Improved Computation of Core Inductance for Fast Transient Analysis of Transformers

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***Abstract*—This letter introduces a simple and accurate formula for the computation of the inductance component due to the flux inside the transformer core for fast transient analysis. When compared with an existing formula, the new formula yields considerably better behavior for a broad frequency range (1 Hz to 10 MHz). In addition, two finite-element method approaches for core inductance computation at different frequencies are evaluated and compared: flux linkage and method of energy. It is demonstrated that these methods can provide substantially different results at high frequencies.**

***Index Terms*—Fast transients, magnetic fields, transformer.**

I. Introduction

Fast front electromagnetic (EM) transient analysis of transformers commonly relies on simulation tools, which require the development of mathematical models. One of the most important parts in the development of these models is parameter determination. Transformer models for fast transient analysis are based on inductive, capacitive, and resistive parameters. Inductive parameters can be divided into three parts: 1) inductance due to the flux inside the core window (usually referred to as air inductance); 2) inductance due to the flux inside the winding conductors (conductor inductance); and 3) inductance due to the flux inside the core (core inductance). The latter inductance is treated in this letter.

At operating frequency, the magnetic flux inside the transformer core is very high due to the large permeability of electrical steel. However, when dealing with fast front transients with high-frequency content, the flux inside the core is considerably reduced because of eddy currents. At very high frequencies (>1 MHz), the flux inside the core becomes negligible (the core walls act as a flux barrier) [1]. For frequencies between operating and 1 MHz, the behavior of the core flux is frequency dependent. Therefore, computing the inductance related to this flux is not an easy task. On the other hand, due to the reduction of magnetic flux density, the core behavior at high frequencies is linear (saturation is not reached).

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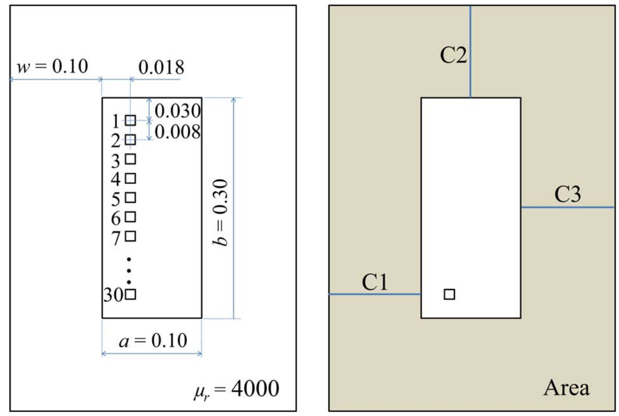

Fig. 1. Cross-sectional view of the transformer: (a) physical dimensions and (b) contours and integration area.

This letter presents a simple formula for the computation of core inductance that focuses on fast front transients, but is able to reproduce the behavior of the core flux for a broad frequency spectrum with considerably better accuracy than the most commonly approach used these days. In addition, the two main choices used in the finite-element method (FEM) for the computation of the core inductance are evaluated in this letter: flux linkage and method of energy. By means of a test case, it is shown that the method of energy is a better alternative, particularly for the high-frequency spectrum.

II. FEM Computations

Two approaches for the computation of core inductance using FEM are considered in this letter: by means of the magnetic flux Φ through surface s

$$\Phi = \int_s \mathbf{B} \cdot d\mathbf{s} = Li \tag{1}$$

and by means of the magnetic energy W_m stored in volume Ω

$$W_m = \int_\Omega u_m d\Omega = \frac{1}{2}Li^2 \tag{2}$$

where $\mathbf{B}$ is the magnetic flux density, u_m is the magnetic energy density, L is the inductance, and i is the excitation current. Considering a 2-D geometry (see Fig. 1), the surface integral over s is replaced by a contour integral per unit of transversal length, while the volume integral over Ω is replaced by a surface integral per unit of transversal length.

For simplicity, a solid core is considered (laminations are not included in the simulations). In order to approximate the skin effect in the core at high frequencies, an equivalent resistivity is

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computed, so that the flux penetration into the core is the same as the penetration in a laminated core.

III. Analytical Computation

A. Core Inductance at Low Frequency

A well-known formula used for the computation of core inductance at nominal frequency (per unit of transversal length) is given by [1]

$$L_{LF,1} = \frac{\mu_0 \mu_r w}{\ell} \tag{3}$$

where μ_0 is the permeability of free space, μ_r is the relative permeability of the core, ℓ is the mean length of the flux trajectory, and w is the core width. Defining a and b as the internal dimensions of the core window [see Fig. 1(a)], ℓ is computed as

$$\ell = 2(a+b) + 4w. \tag{4}$$

Equation (3) assumes that the magnetic flux penetration into the core is uniform. However, even at low frequencies, the longitudinal flux density is not uniform, but a function of the horizontal position x

$$B(x) = \frac{\mu_0 \mu_r i}{2(a+b) + 4x}. \tag{5}$$

Applying (5) in (1) it follows that:

$$L_{LF,2} = \frac{\mu_0 \mu_r}{4} \ln\left(\frac{a+b+2w}{a+b}\right). \tag{6}$$

Equation (6) is the formula proposed in this letter. It considers the nonuniformity of the flux density, and it does not require defining a mean length of flux trajectory.

