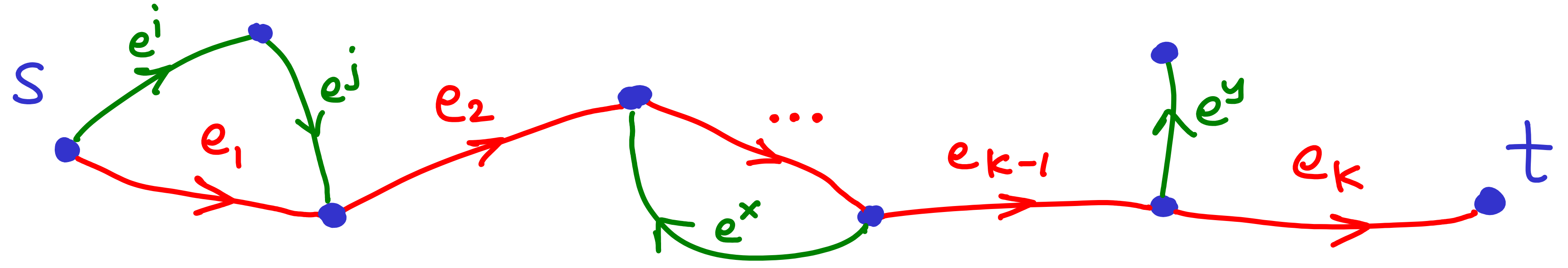
Assume this is a shortest path from s to t unknown but exists



Suppose we have an algorithm based on relaxing edges.

If we relax e_1 before e_2 before ... before e_{k-1} before e_k

then we will correctly compute $d(t)$ $\boxed{\text{by INDUCTION}}$

Relax sequence : $e^x\ e_1\ e^j\ e^y\ e_2\ e^x e^i\ e_k\ e_{k-1}\ e_1 e^x\ e_k\ e^y$: OK

(don't care if we relax other edges or the same ones repeatedly)

BELLMAN-FORD ALGORITHM

simple, but made even simpler

(not identifying negative cycles)

A. Shimbel (1954) → slower variant

L. Ford (1956) → slower variant

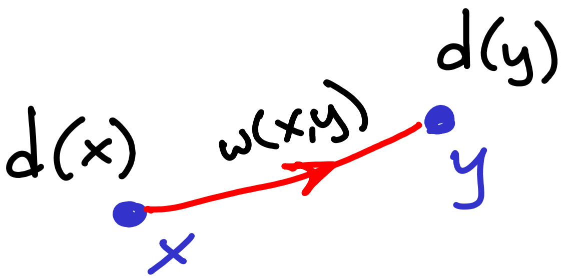
E. Moore (1957) → non-negative weights

R. Bellman (1958)

Finds a shortest path from s to ALL vertices

BELLMAN-FORD ALGORITHM $O(V \cdot E)$

1) set score of s : zero
set score of $\neq s$: ∞
set parent of $\neq s$: null



2) for $i = 1$ to $V-1$

RELAX every edge in G

Why does this work?

