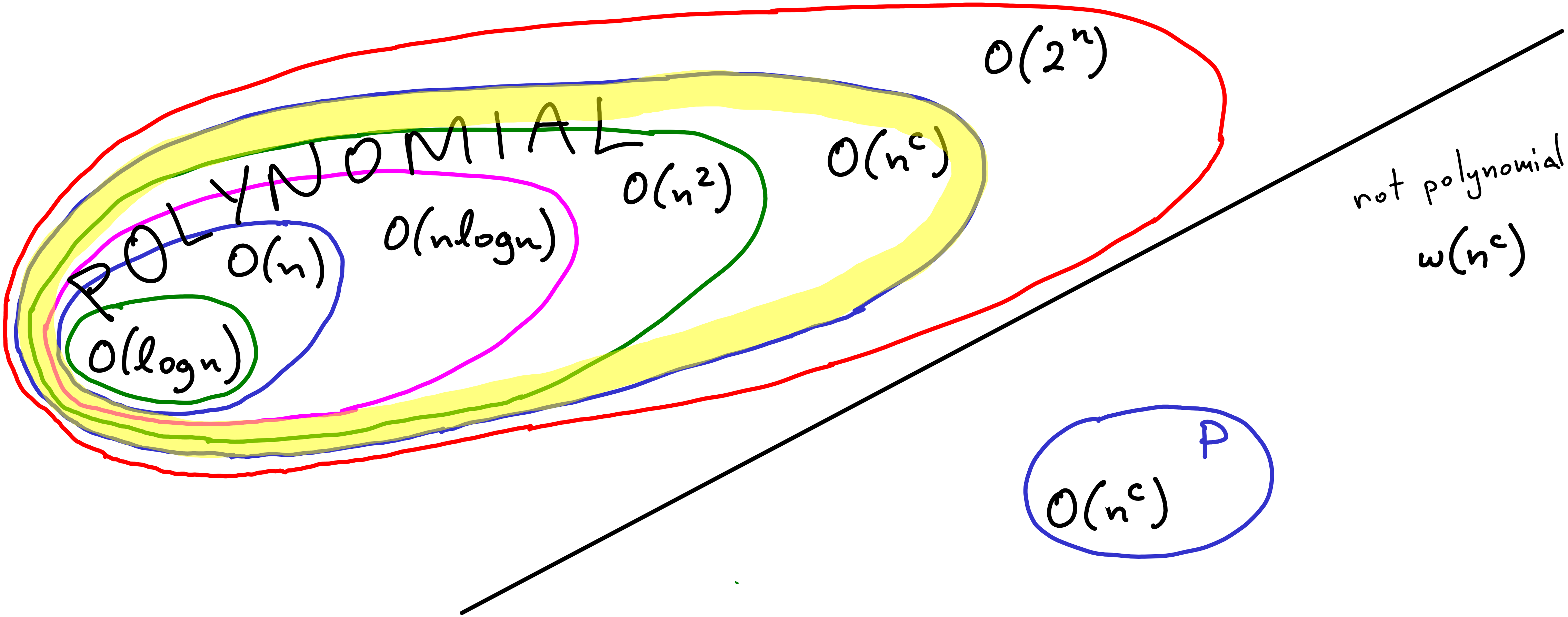
NP-COMPLETENESS: a brief informal introduction

we've seen algorithms with several time complexities:



