DIJKSTRA'S ALGORITHM

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Grow SSSP tree greedily.

Maintain set of vertices to add.

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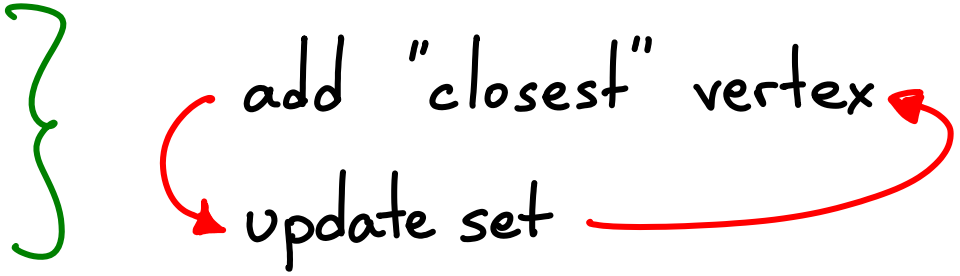

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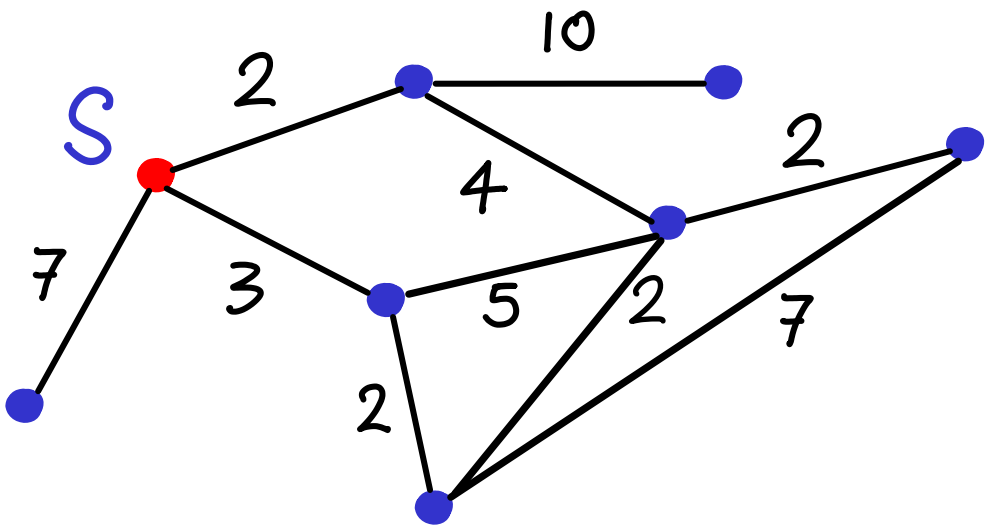
add "closest" vertex

update set



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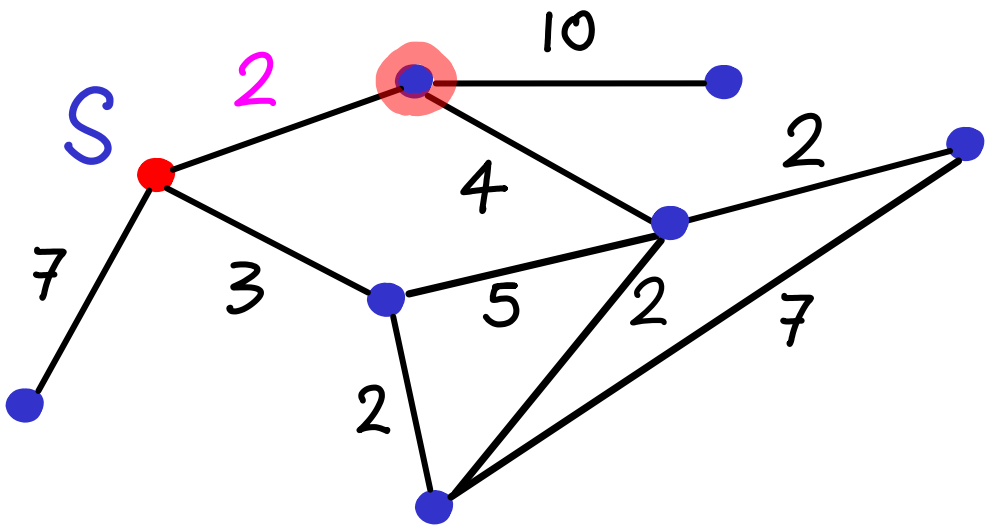

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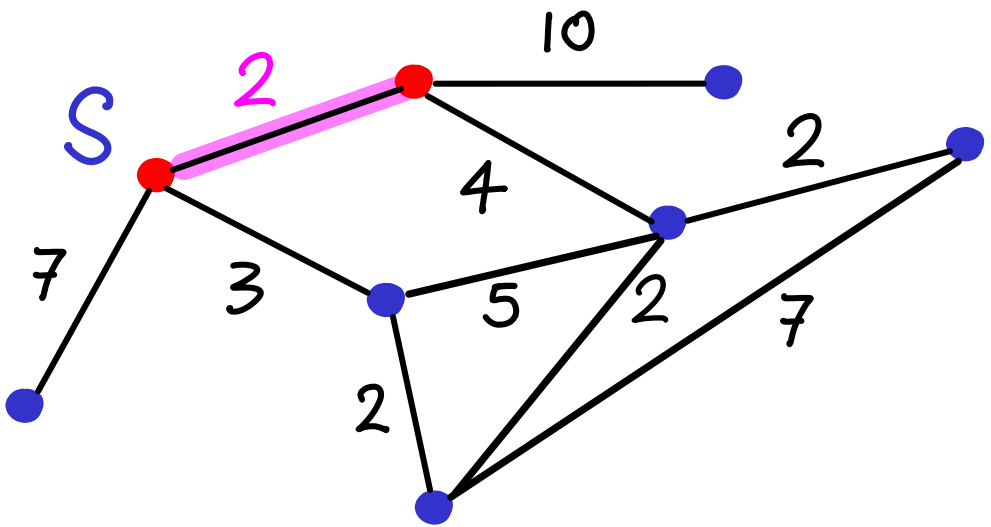

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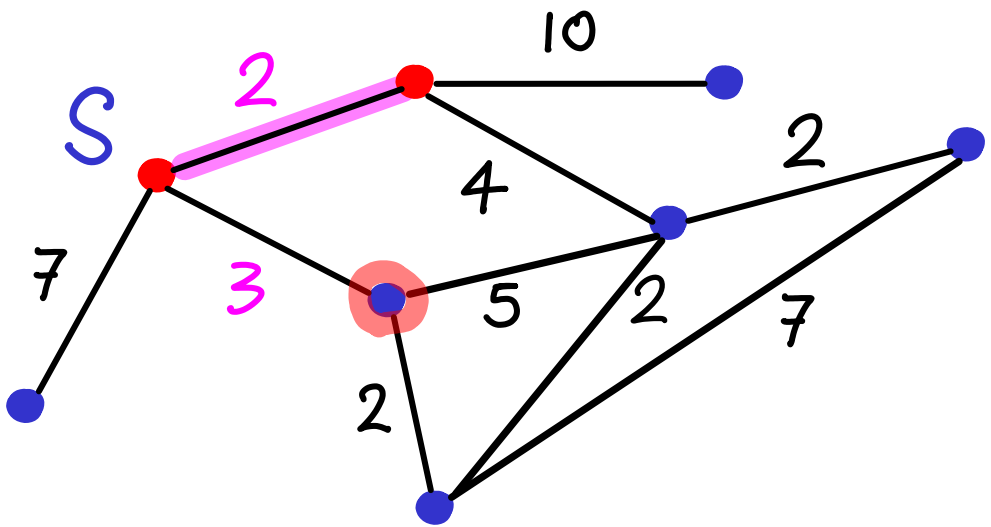

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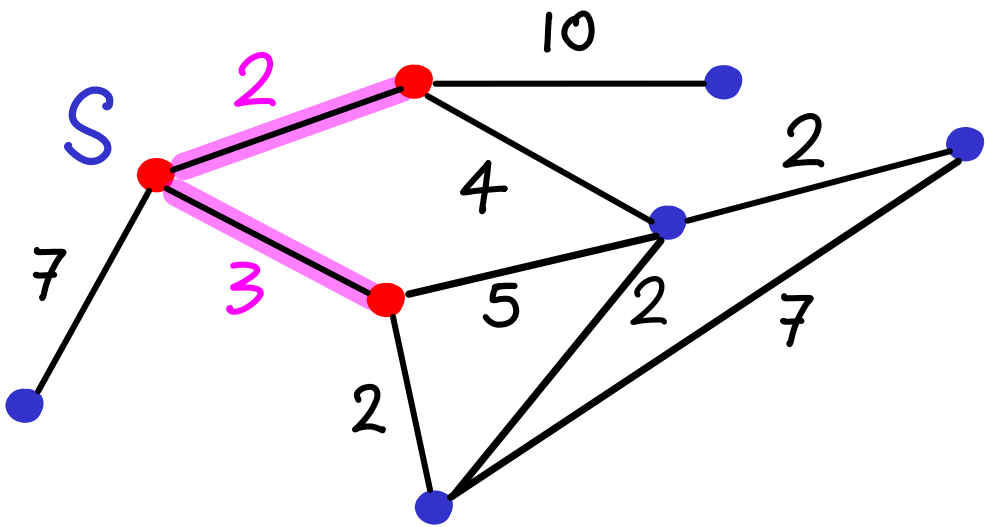

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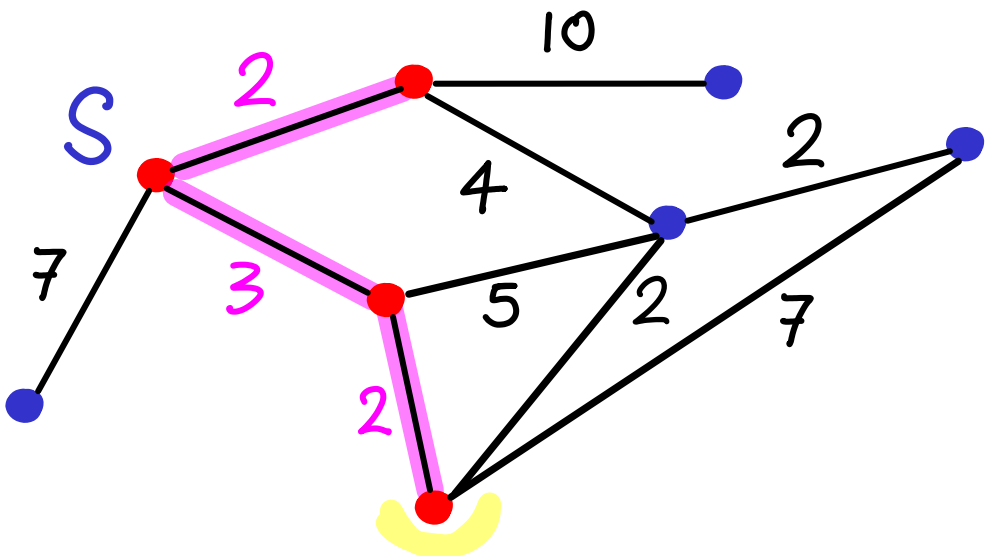

