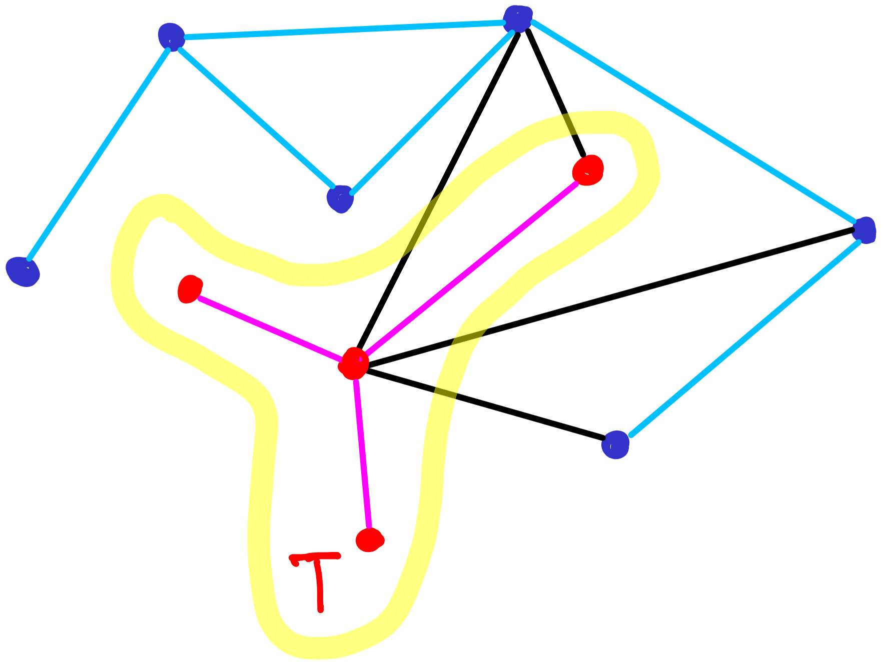
PRIM'S ALGORITHM for MST

(R. Prim 1957, but also V. Jarnik 1930)

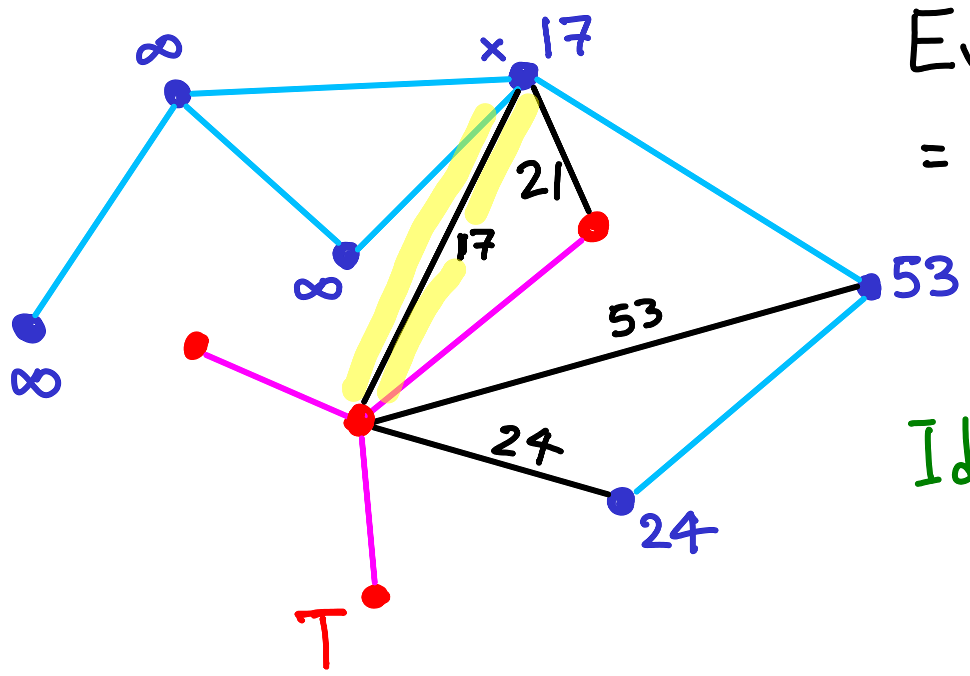


Uses basic principle:

Given a subtree T of MST, the "lightest" edge connecting to a vertex not in T can be added to T.