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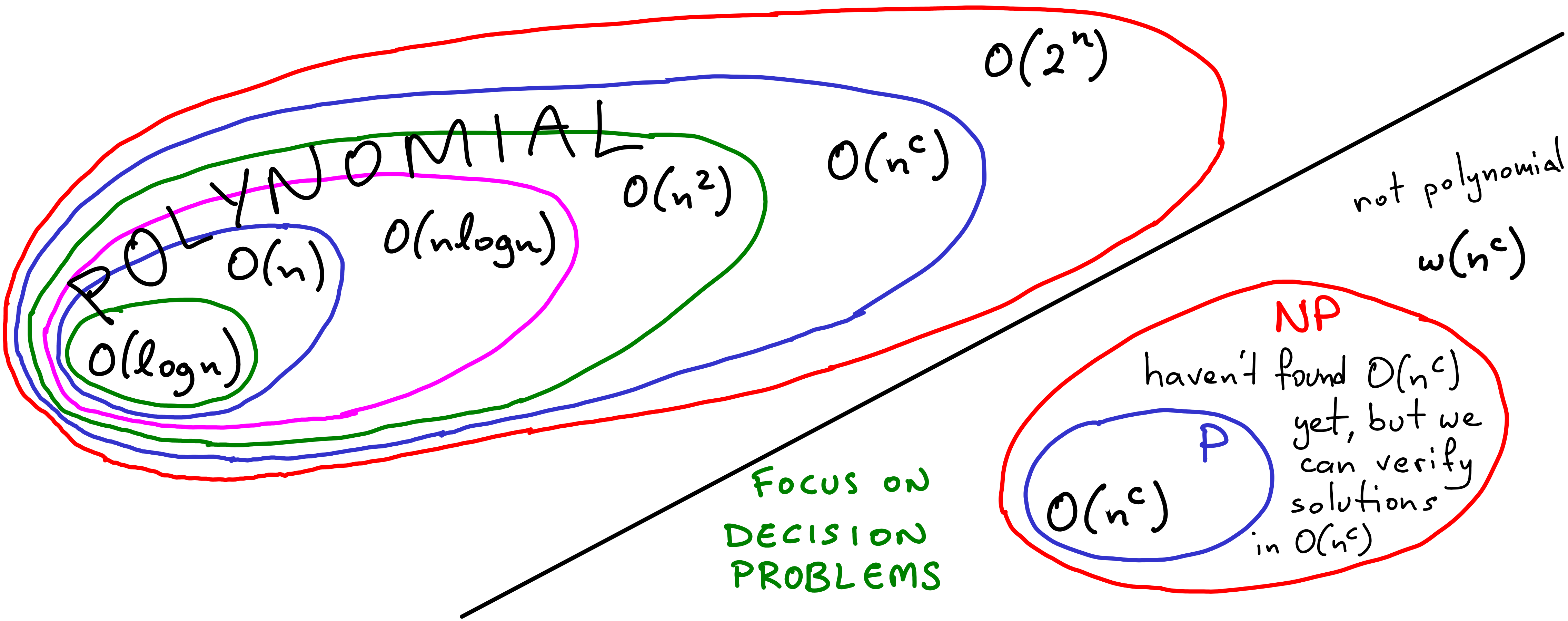
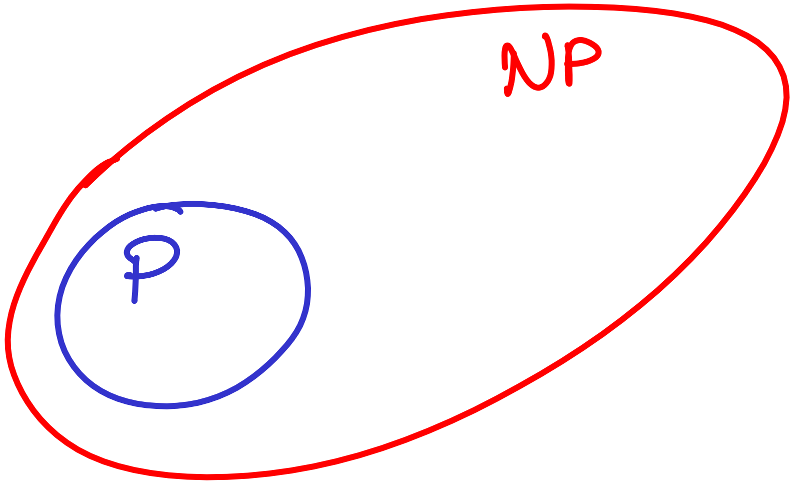
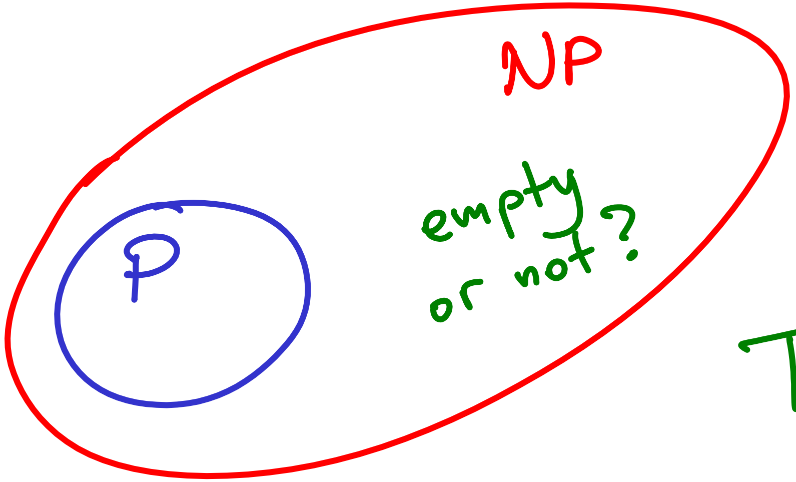




NP: nondeterministic polynomial

(NOT "non-polynomial")