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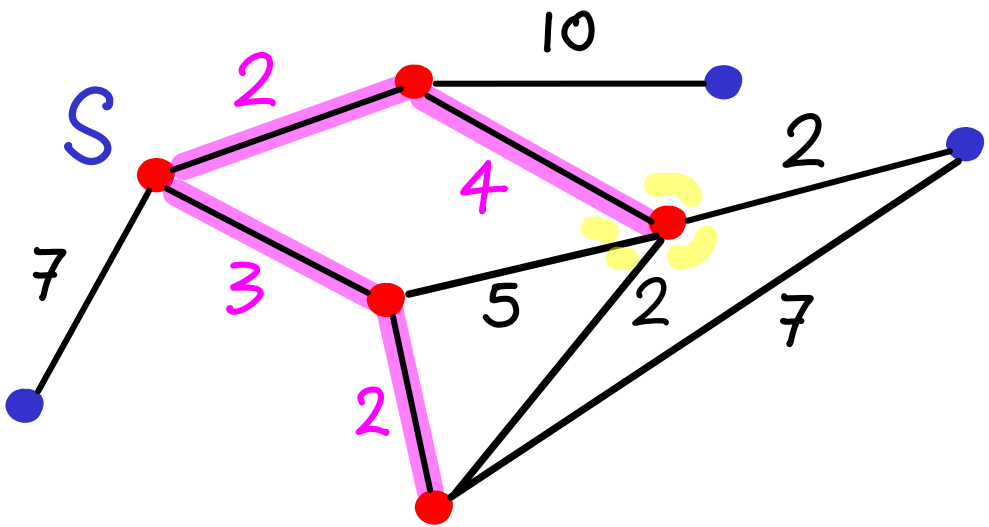

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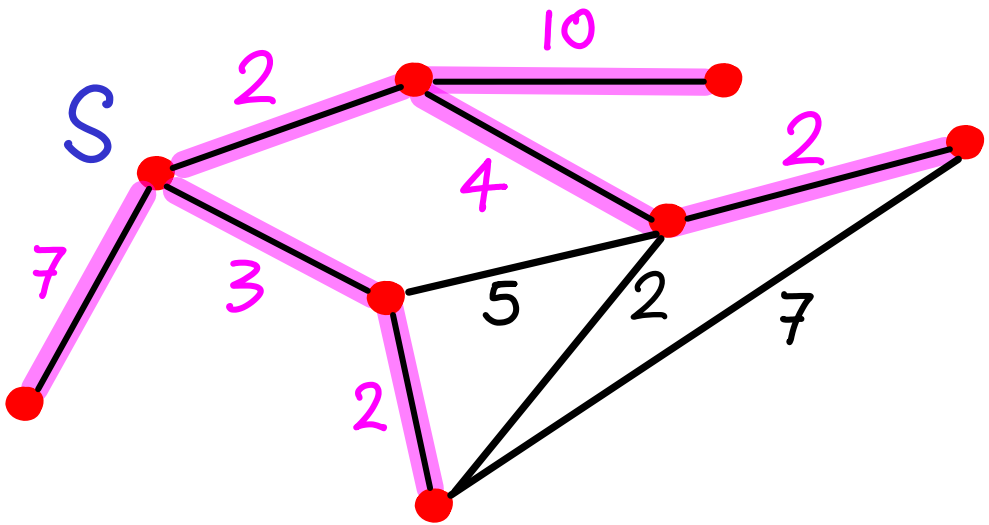
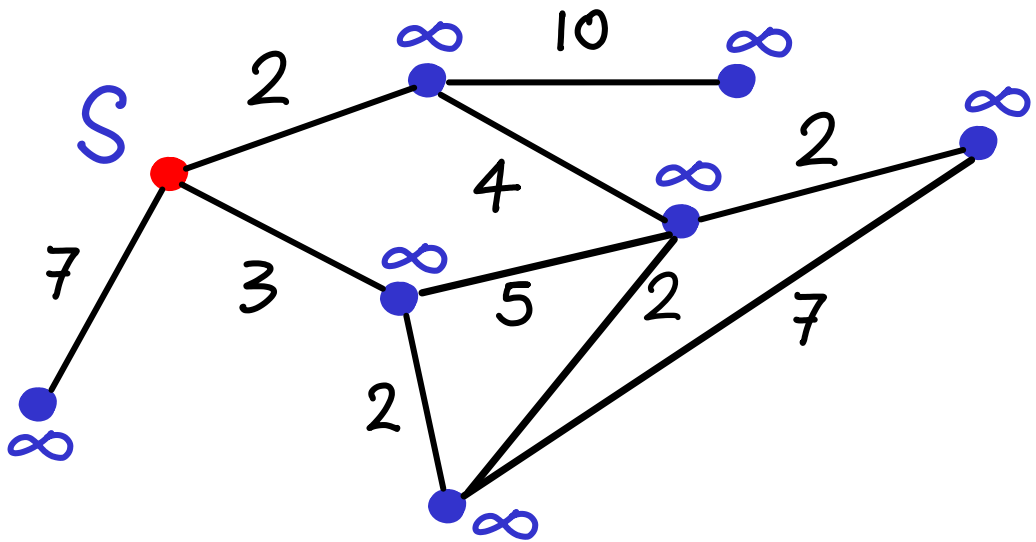


Initialize : $S=0$, others $=\infty$



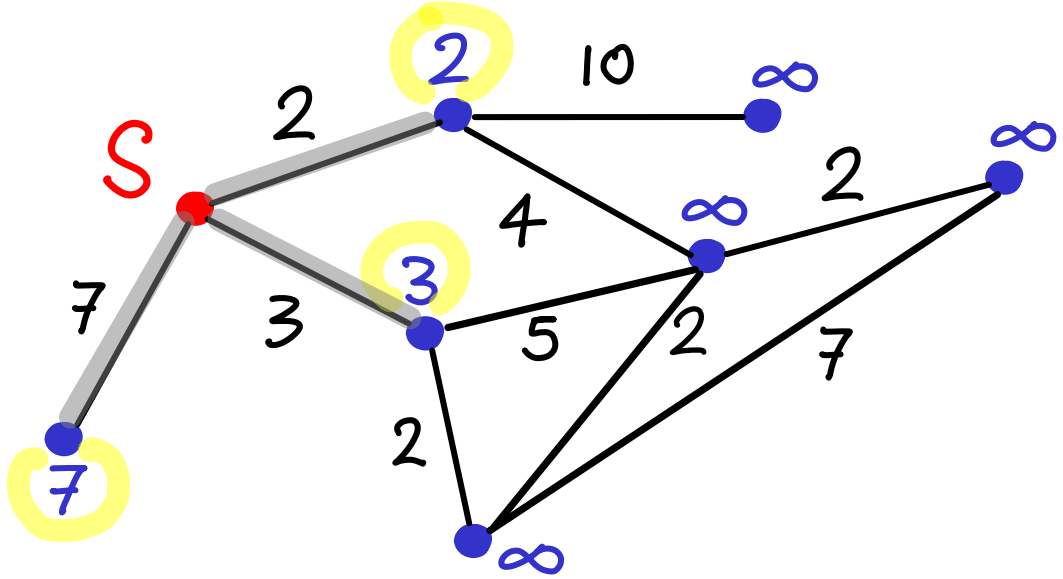
Initialize : $S=0$, others $=\infty$

add "closest" vertex

update set

= extract lowest score

= relax incident edges



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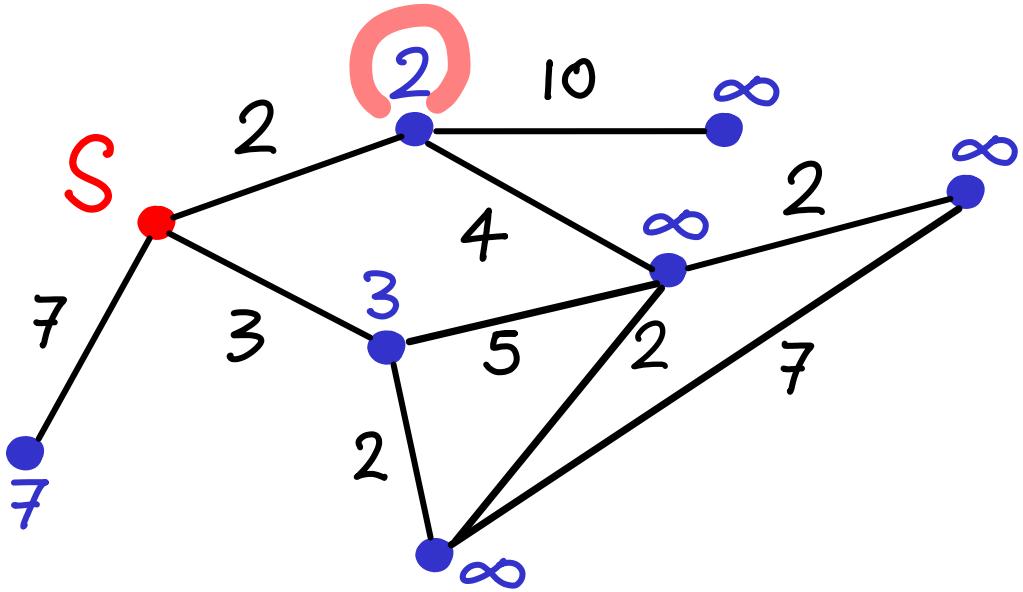

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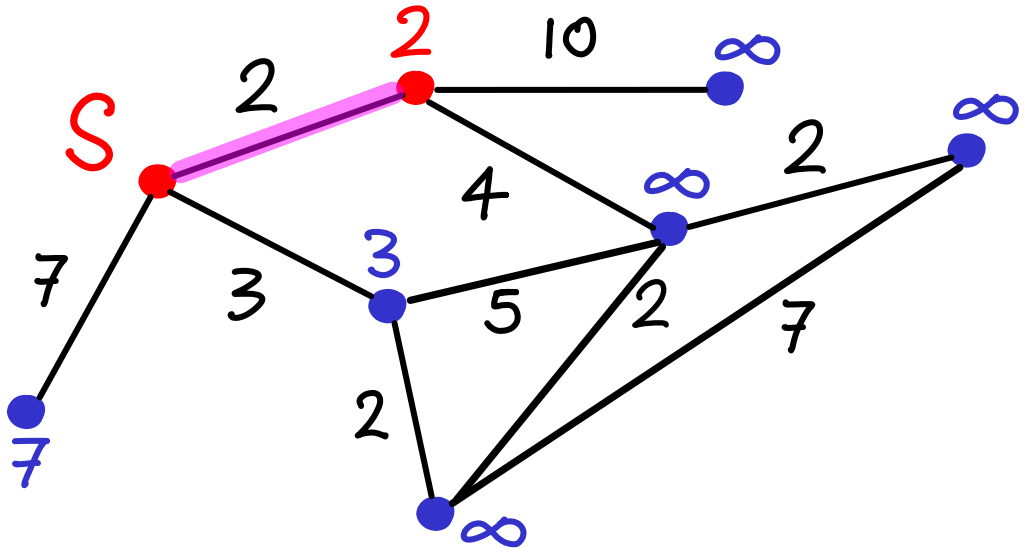