FOR ANY t

In iteration i

we will get $d(p_i^t)$

on some shortest path p^t

$= \{p_1^t, p_2^t, p_3^t, \ldots, p_k^t = t\}$

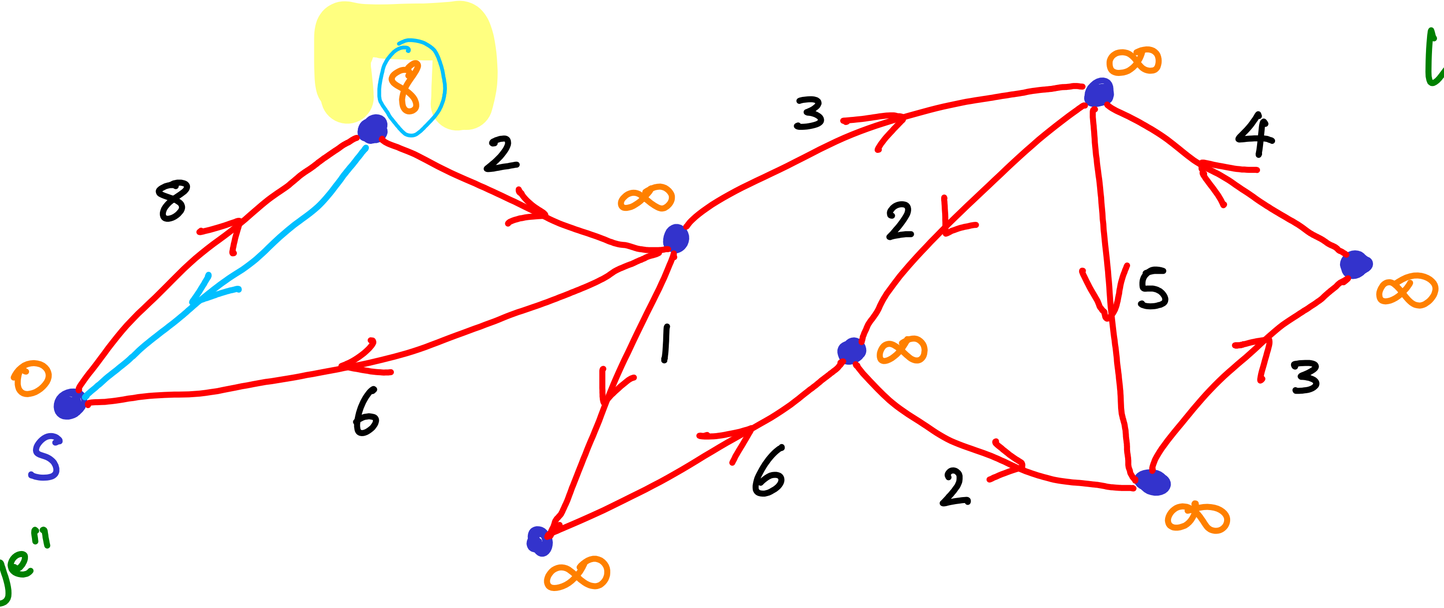
from $s \to t$

RELAX an edge $x \to y$

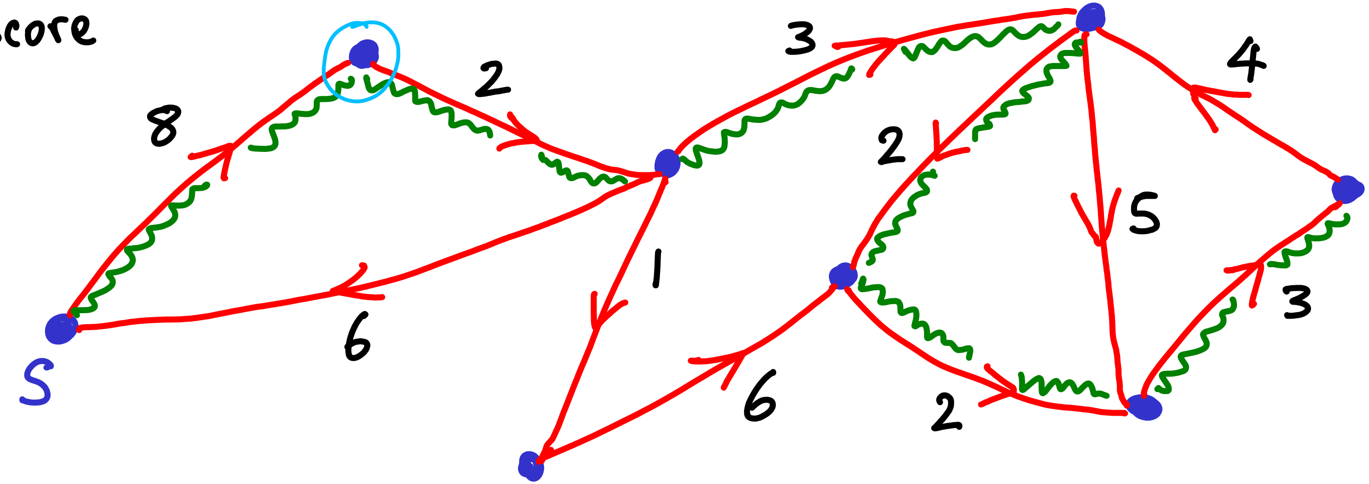
if $d(x) + w(x,y) < d(y)$

then $d(y) = d(x) + w(x,y)$