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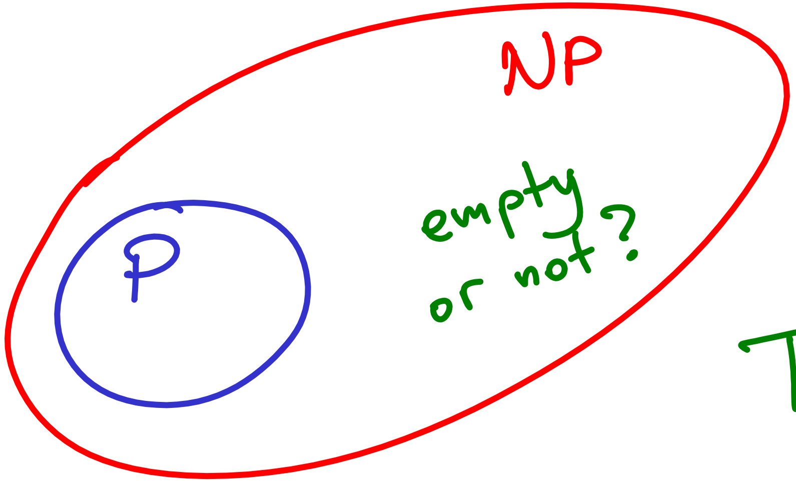
The $1,000,000 question...

P v. NP : = OR ≠ ?

Is it ever "much" harder to solve a decision problem than it is to verify a solution, if the verification takes poly-time?



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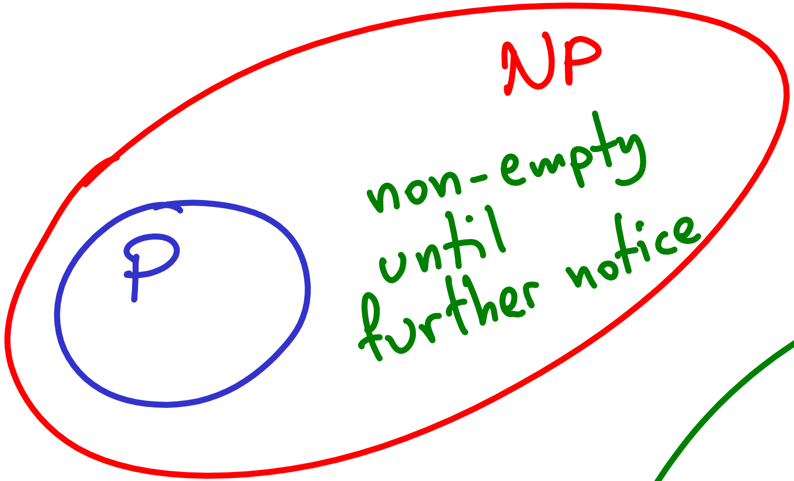
within a polynomial factor } considered ~equivalent

$$T(\text{verify}) = o\left(n^c \cdot T(\text{solve})\right)$$
$$T(\text{solve}) = \omega\left(n^c \cdot T(\text{verify})\right)$$
?