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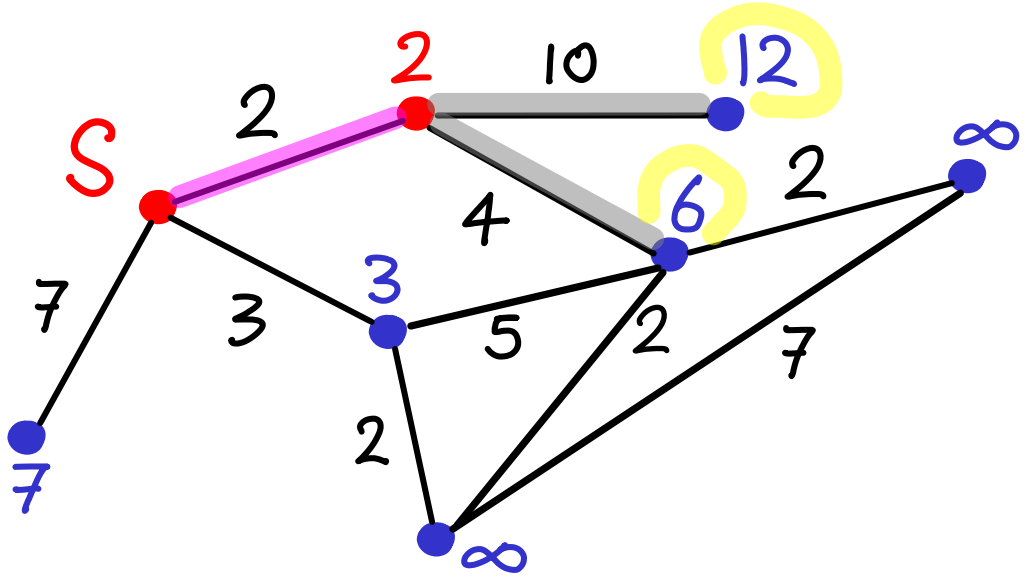

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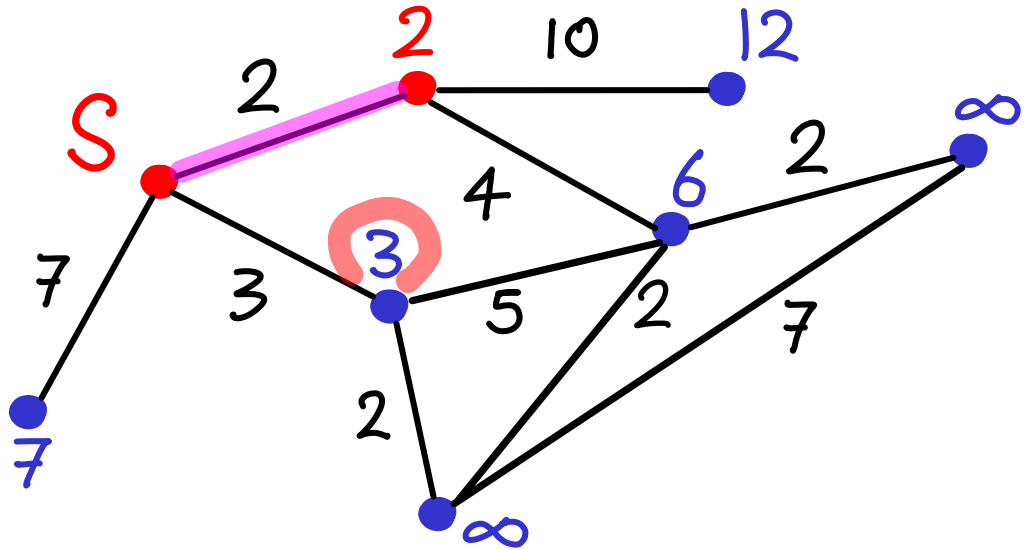

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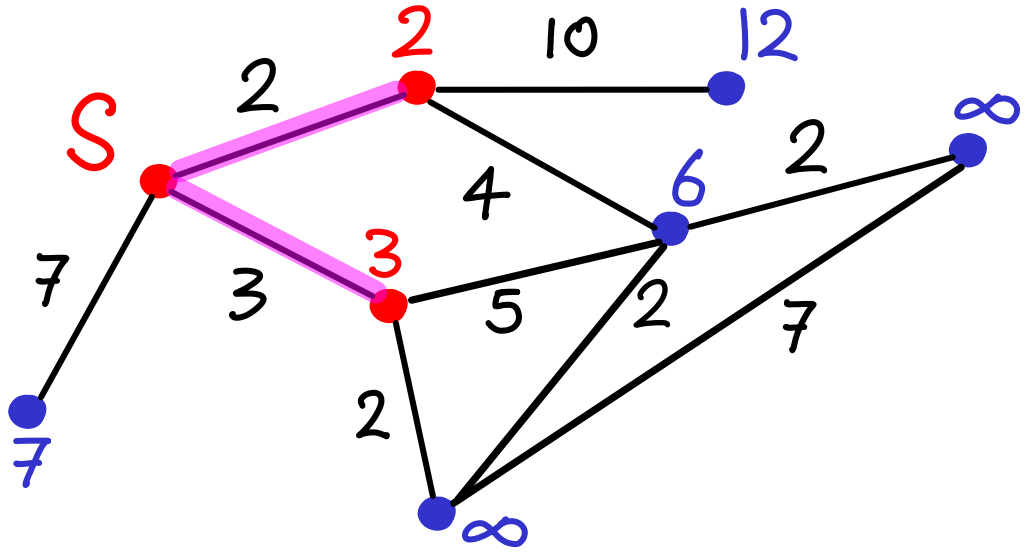

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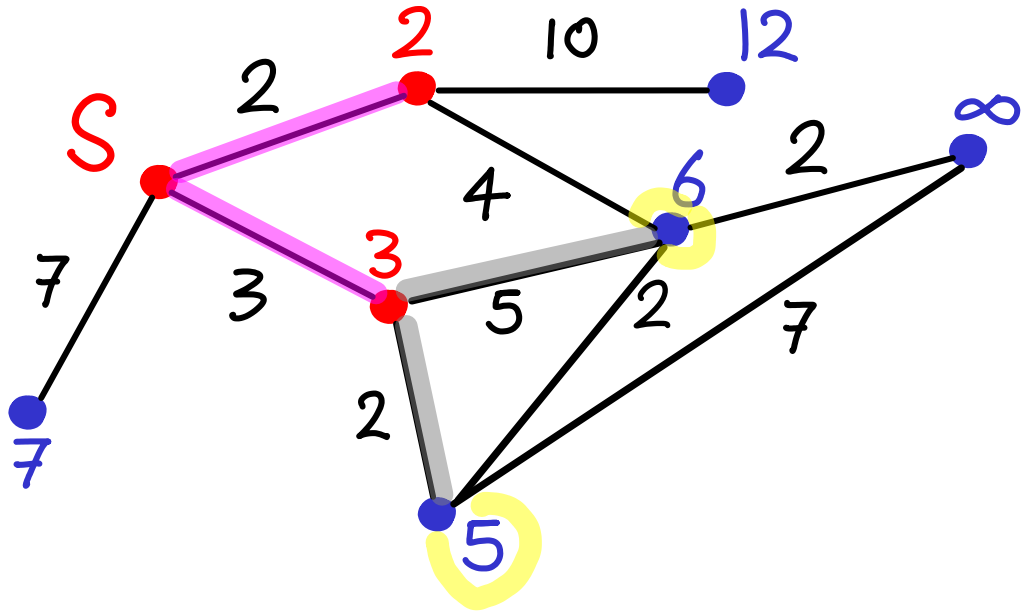

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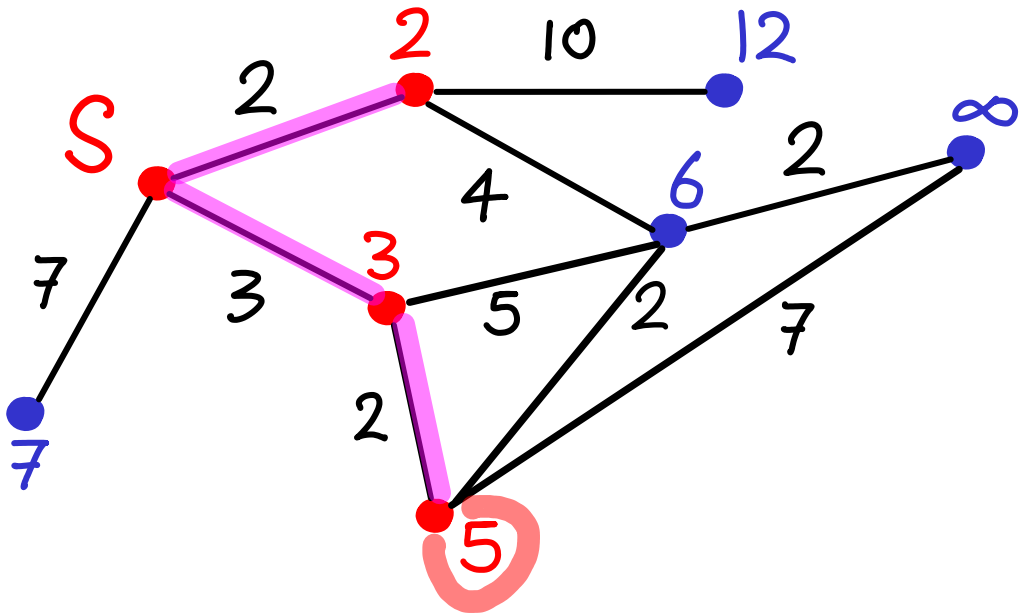

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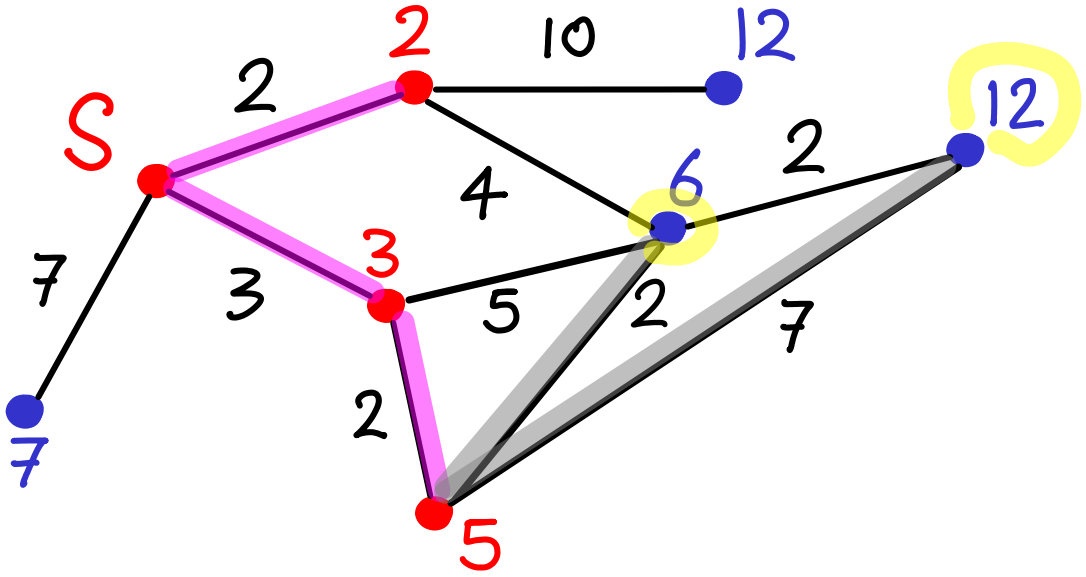