Grow one tree, incrementally adding one edge (& vertex)

PRIM'S ALGORITHM for MST



Every vertex not in T has a score =
= lightest edge weight connecting it to T

Identify lightest edge crossing cut:

1) identify min-score vertex, x
2) identify lightest edge from x to T

Brute force: O(V) per MST edge

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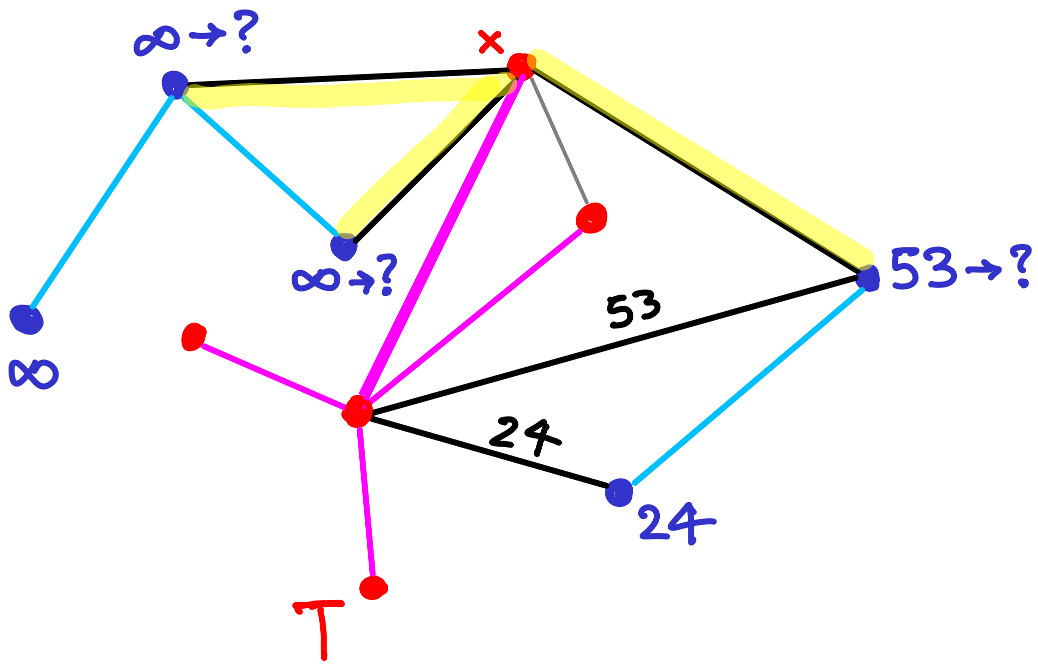

Update scores when x joins T:

For each neighbor v_i of x

 if v_i not in T

  $c = \text{score}(v_i)$

  $\text{score}(v_i) \leftarrow \min\{c, \underbrace{w(x,v_i)}_{\text{new option}}\}$

Need to extract min score & decrease scores. How?

