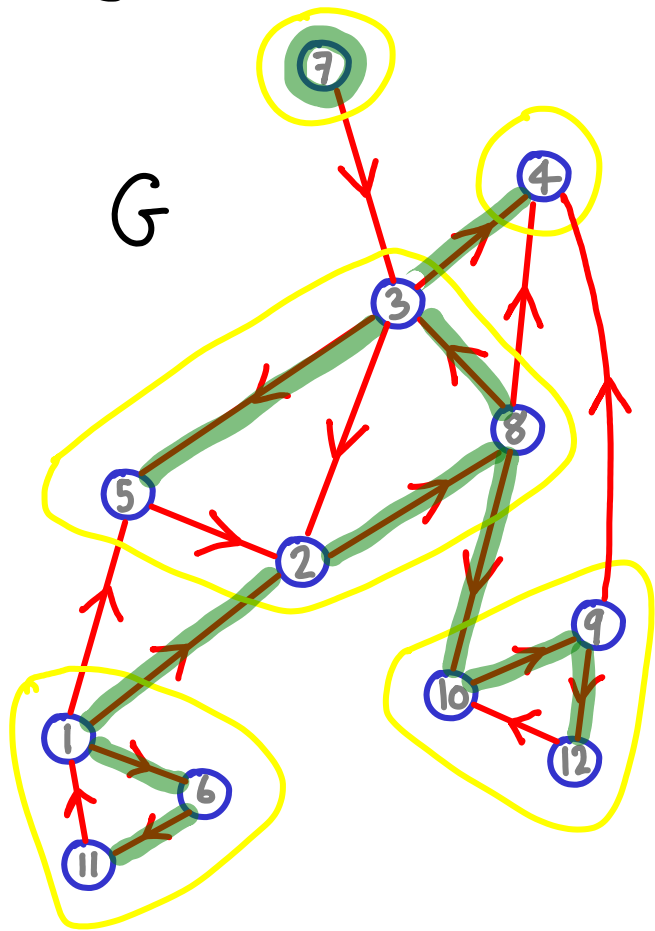
DFS from arbitrary vertex



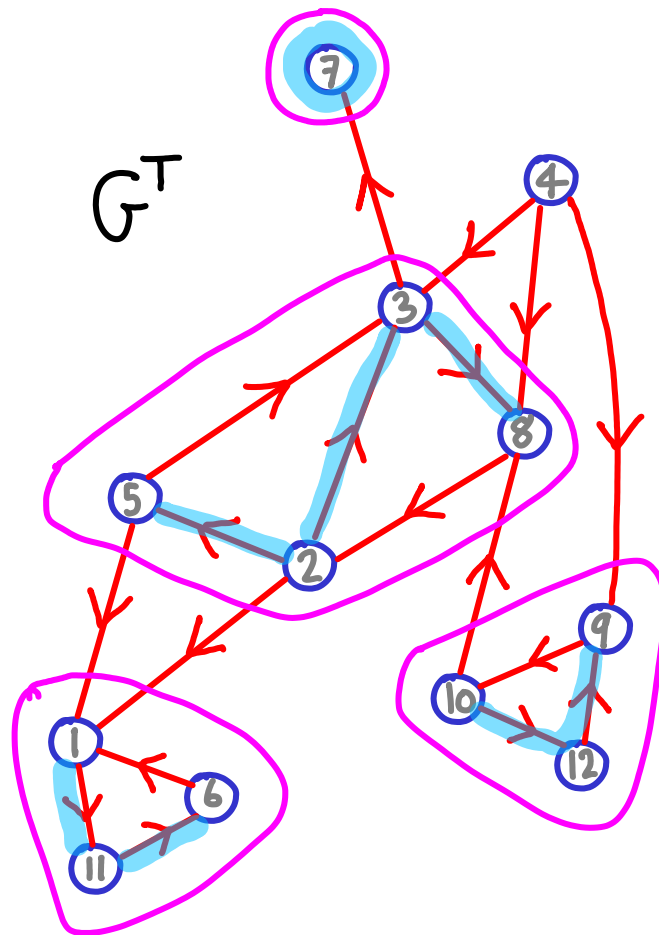


Finishing times:  $\boxed{7}$   $\boxed{1\ 6\ 11}$   $\boxed{2\ 8}$   $\boxed{10\ 9\ 12}$   $\boxed{3}$   $\boxed{4}$   $\boxed{5}$  finished first