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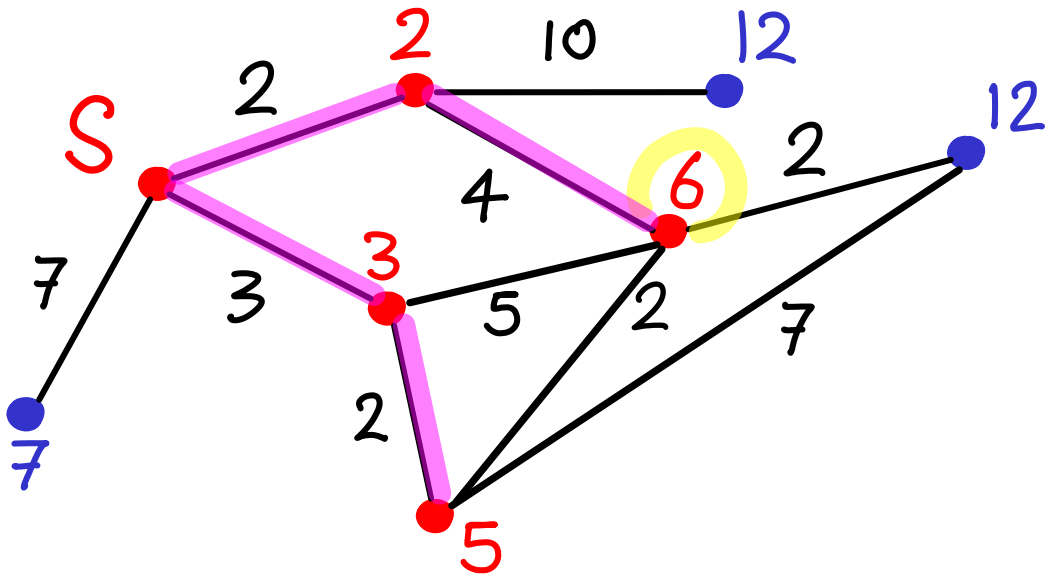

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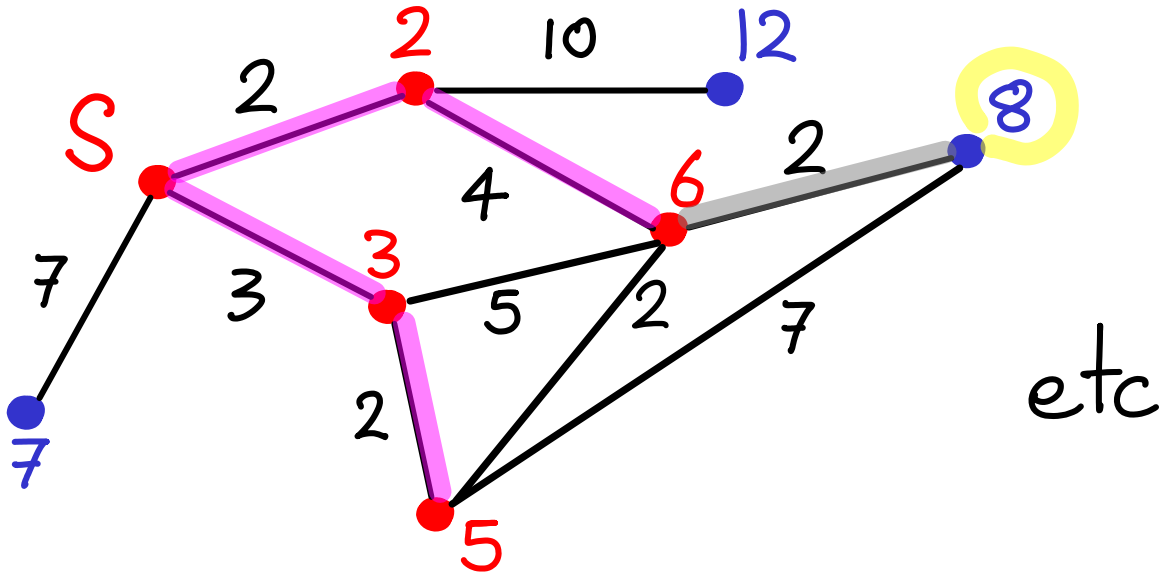
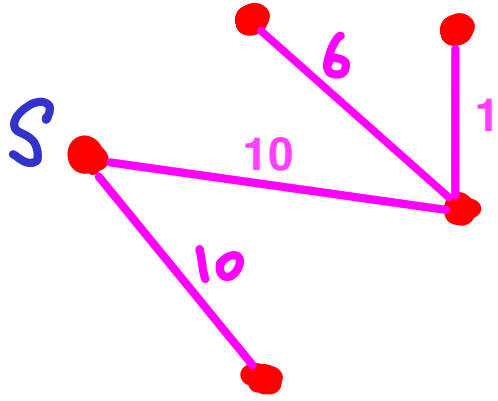


etc

Correctness:

assume we have shortest paths
to a set of red vertices



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assume we have shortest paths
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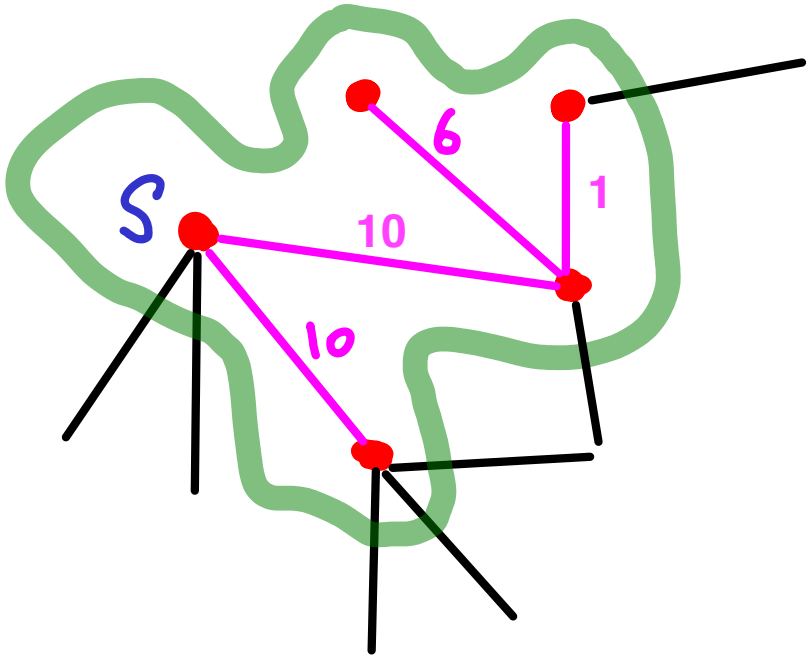
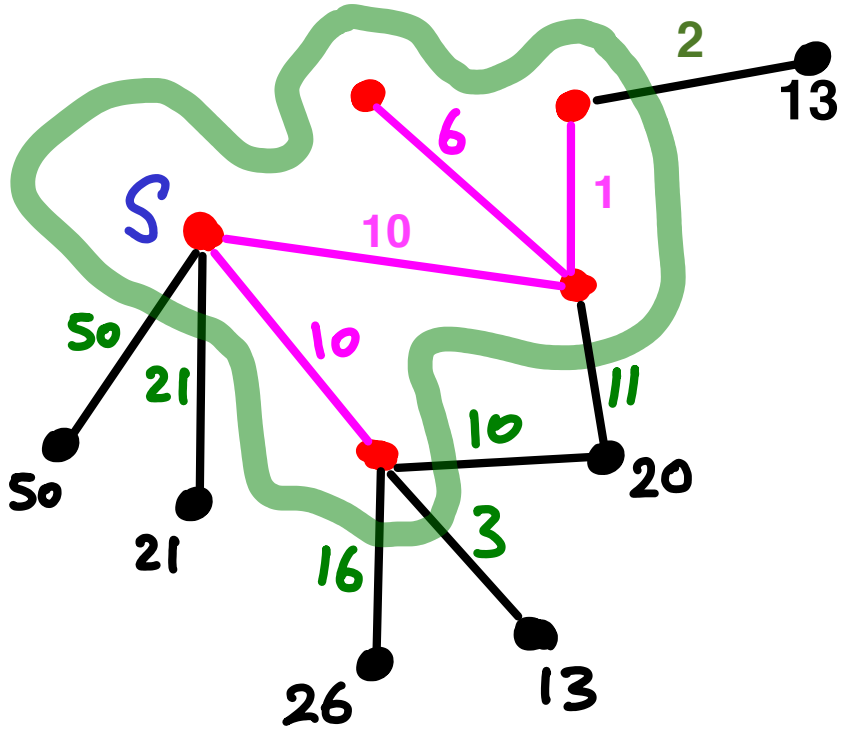
CUT



Correctness:

assume we have shortest paths
to a set of red vertices

Just across CUT,
vertices have finite scores
= [a path in known set] + black edge



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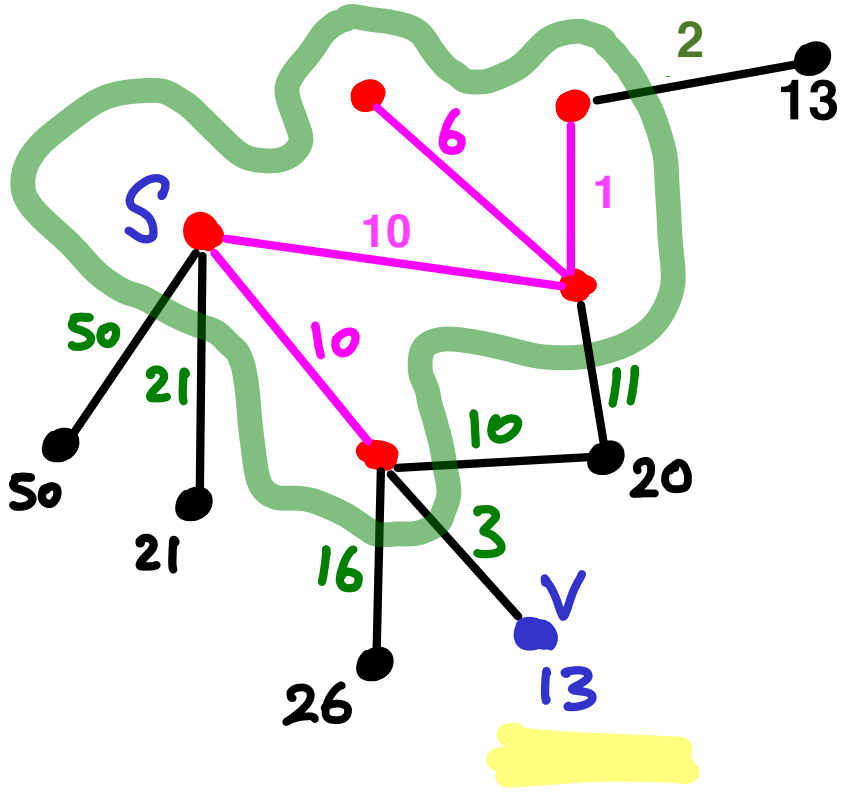