$\quad$ parent$(y) = x$

$i = 1$



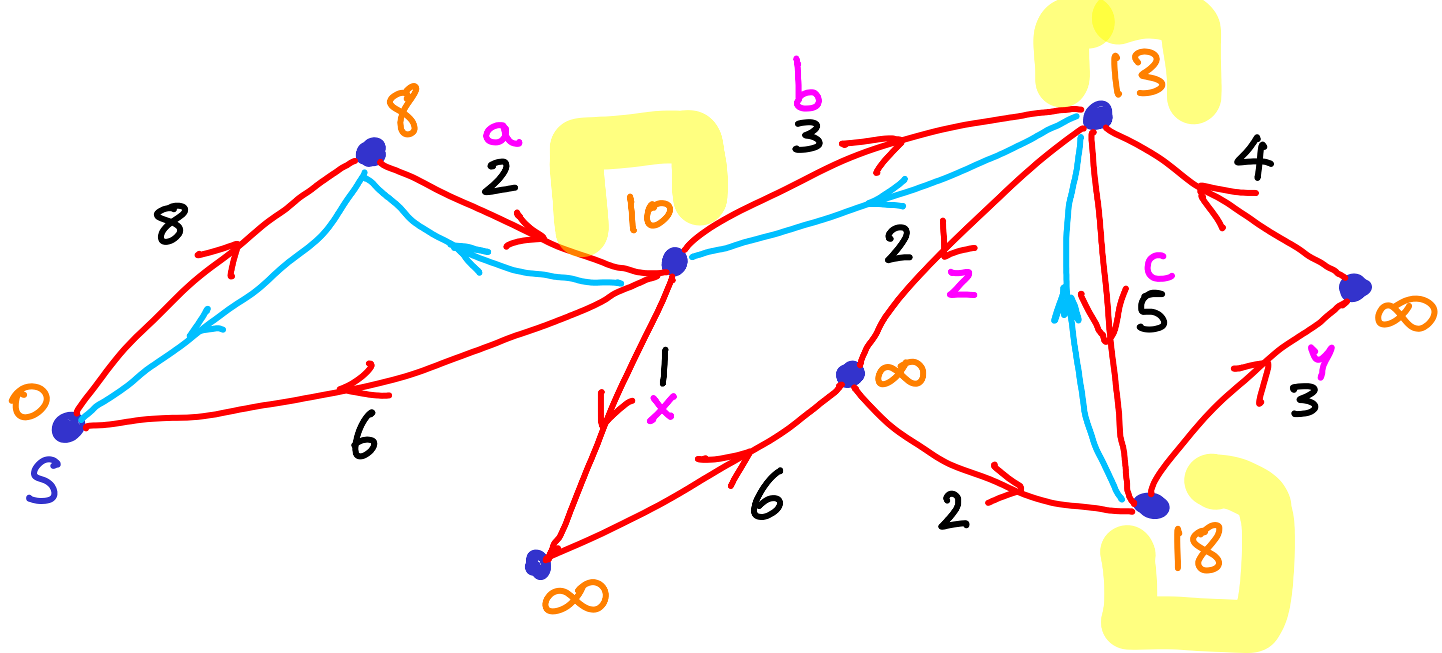
depending on order of processing edges in "RELAX every edge" we might get only 1 finite score or actually be done.



What matters is that we get final score of a vertex on shortest path to target

& in fact we extend a chain of such vertices.