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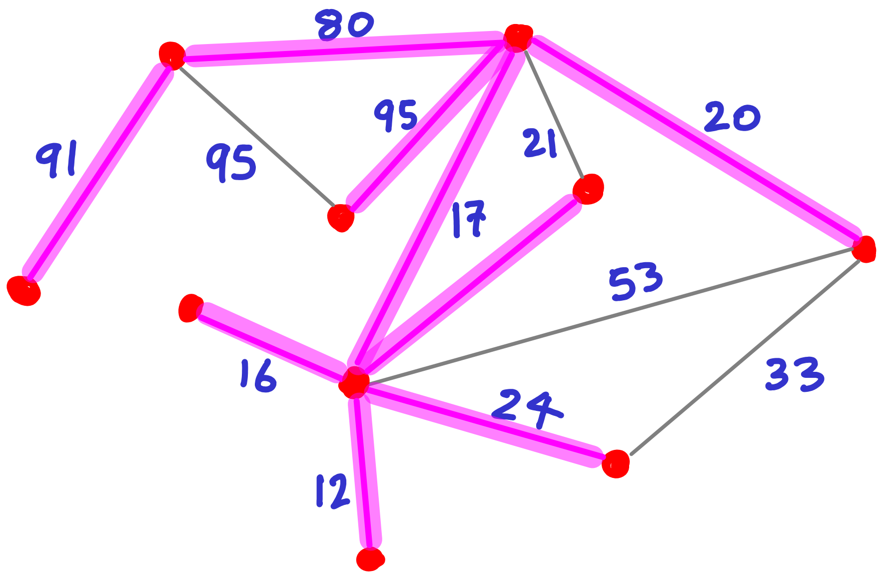
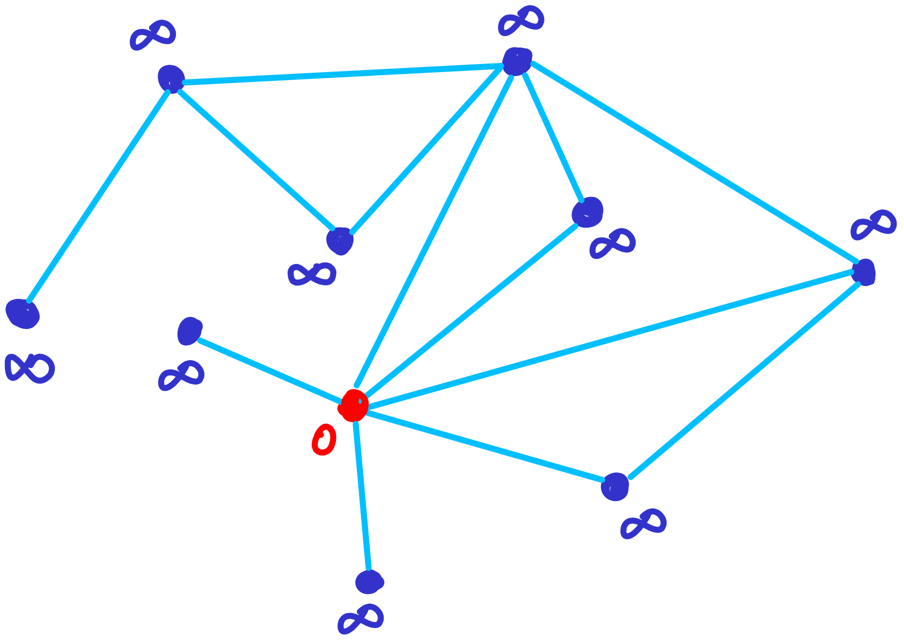


For a detailed example of Prim's algorithm on this graph, please see full version of class notes.

Summary follows.