V = "closest" to S

Correctness:

assume we have shortest paths
to a set of red vertices

Just across CUT,
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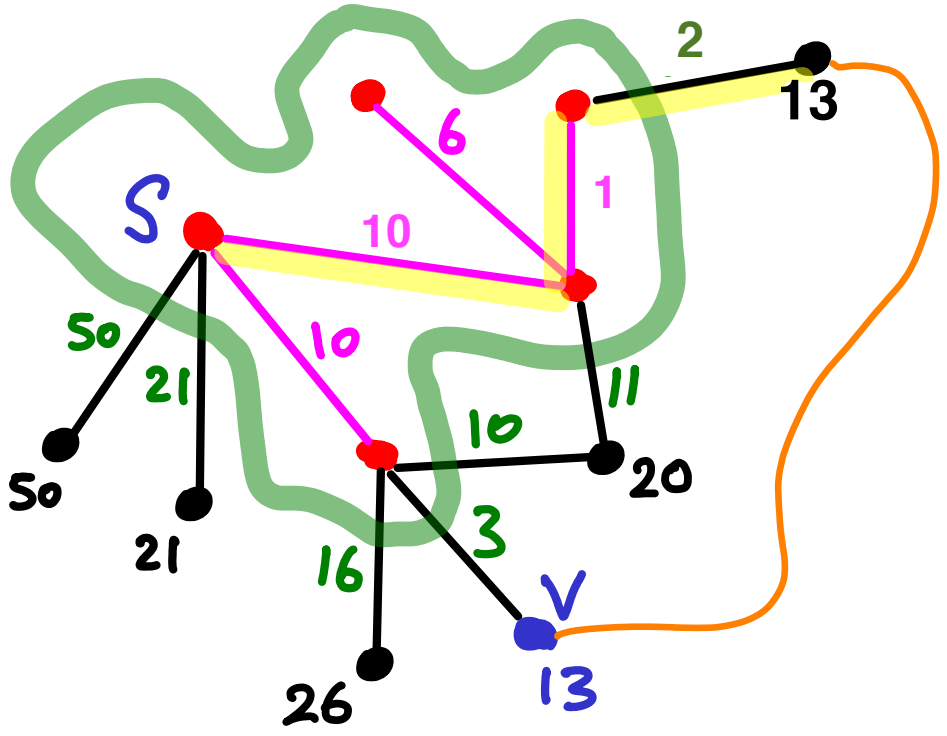

v = "closest" to S

Any other path s ~~~ v
will cost at least as much

Correctness:

assume we have shortest paths
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Just across CUT,
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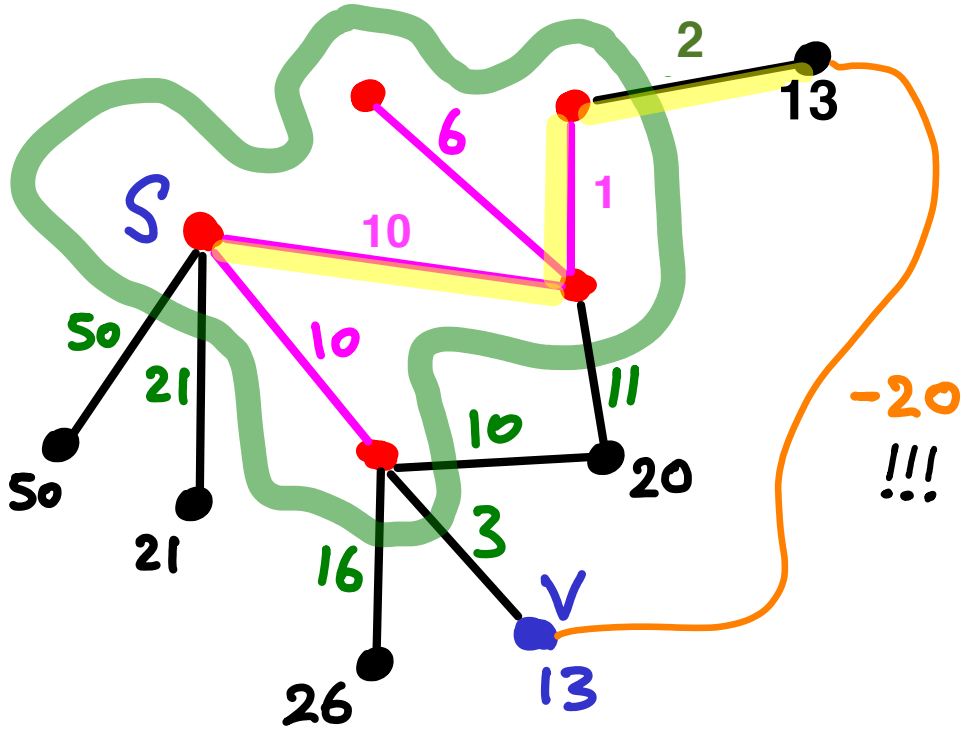

v = "closest" to S

Any other path s ~~~ v
will cost at least as much

} assuming weights ≥ 0

Correctness:

assume we have shortest paths
to a set of red vertices

Just across CUT,
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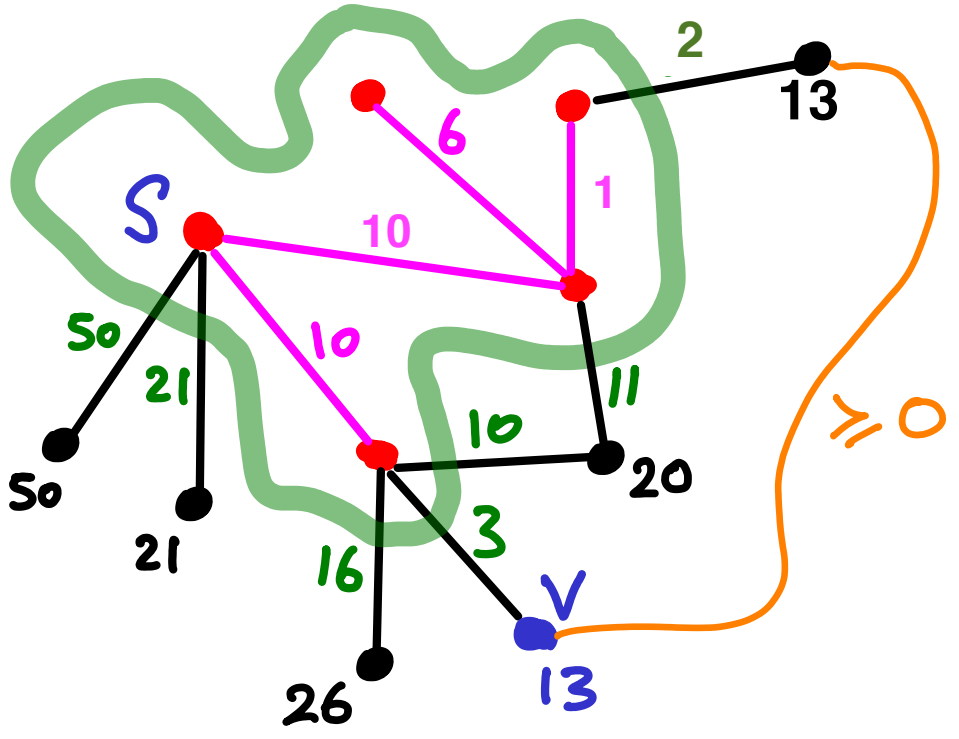


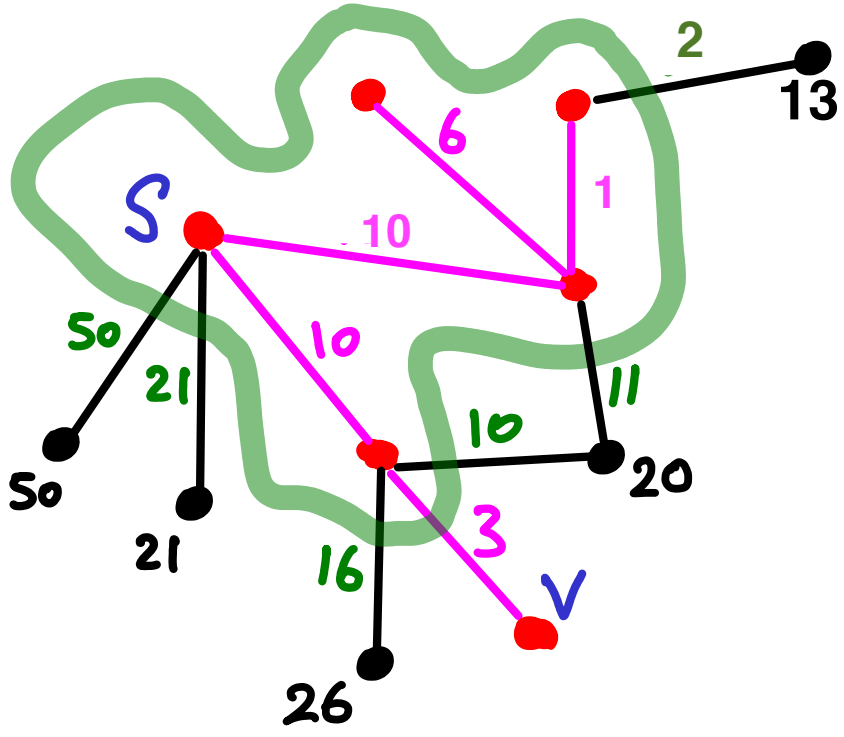
v = "closest" to S → safe to add

Any other path s ~~~ v
will cost at least as much

assuming weights ≥0

Correctness:

assume we have shortest paths
to a set of red vertices



Just across CUT,
vertices have finite scores
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v = "closest" to S → **safe to add**

Any other path s ~~~ v
will cost at least as much

} assuming weights ≥ 0

Correctness:

assume we have shortest paths
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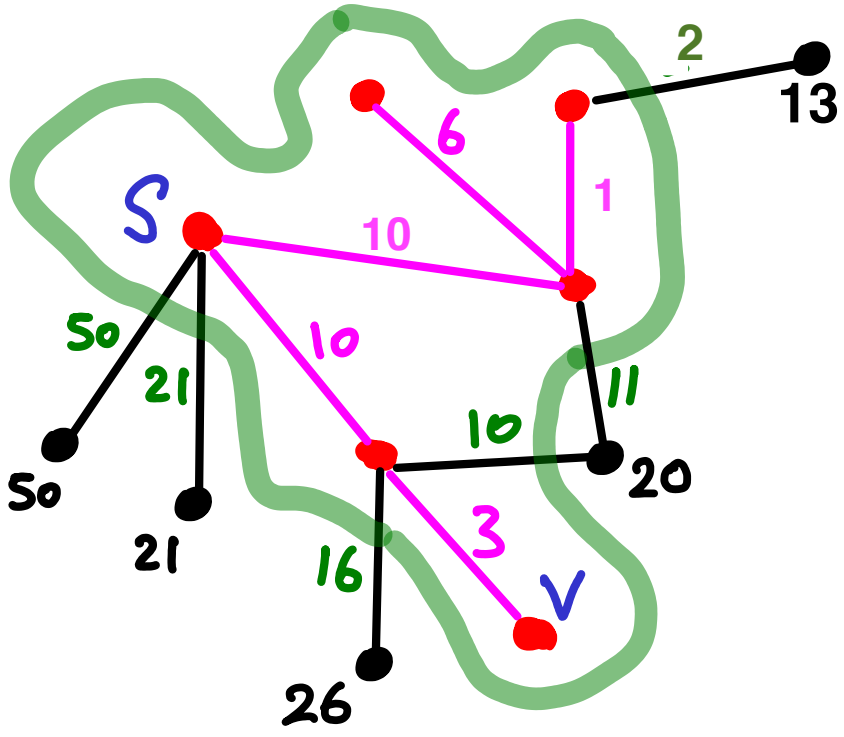