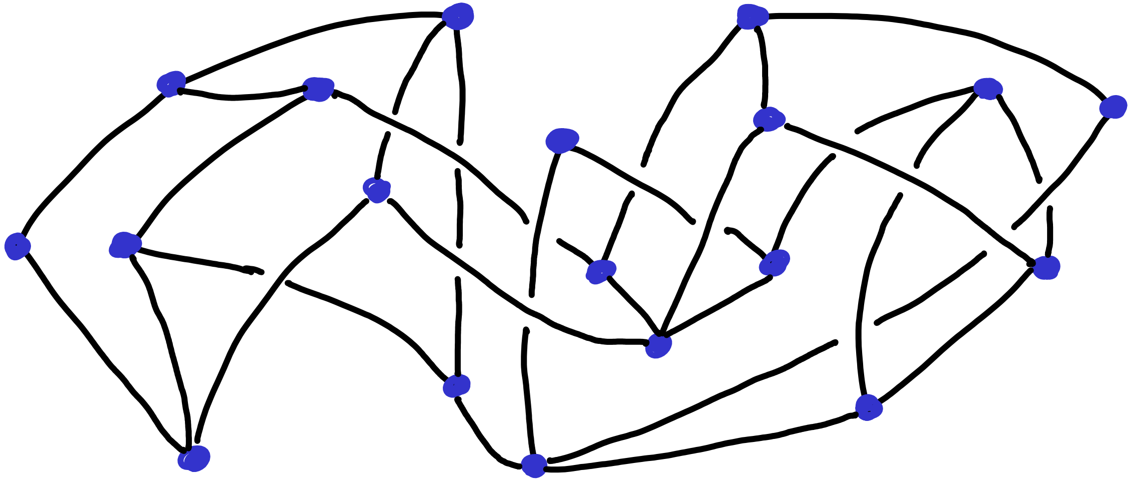


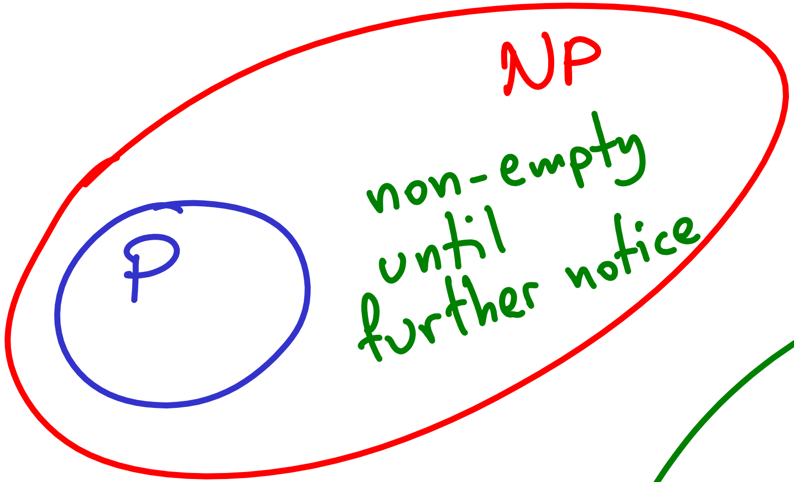
There are thousands of problems for which no known polynomial-time solution is known, yet we can verify proposed solutions in poly-time.

e.g. Hamiltonian cycle

given a graph, find a cycle that visits each vertex exactly once.

↳ decide if one exists





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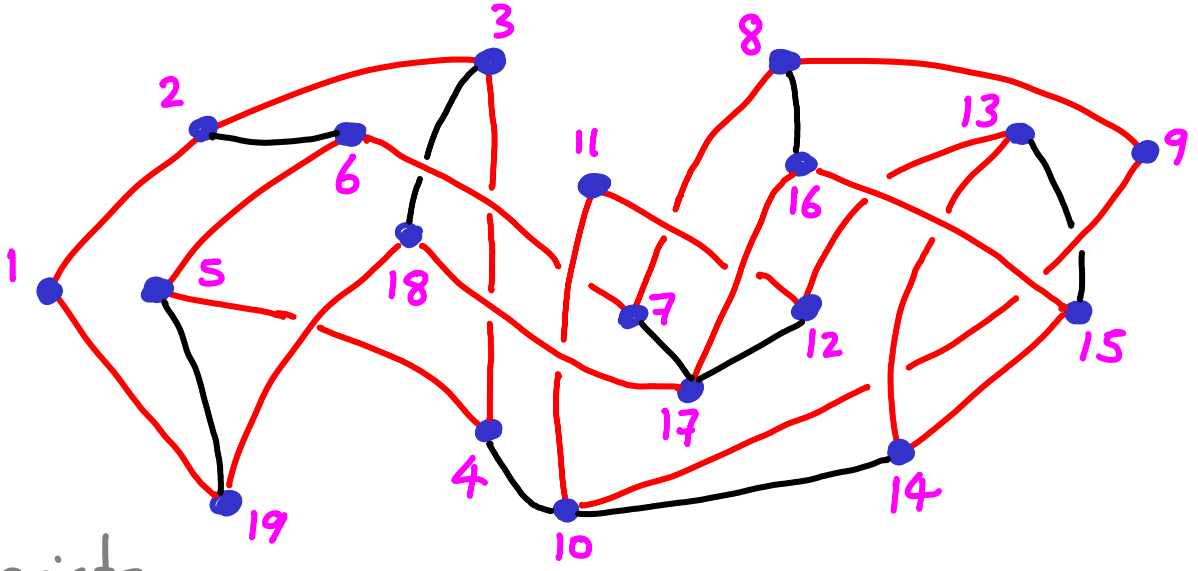
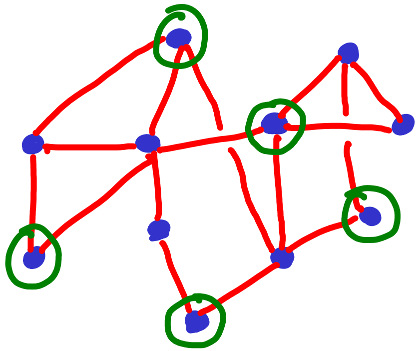


"is there a set of
k independent vertices?"



(independent: no neighbors)

DECISION PROBLEM

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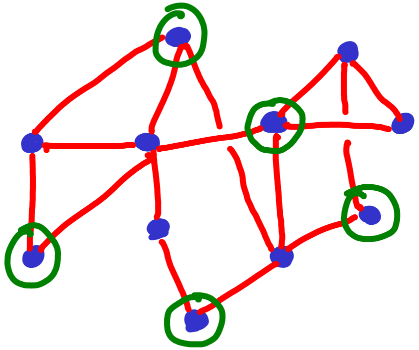

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OPTIMIZATION PROBLEM

"find the largest independent set"

(size)