$i = 2$

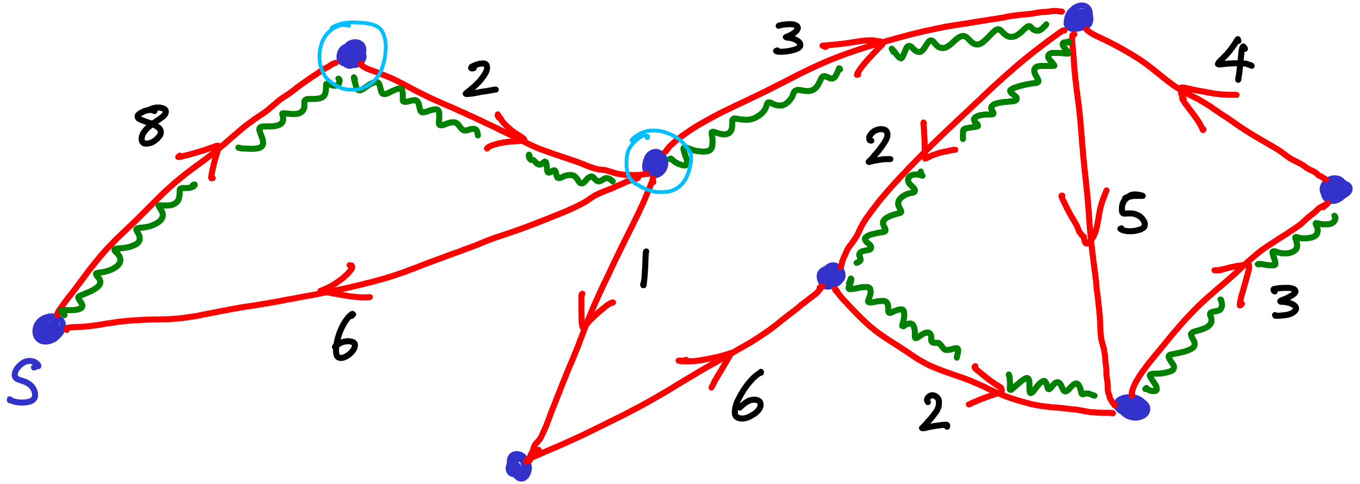


order of relaxing?