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Any other path s ~~~ v
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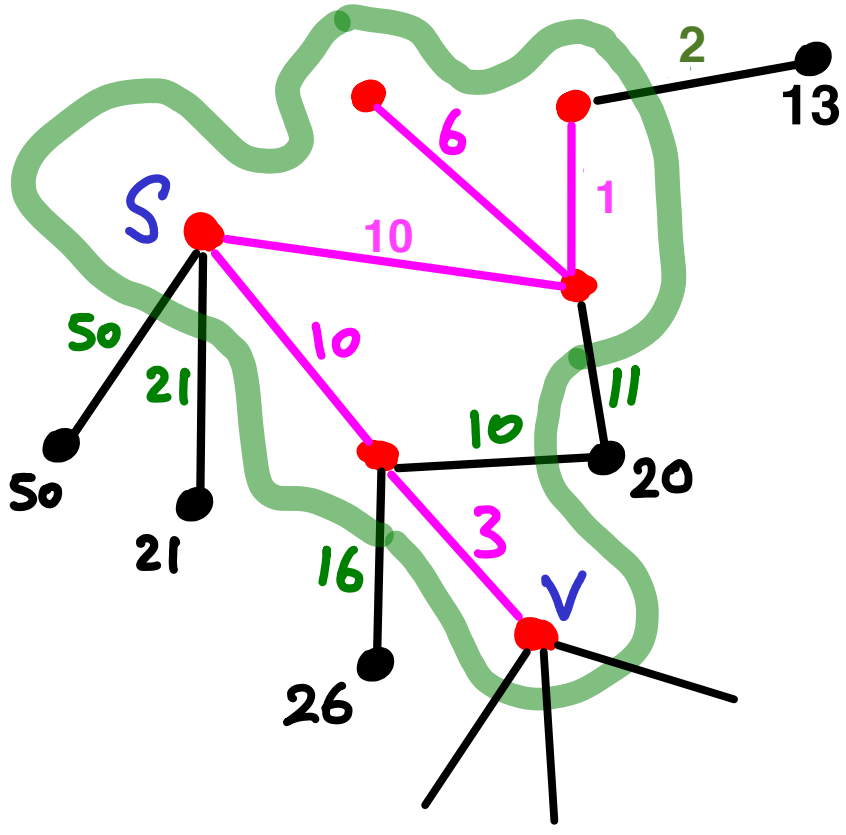


v = "closest" to S → safe to add

Any other path s ~~~ v
will cost at least as much

assuming weights ≥ 0

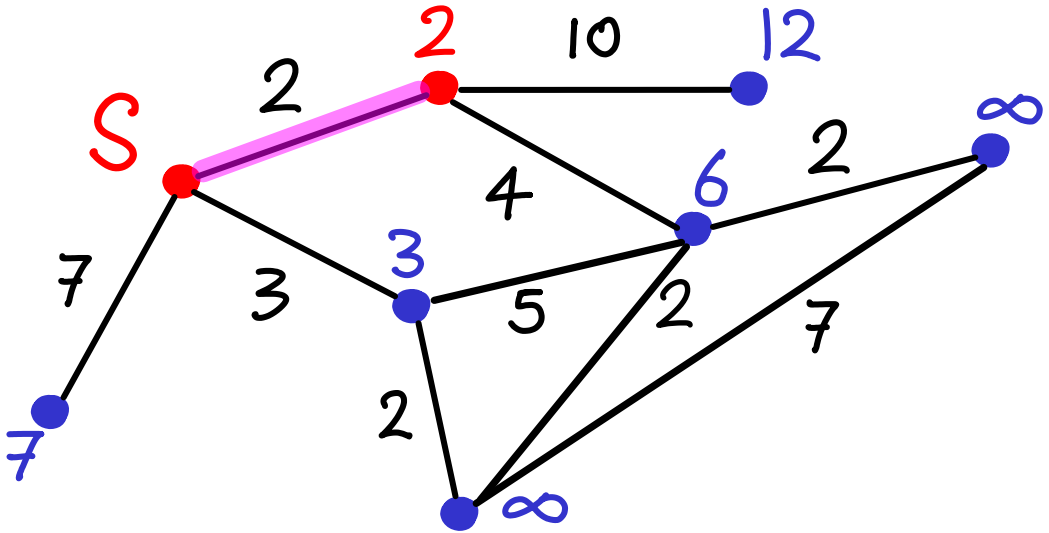
add "closest" vertex

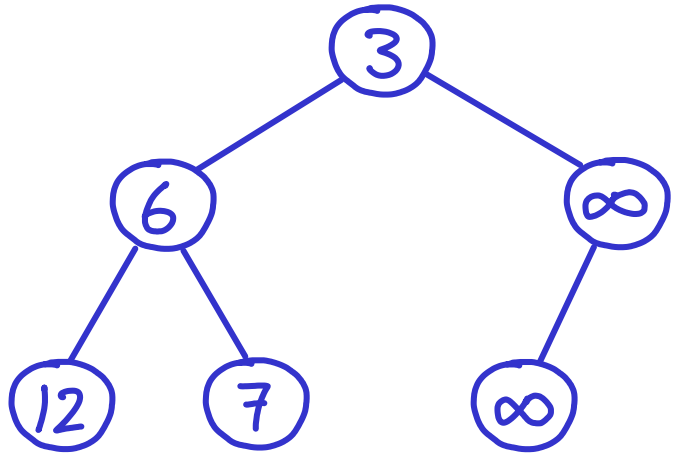
update set

= extract lowest score

= relax incident edges

Use a priority queue: heap



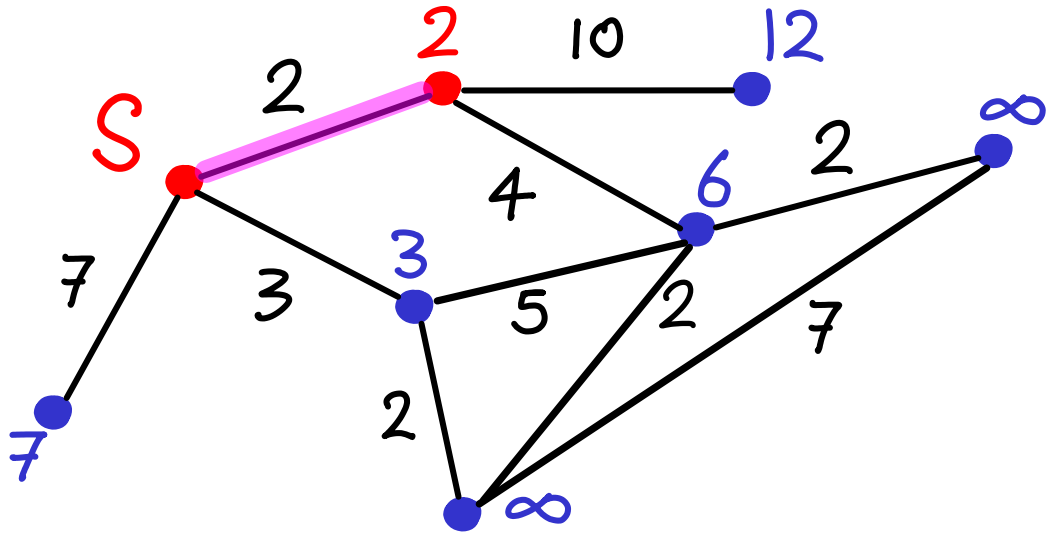


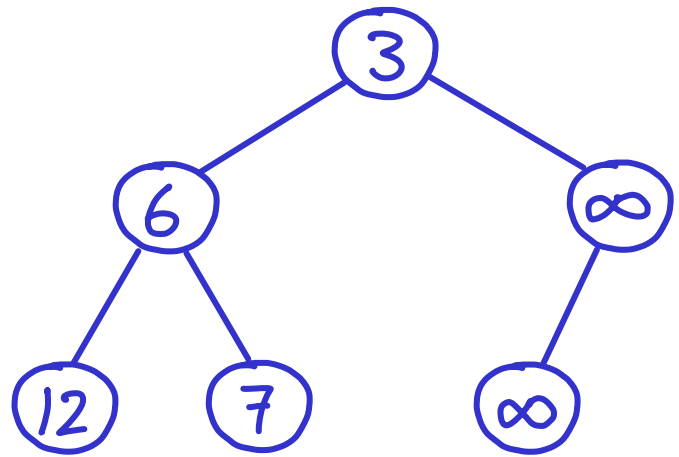
while priority queue not empty

| x: extract min (add to SSSP)

| for each neighbor y of x

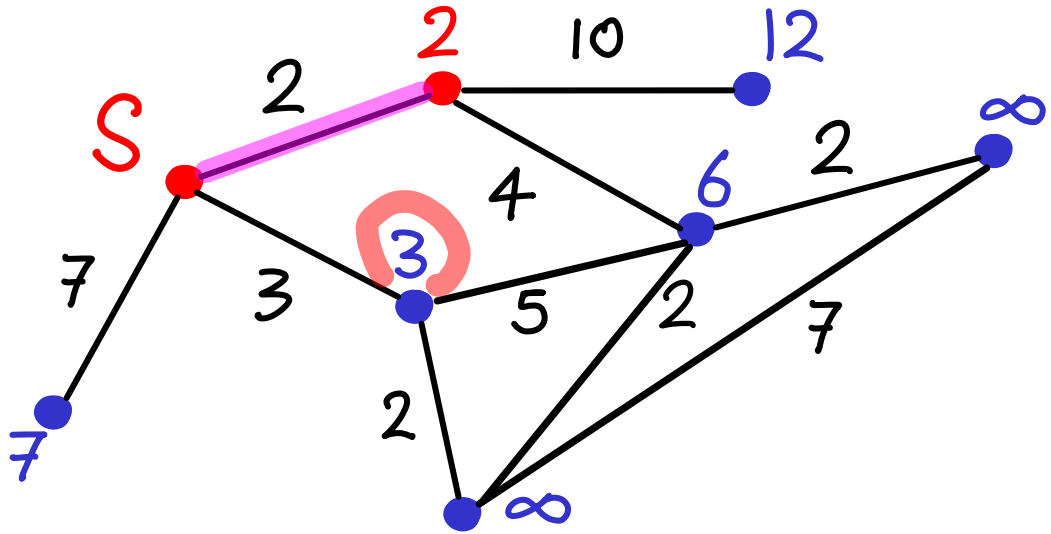
| RELAX(x,y)

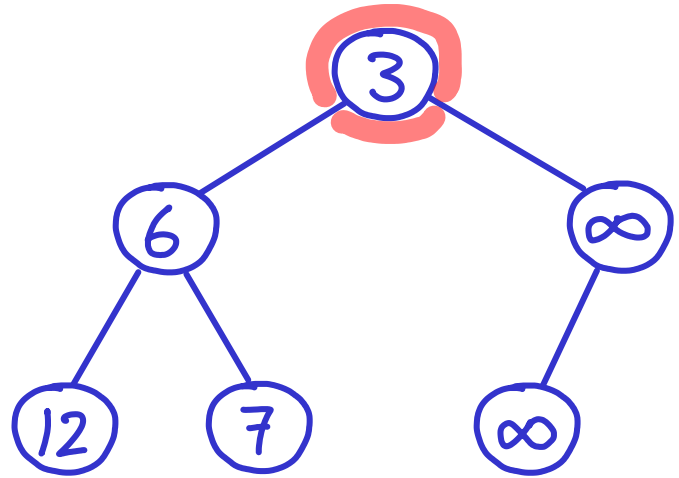




while priority queue not empty

- x: extract min (add to SSSP) → time?
  - for each neighbor y of x
    - RELAX(x,y)