DECISION PROBLEM

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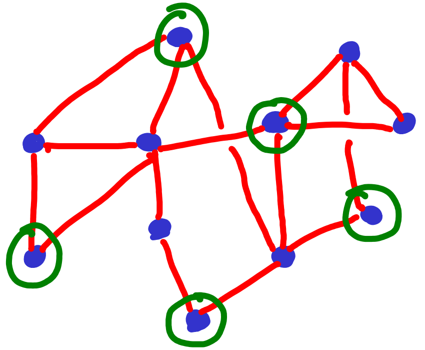


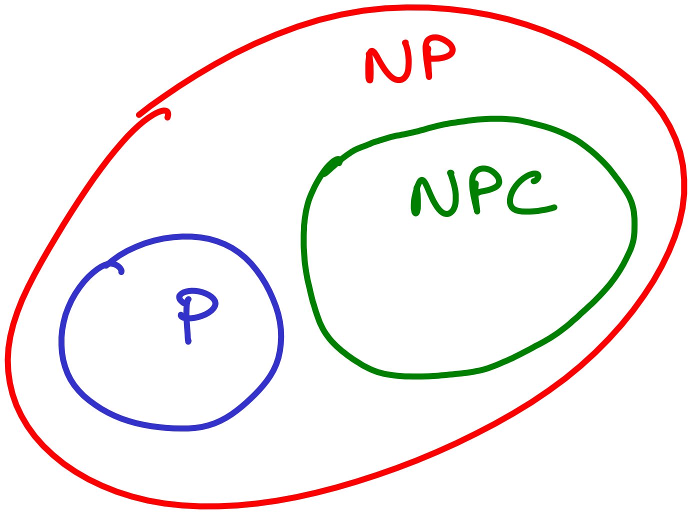
(independent: no neighbors)

OPTIMIZATION PROBLEM

"find the largest independent set" (size)

binary search on k: 0...|V|

Often, optimization problems are not polynomially harder than decision.



NP-COMPLETE PROBLEMS

1) in NP, & not known to be in P

(decision problems with solutions that can be verified in poly-time, but for which no poly-time algo is known)

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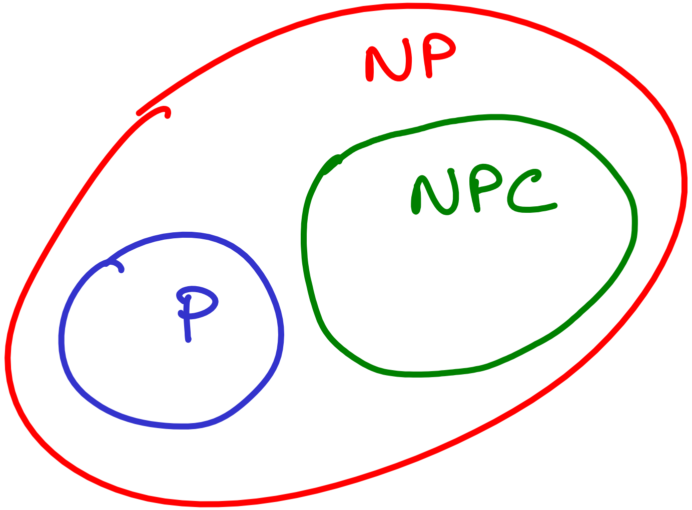

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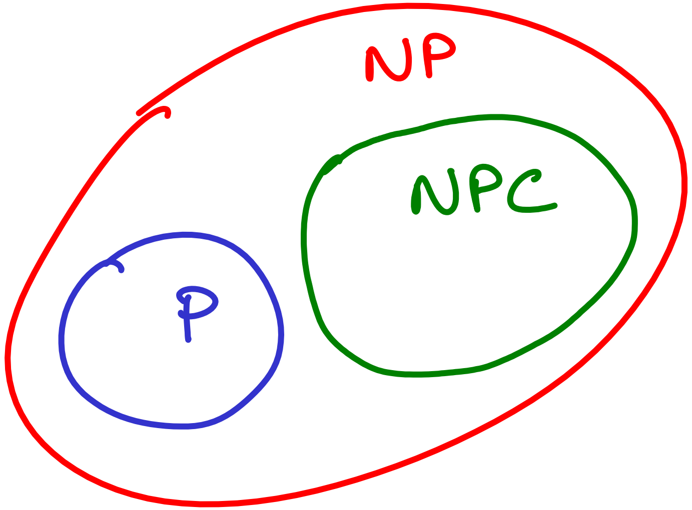
2) if you ever find a polynomial-time solution for <u>any</u> NPC problem, this implies the same for <u>every</u> problem in NP. → P = NP