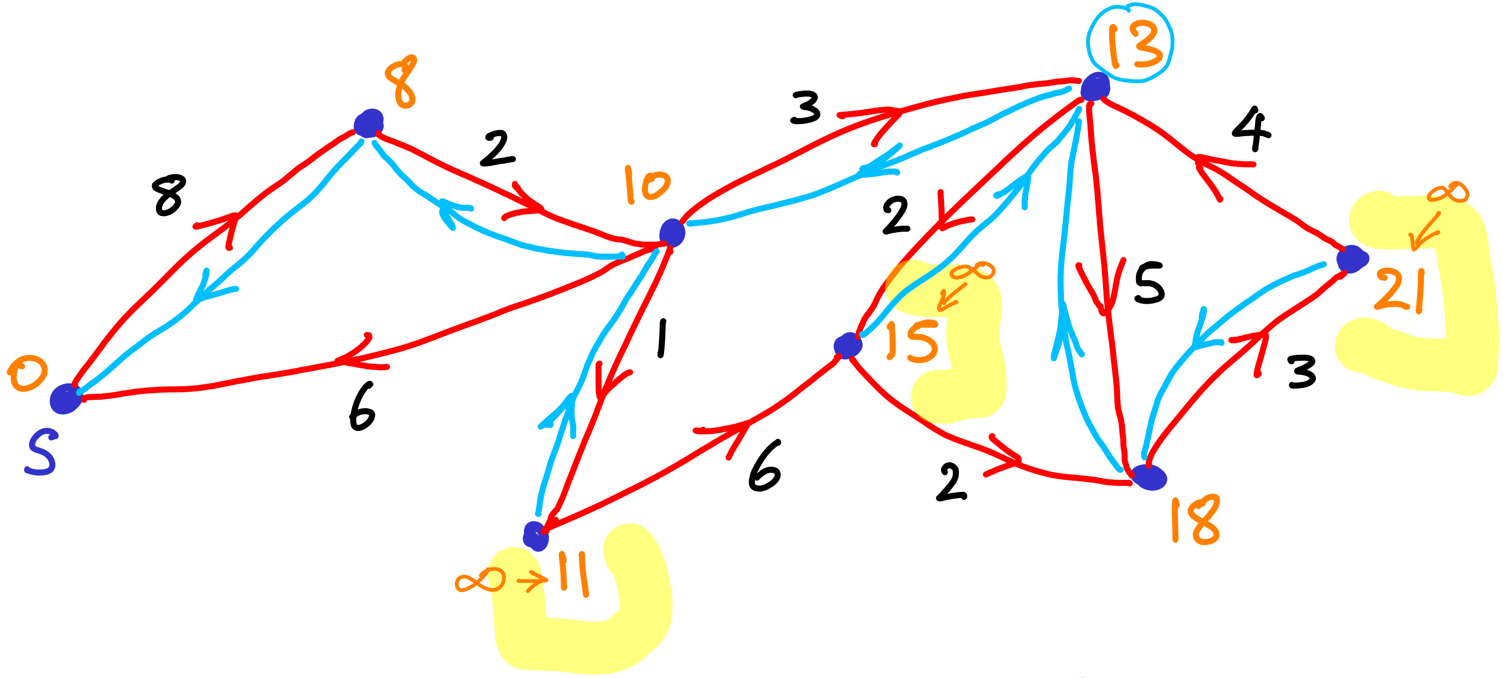
x...a...b...c

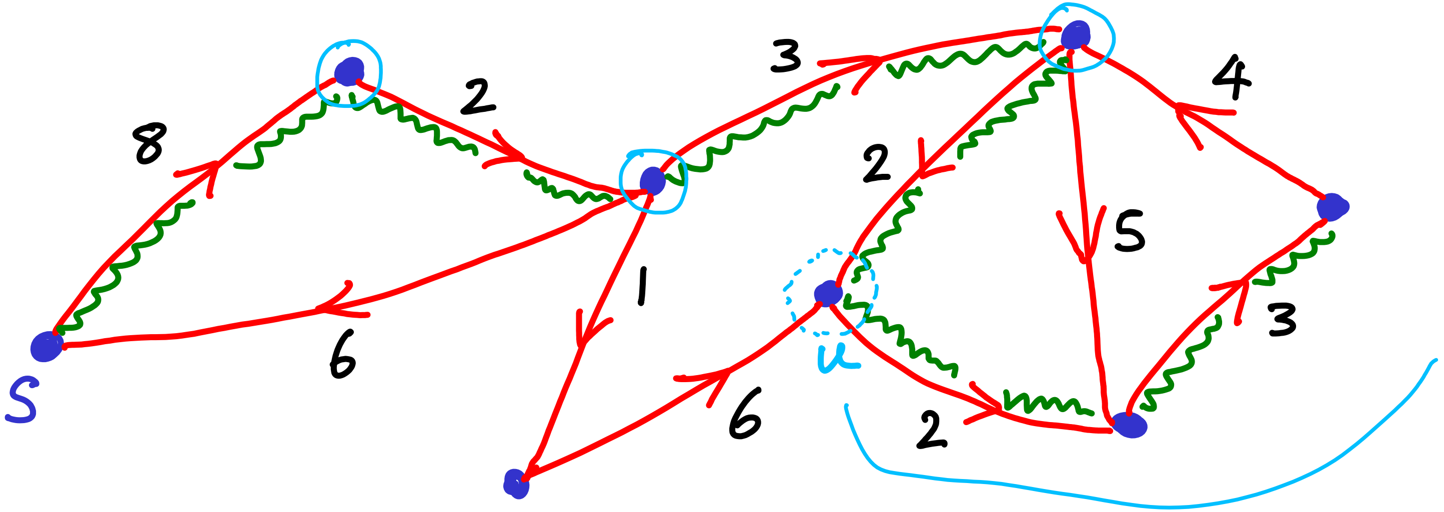
y...c

z...b



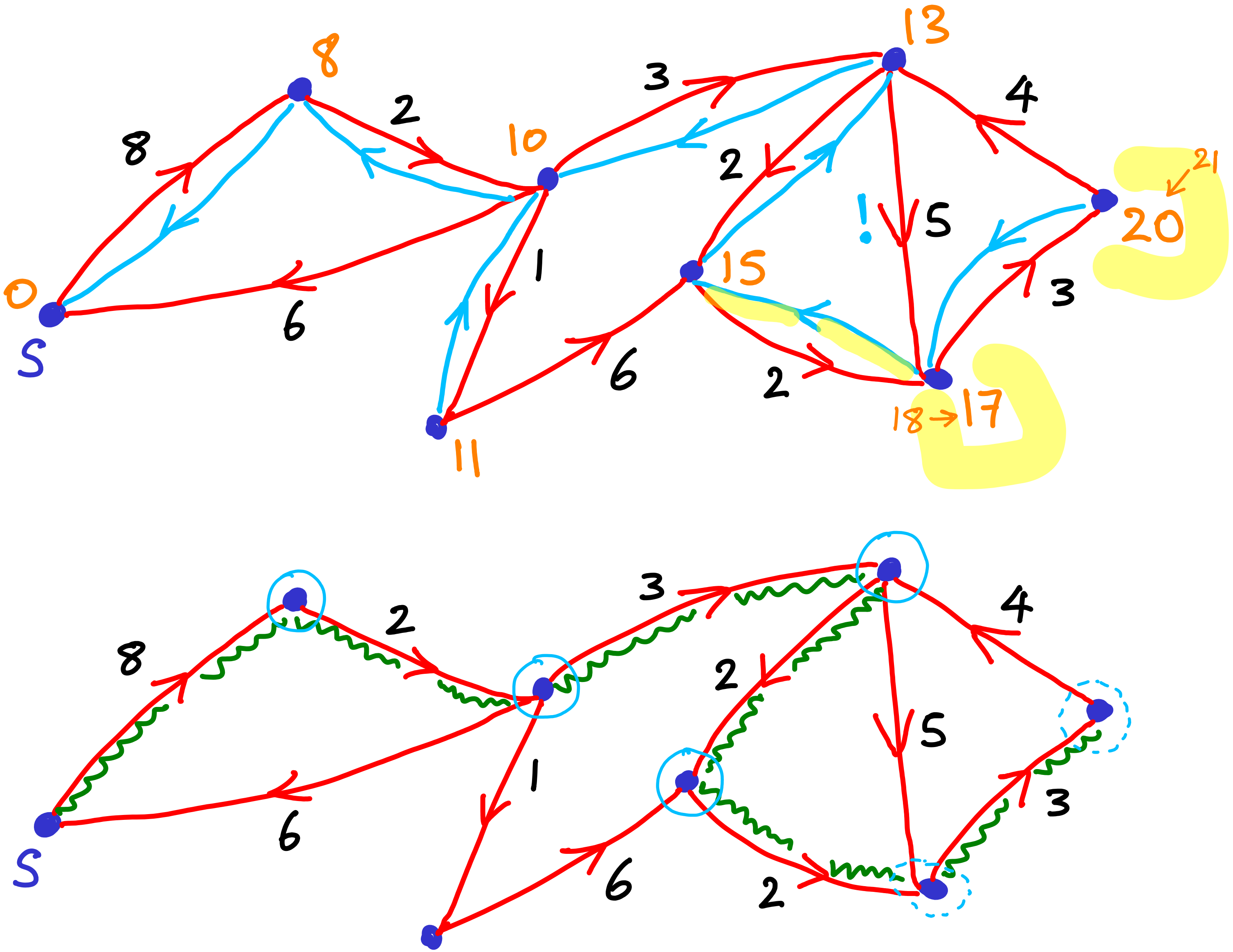