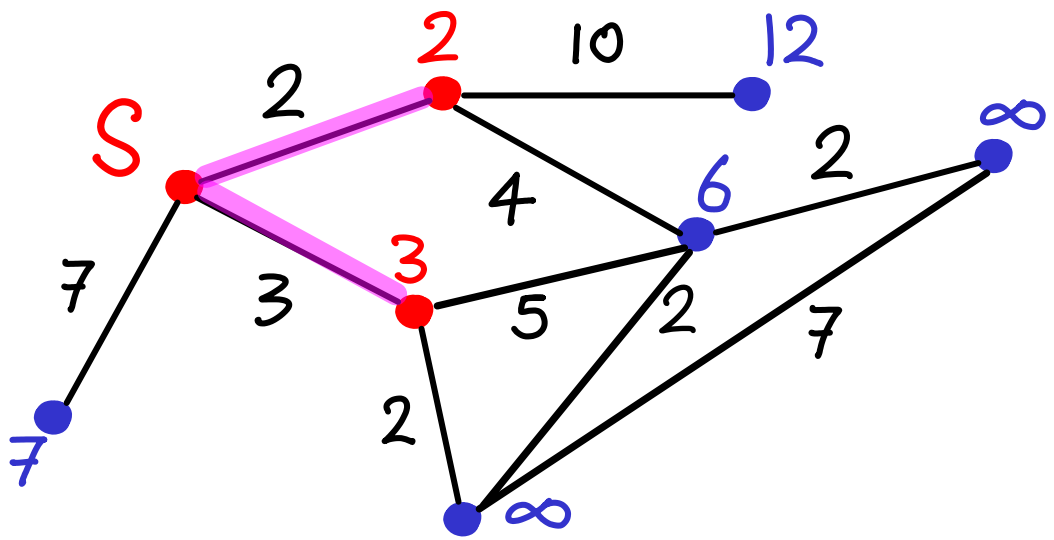


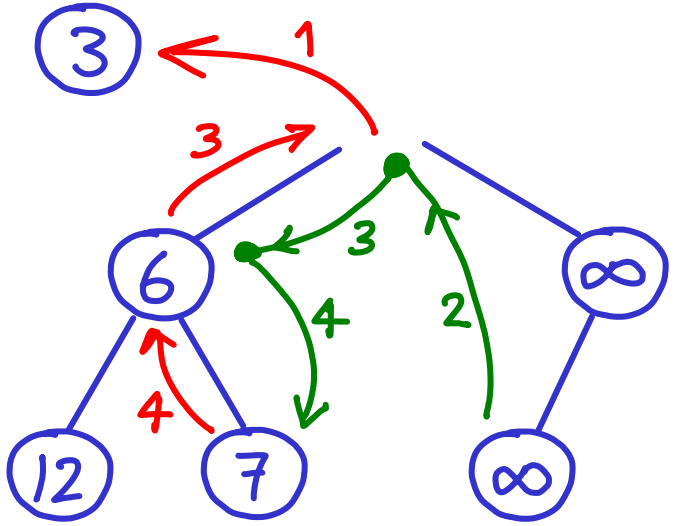
while priority queue not empty

| x: extract min (add to SSSP) → O(logV)

| for each neighbor y of x

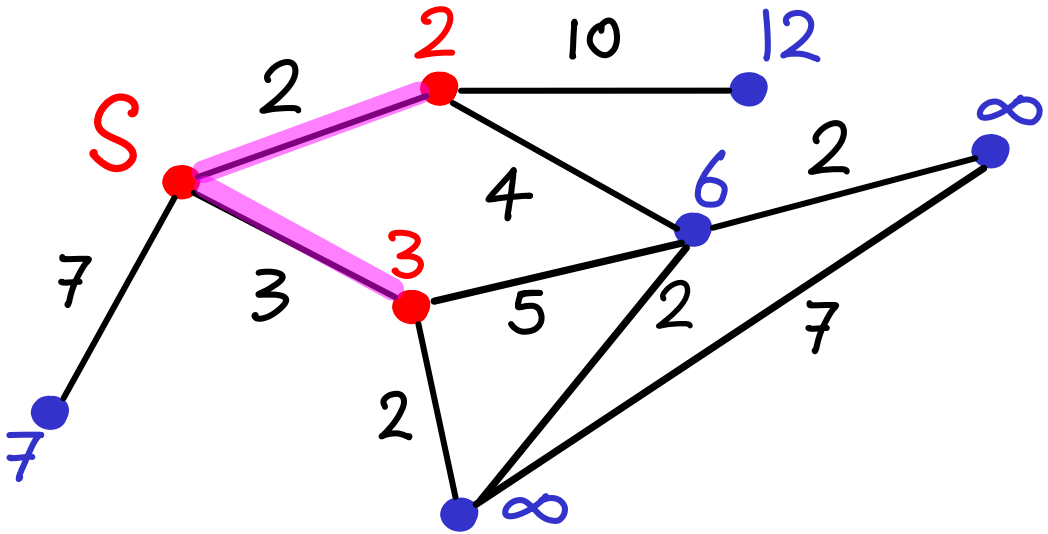
| RELAX(x,y)

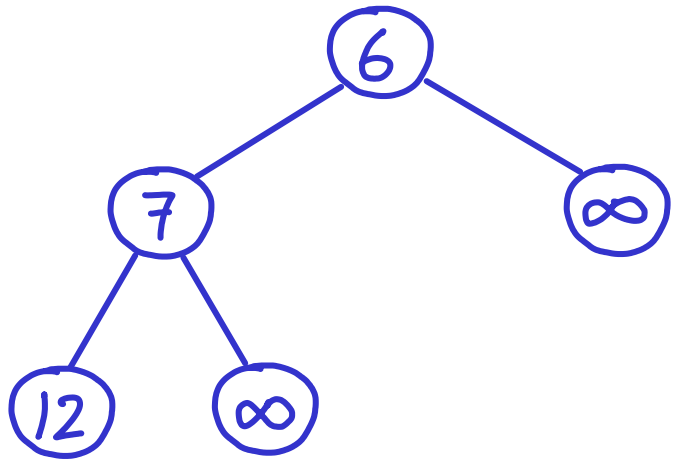




while priority queue not empty

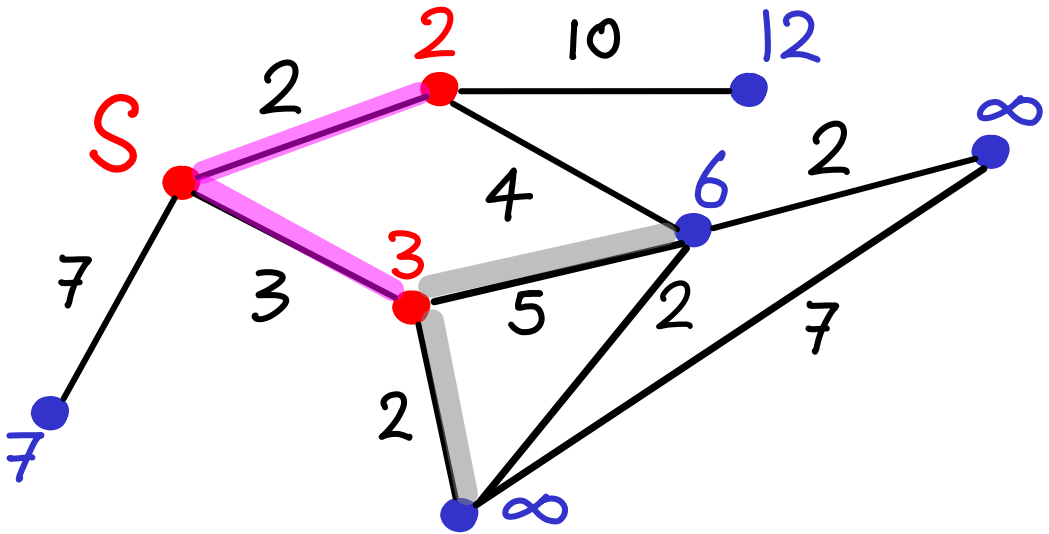
| x: extract min (add to SSSP) → O(logV)
| for each neighbor Y of x → how many?
| RELAX(x,Y)

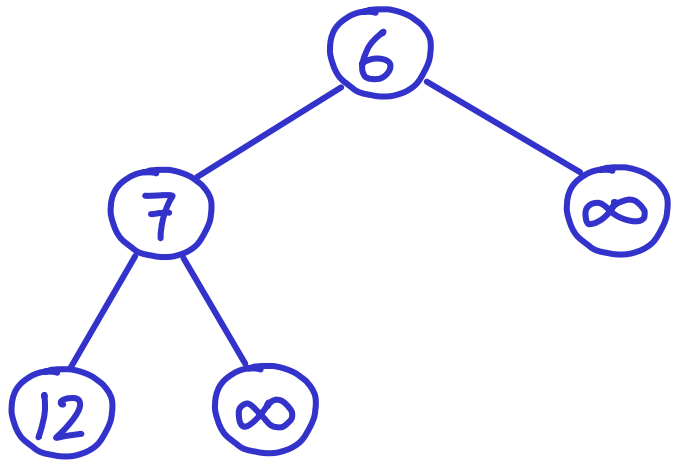




while priority queue not empty

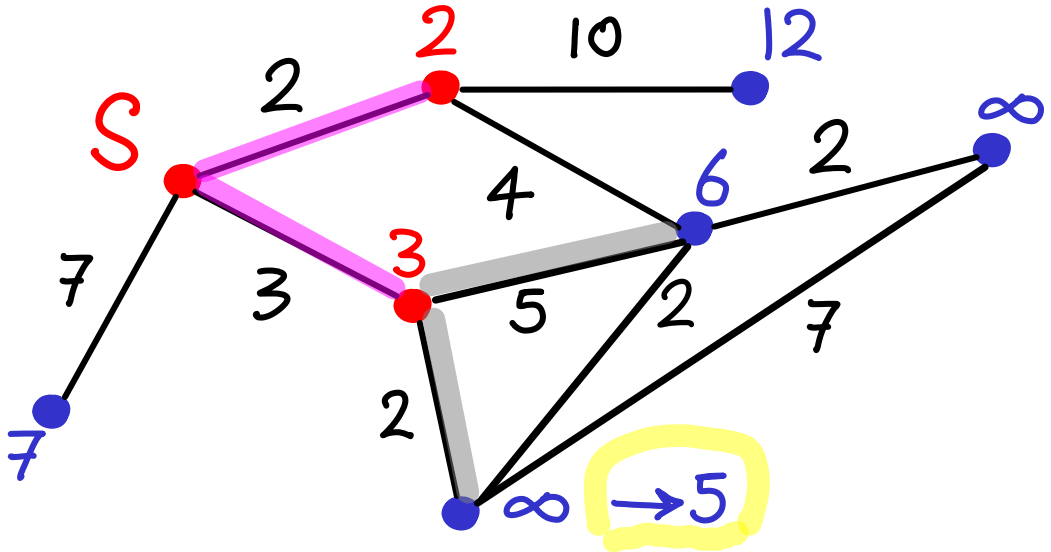
| x: extract min (add to SSSP) → O(logV)
| for each neighbor Y of x → O(deg(x))
| RELAX(x,Y) → cost?