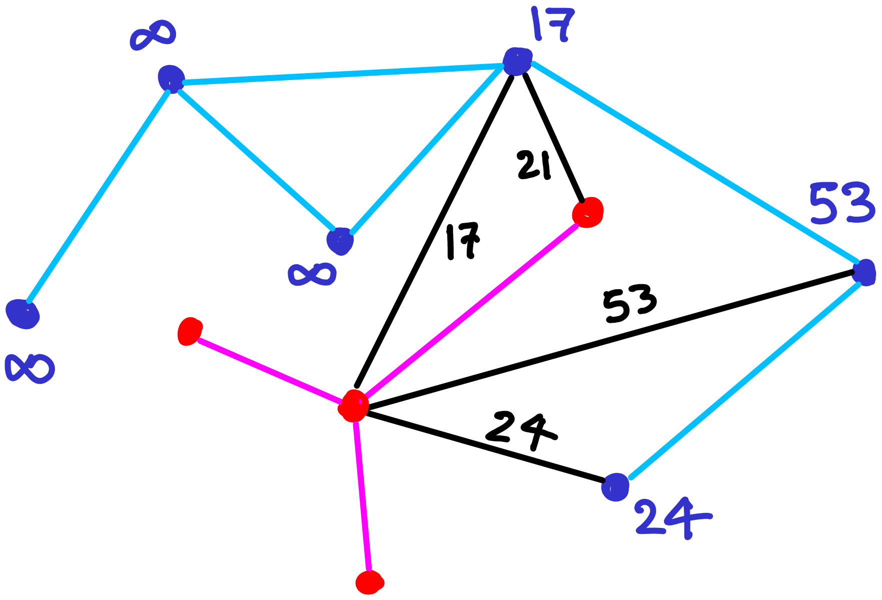
PRIM'S ALGORITHM for MST



1) start w/ any vertex s; set $w(s)=0$

2) set $w(\neq s) = \infty$ & put all in pr.queue

PRIM'S ALGORITHM for MST



1) start w/ any vertex s; set $w(s)=0$
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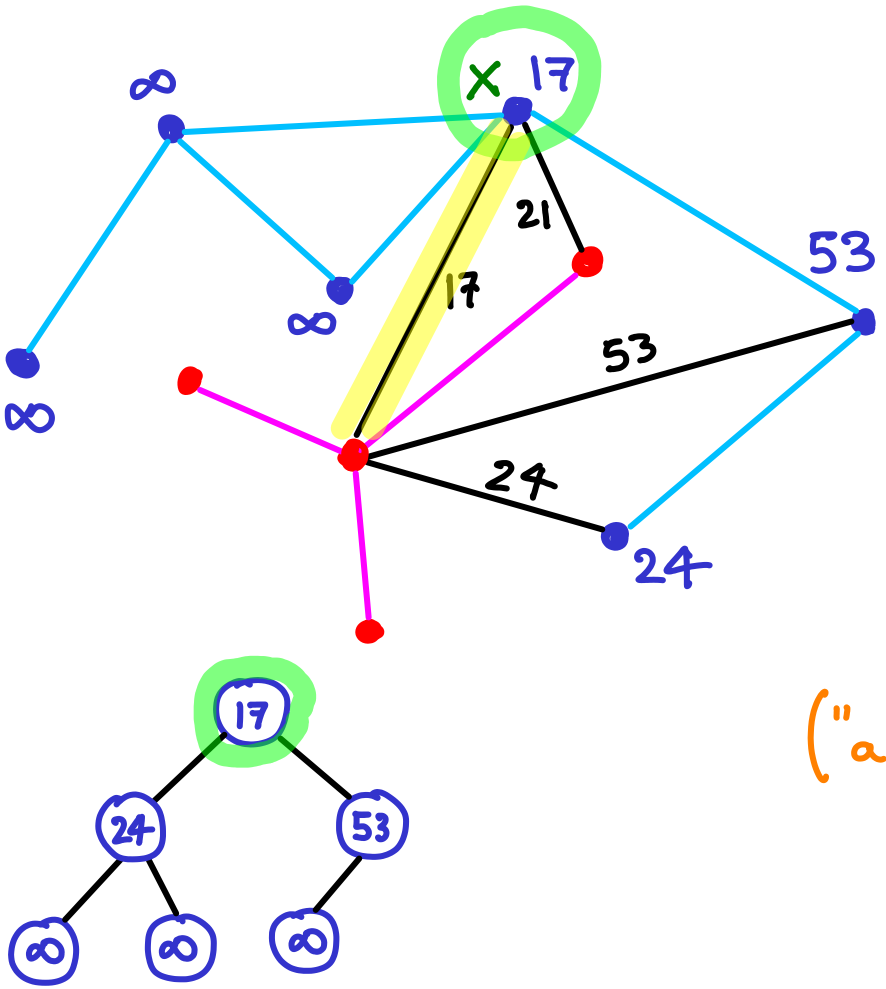
3) while pr.queue not empty

$\hookrightarrow$ $|V|$ rounds

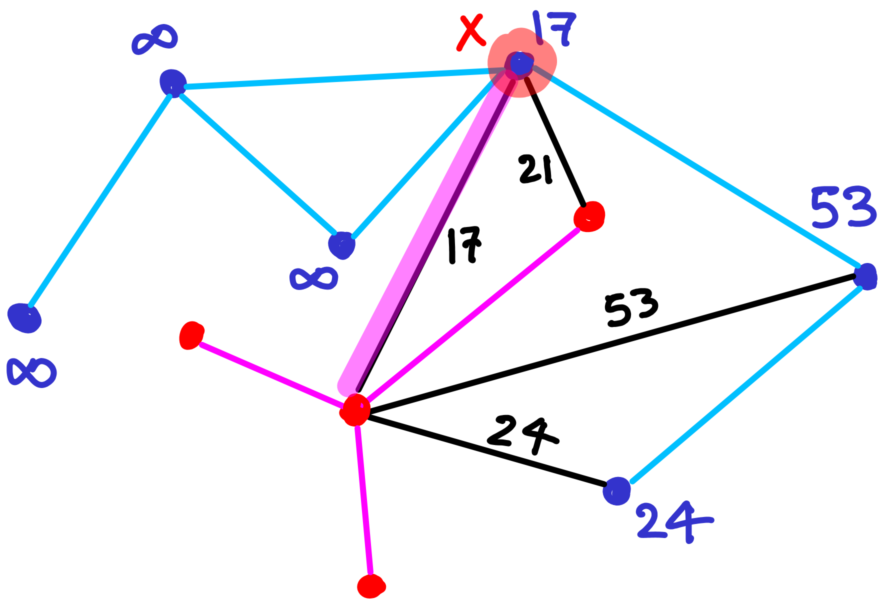
PRIM'S ALGORITHM for MST



1) start w/ any vertex s; set w(s)=0
2) set w(≠s) = ∞ & put all in pr.queue
3) while pr.queue not empty
   x: extract-min & add edge to T
   mark x → in T.