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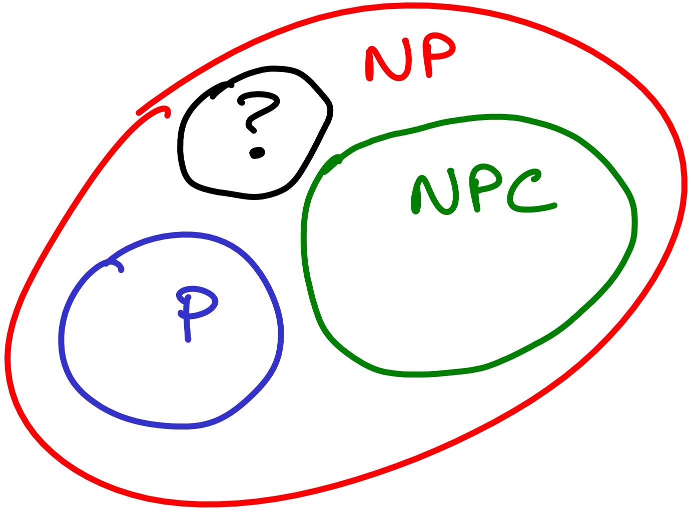
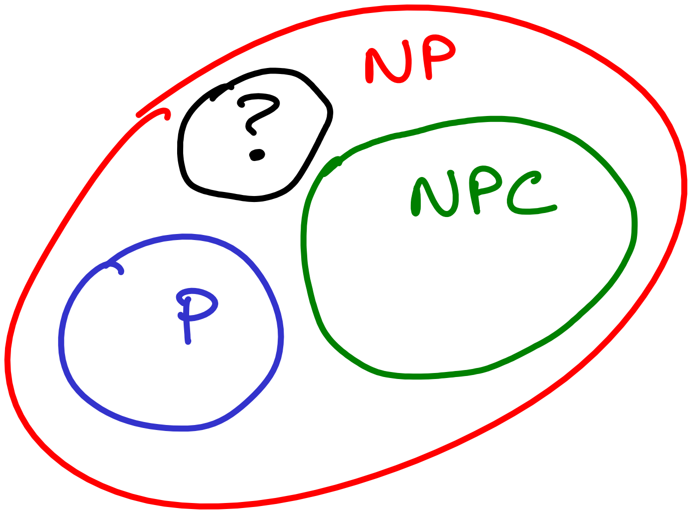
↳ if you ever prove that an NPC problem has no poly-time algo, then no NPC problem does → $P \neq NP$



Are there other problems in NP but not in P or NPC ?



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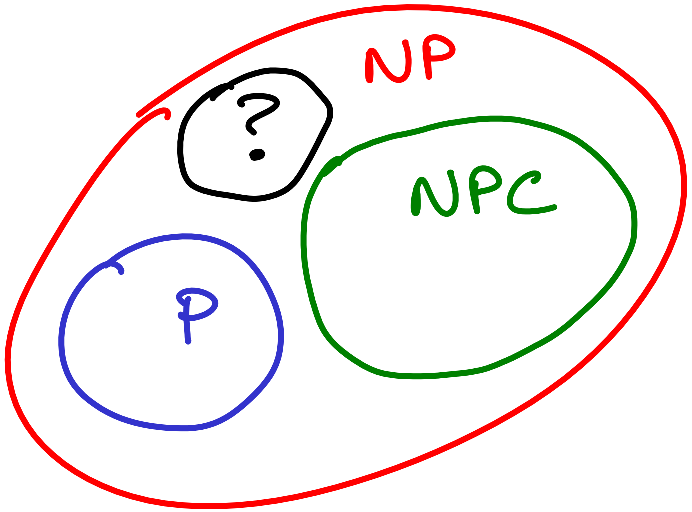

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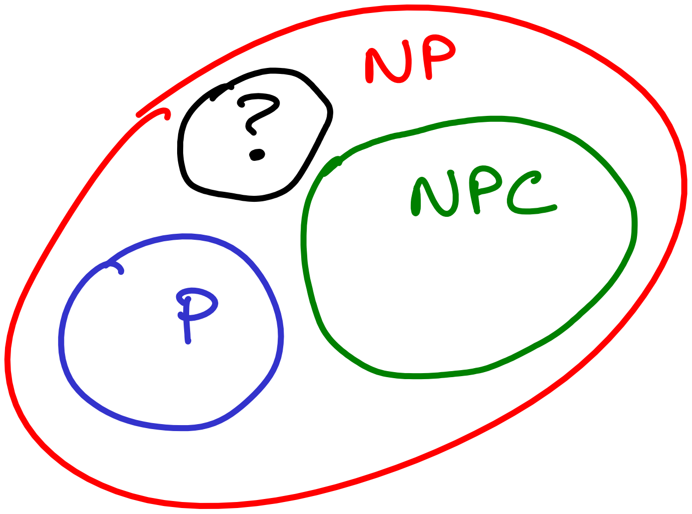
- if P≠NP then yes. [theorem]

↳ few "natural" problems (almost everything in NP is P or NPC)