DFS( $G^T$ ) in this order



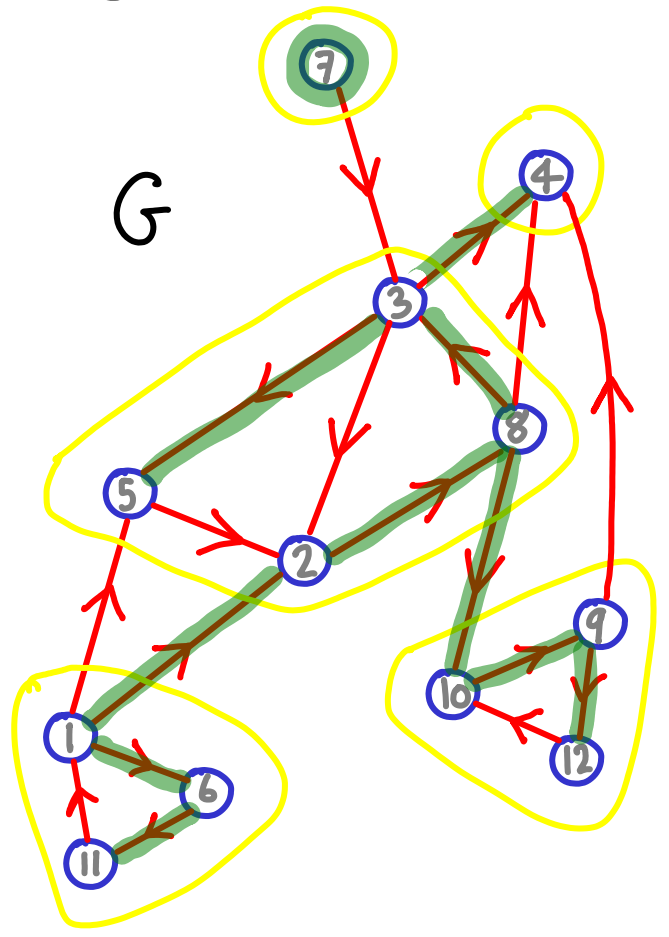
DFS from arbitrary vertex



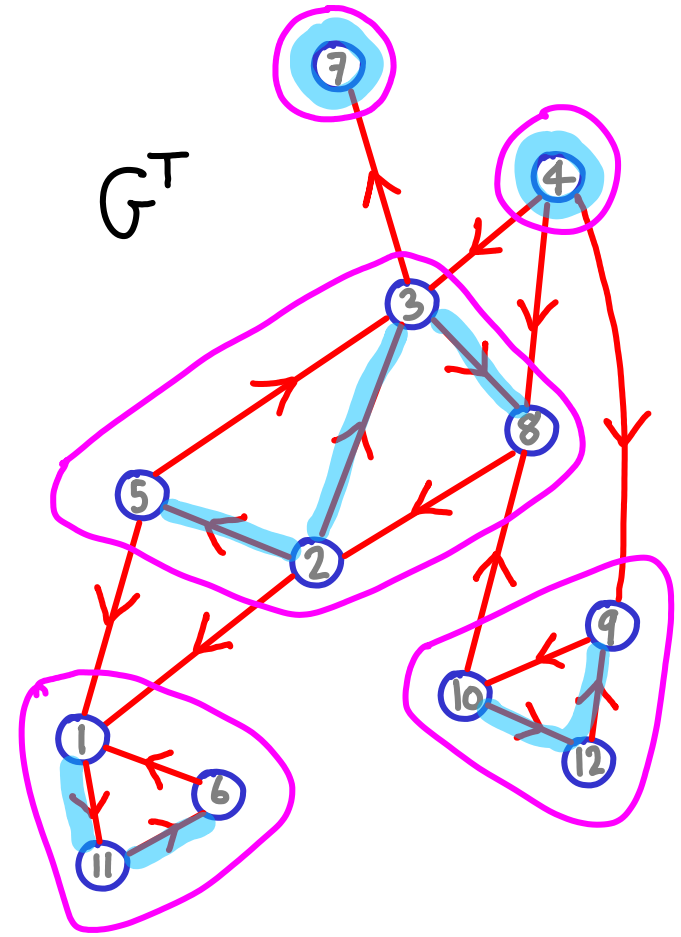
Finishing times:  $\{7\}$   $\{1, 6, 11\}$   $\{2, 8\}$   $\{10, 9, 12\}$   $\{3\}$   $\{4\}$   $\{5\}$

DFS( $G^T$ ) in this order

finished first



DFS from arbitrary vertex



For correctness, see full course notes

(correctness is not part of exam material unless specified)