$i = 3$





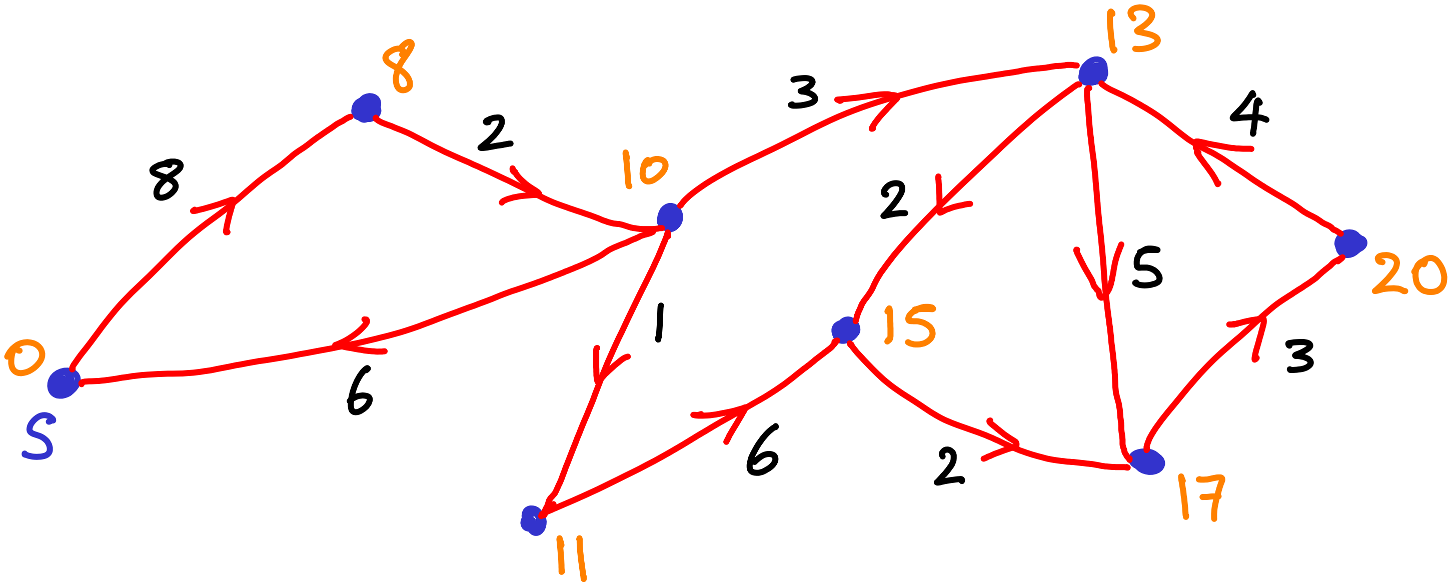
we have found shortest path to u ahead of schedule

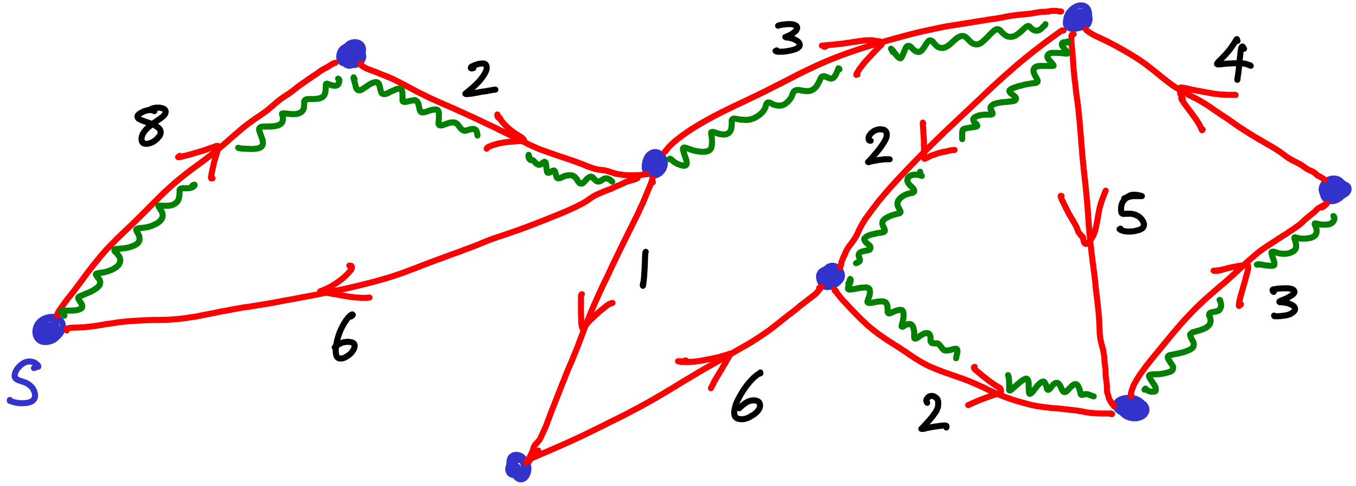
$i = 4$



$i = 5, 6, 7$

redundant





BELLMAN-FORD ALGORITHM

Works for negative weights
& can detect negative cycles.

scores will still be changing after $i = V-1$

(see CLRS)