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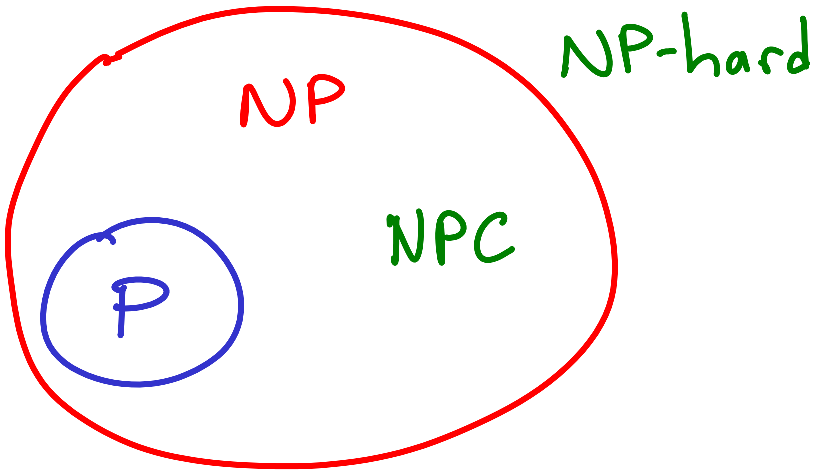
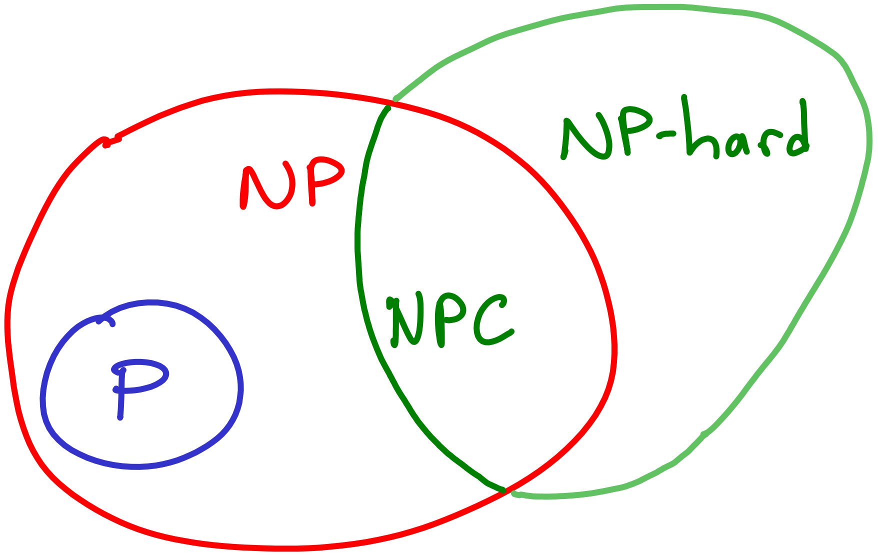
If we solved such a problem in poly-time, it would just move into P without dragging everything else along.



NP-hard problems

↳ as hard as any NP problem.



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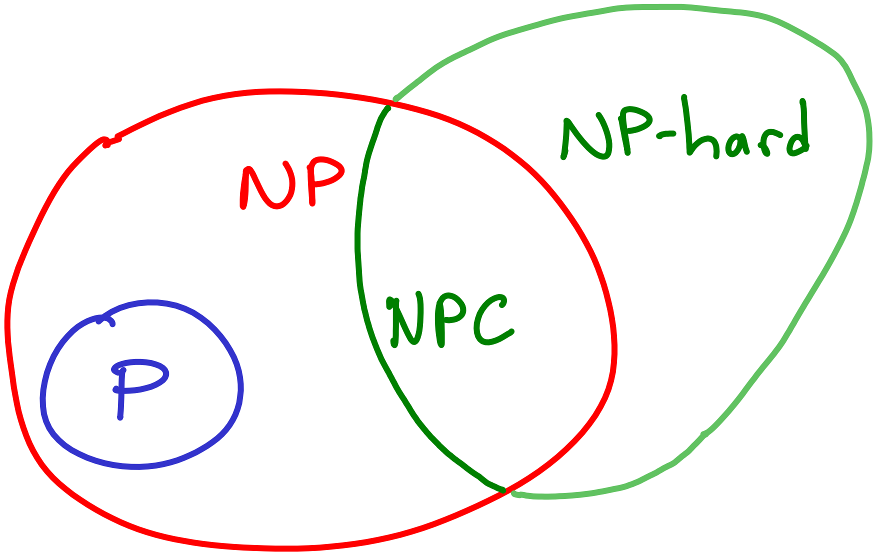