("add edge" → find an edge from x to T, w/ min weight)
(if x=s, no edge to add)

PRIM'S ALGORITHM for MST



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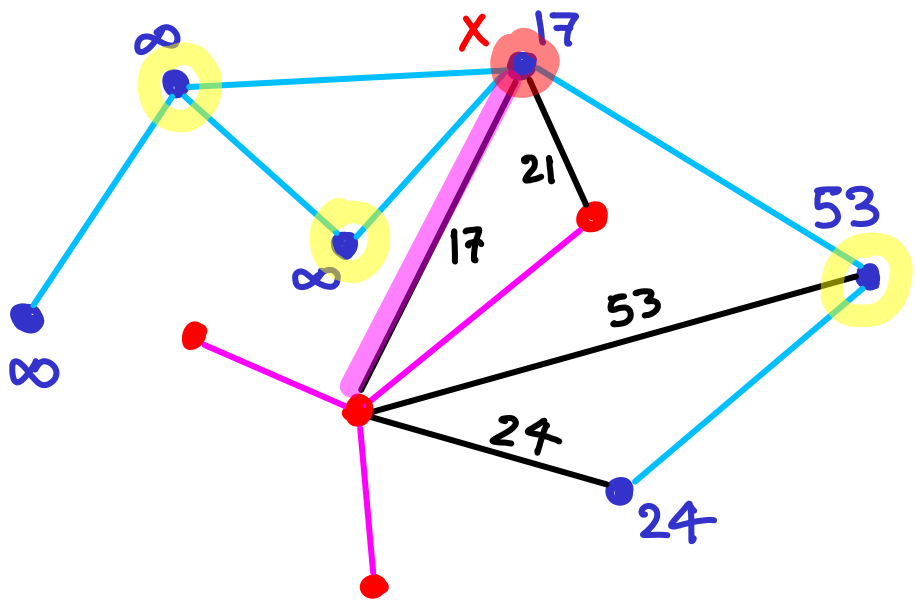

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3) while pr.queue not empty
    x: extract-min & add edge to T
    mark x → in T.
    for each unmarked neighbor q of x
        if w(q) > w(q,x) then decrease.

PRIM'S ALGORITHM for MST

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|V| rounds

$$\sum_{x \in V} \left(O(\log V) + O(\text{degree}(x)) \right) = O(V \log V) + O(E)$$

$$\sum_{x \in V} O(\text{degree}(x)) \cdot O(\log V) = O(E) \cdot O(\log V)$$

dominates

Using adjacency list

TOTAL = $O(E \log V)$

PRIM'S ALGORITHM for MST

with Fibonacci heap
(beyond scope of COMP160)

1) start w/ any vertex s; set $w(s)=0$
2) set $w(\neq s) = \infty$ & put all in pr.queue
3) while pr.queue not empty

$|V|$ rounds

x: extract-min & add edge to T
mark x → in T.
for each unmarked neighbor q of x
if $w(q) > w(q,x)$ then decrease.

O(1) amortized

amortized

$$\sum_{x \in V} \left(O(\log V) + O(\text{degree}(x)) \right)$$

$$= O(V \log V) + O(E)$$

$$\sum_{x \in V} O(\text{degree}(x)) \cdot O(\cancel{\log} V)$$

$$= O(E) \cdot O(\cancel{\log} V)$$

Using adjacency list

TOTAL = $O(E + V \log V)$

PRIM'S ALGORITHM for MST

Using adj. matrix
w/ weighted entries

& no pr. queue

1) start w/ any vertex s; set $w(s)=0$

2) set $w(\neq s) = \infty$ & put all in ~~pr.queue~~ array

$|V|$ rounds 3) while ~~pr.queue not empty~~ $\exists$ v not in T

scan array $O(V)$ { x: extract-min & add edge to T
mark x → in T.

scan row(x) in matrix $O(V)$ { for each unmarked neighbor q of x
if $w(q) > w(q,x)$ then decrease.

$O(V^2)$ time & space