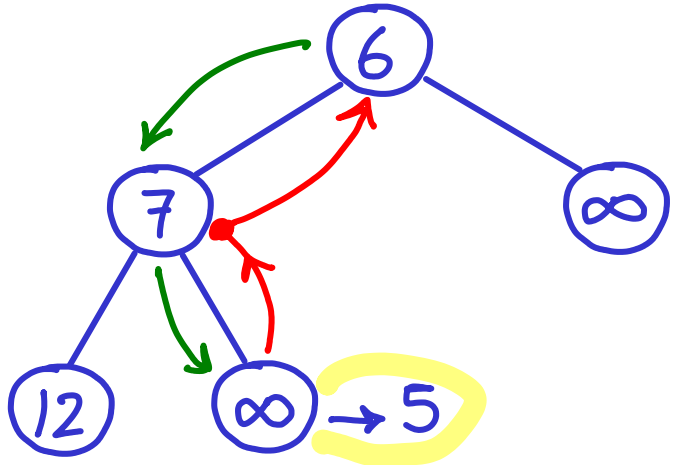




while priority queue not empty

- x: extract min (add to SSSP) → $O(\log V)$
- for each neighbor y of x → $O(\deg(x))$
  - RELAX(x,y) → decrease key → $O(\log V)$

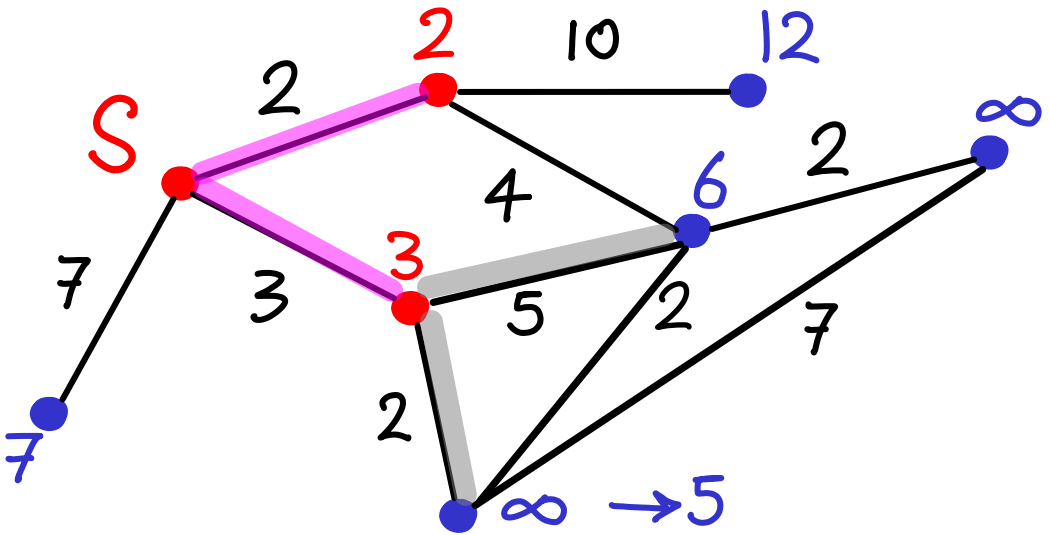


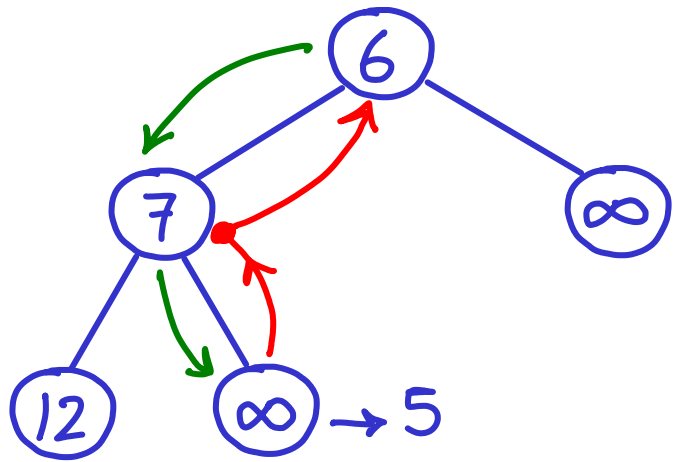


while priority queue not empty

TOTAL

x: extract min (add to SSSP)	→ O(logV)	O(VlogV)
for each neighbor y of x	→ O(deg(x))	?
RELAX(x,y) → decrease key	→ O(logV)	?





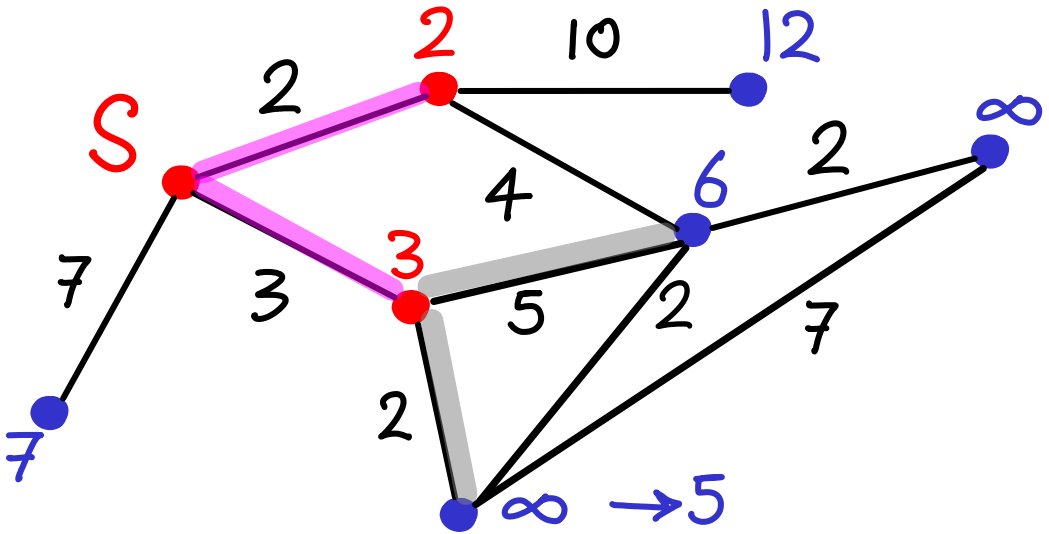
while priority queue not empty

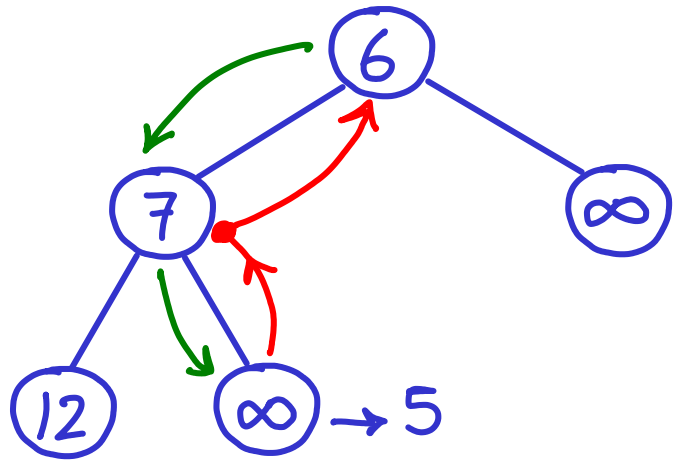
TOTAL

- x: extract min (add to SSSP) → O(logV) —— O(VlogV)
- for each neighbor y of x → O(deg(x)) }
  - RELAX(x,y) → decrease key → O(logV) }

O(ElogV)

if adj. list





add "closest" vertex

update set

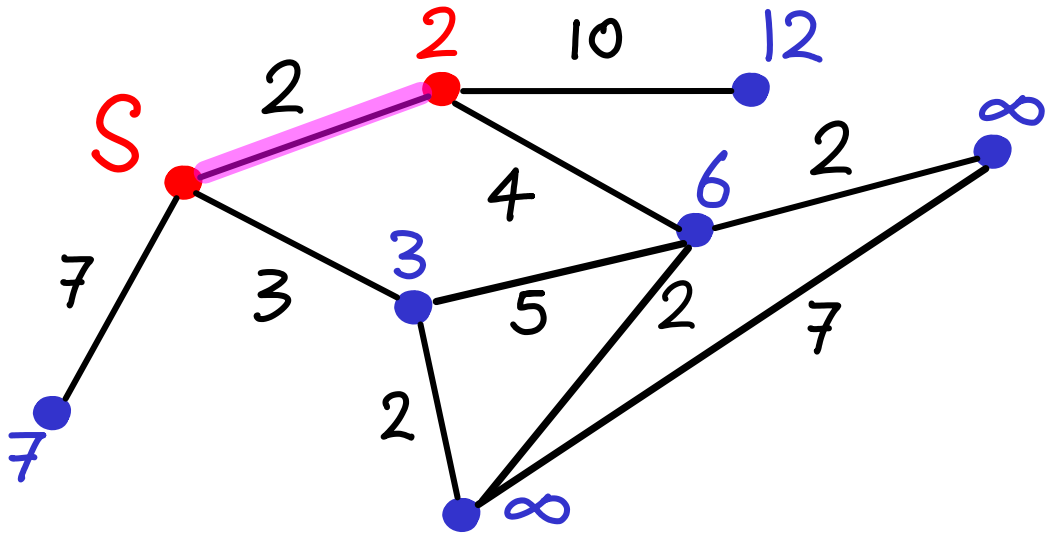
= extract lowest score

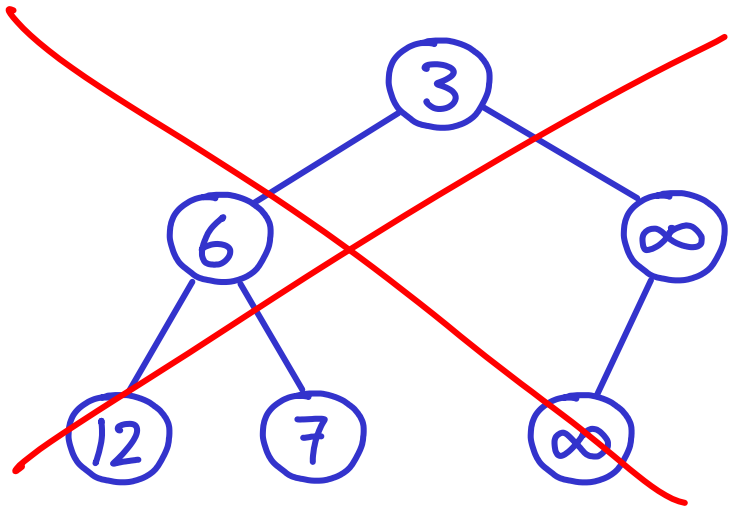
= relax incident edges

= O(V)

= O(V)

If adjacency matrix, don't use queue → $O(V^2)$





DIJKSTRA'S ALGORITHM

Summary: $O(E \log V)$ or $O(V^2)$

$\downarrow$

$V \log V + E \cdot (\text{decrease key})$

DIJKSTRA'S ALGORITHM

Summary: $O(E \log V)$ or $O(V^2)$

$\downarrow$

$V \log V + E \cdot (\text{decrease key})$

FYI: Fibonacci heap does decrease-key in $O(1)$ amortized

↳ $O(E + V \log V)$