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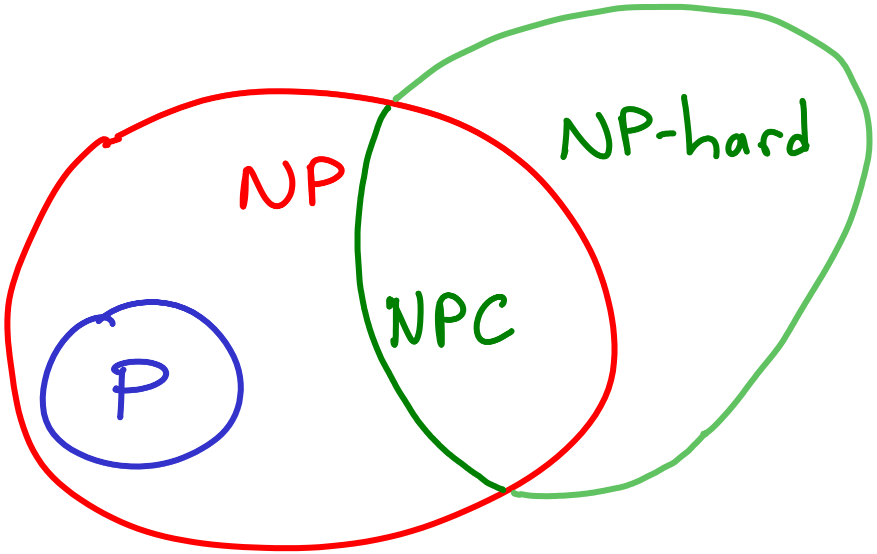
  ↳ might not be decision problems

independent set → decision ($\leq k$?) NPC
independent set → optimization (max k) NP-hard



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not many of these

independent set

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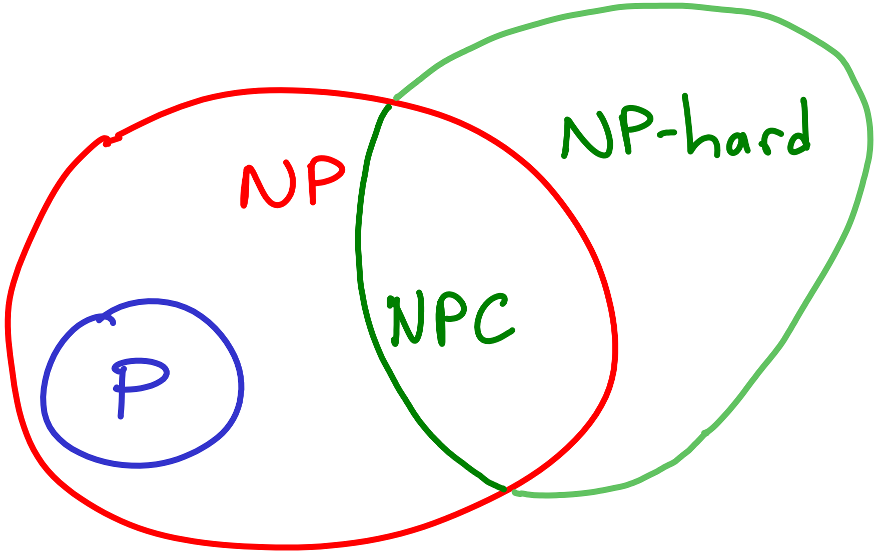

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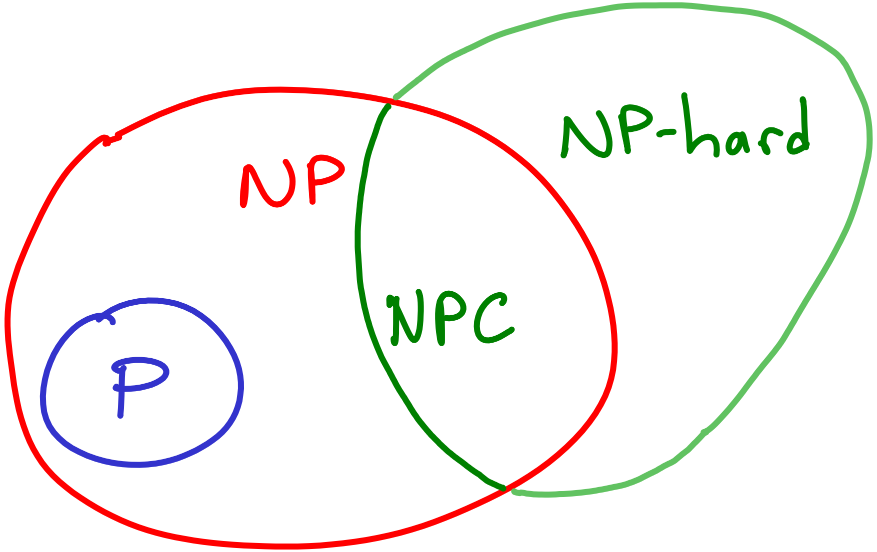

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NPC = NP-hard & in NP

AN IMPORTANT DETAIL just mentioned here

For NPC problems, we measure input in terms of a finite alphabet [e.g. binary: represent k with $\Theta(\log_2 k)$ bits]

– unlike our treatment of constants so far