B. Frequency-Dependent Core Inductance

Due to the skin effect, flux penetration into the core is inversely proportional to frequency. A simple way to take this into account is by means of a modification of (3) [with ℓ defined by (4)] and (6) that considers, instead of w, a fictitious frequency-dependent width p based on the concept of complex penetration depth [2]. This results in the definition of a core impedance (core losses related to eddy currents are also accounted for, but not hysteresis losses)

$$Z_{\text{core},1} = \frac{j\omega\mu_0\mu_r p}{2(a+b)+4p} = R_{\text{core},1} + j\omega L_{\text{core},1} \tag{7a}$$

$$Z_{\text{core},2} = \frac{j\omega\mu_0\mu_r}{4} \ln\left(\frac{a+b+2p}{a+b}\right) = R_{\text{core},2} + j\omega L_{\text{core},2} \tag{7b}$$

where

$$p = \delta \tanh\left(\frac{w}{\delta}\right), \delta\sqrt{\frac{1}{j\omega\mu_0\mu_r\sigma_c}}. \tag{8a-b}$$

ω is the angular frequency and σ_c is the conductivity of the core. Equation (7b) is the proposed formula for a broad frequency range. In (8a), factor $\tanh(w/\delta)$ precludes the penetration depth from exceeding w, since $|p|$ is a decreasing function of frequency with a limiting value of w at 0 Hz.

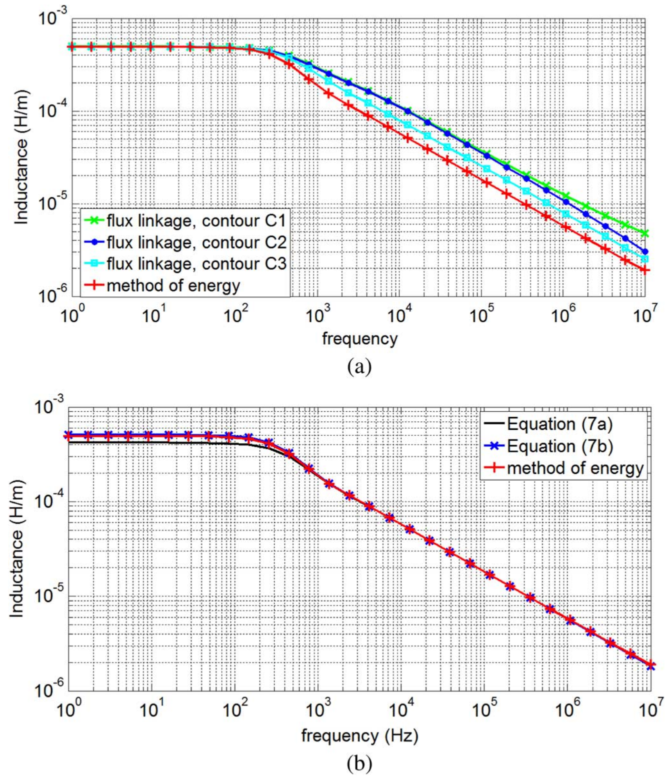

Fig. 2. Core inductance: (a) computed from FEM and (b) computed from analytical expressions and compared with the method of energy.

IV. Comparisons

Turn 30 of the transformer shown in Fig. 1(a) is excited in order to compare the results from FEM and applying analytical expressions (7a) and (7b). The contours and area considered for the flux linkage and energy method are shown in Fig. 1(b). The core inductance obtained with FEM is shown in Fig. 2(a). It can be seen that the results from the flux linkage method integrating at different contours are very similar at low frequencies but they tend to differ as frequency increases. At high frequencies, the flux measured at individual contours is different. The method of energy does not have this issue since it considers the complete core area.

Fig. 2(b) shows a comparison of the results from (7a) and (7b). Both formulas provide very similar results to the method of energy, except at the low-frequency range. Relative differences of (7a) and (7b) at such ranges are 17.35% and 3.52%, respectively. Thus, the proposed formula provides considerably better behavior for the complete frequency spectrum. Although not shown here due to space constraints, the behavior of the core inductance when exciting other turns is very similar.

V. Conclusions

Numerical and analytical computation of the core inductance for fast front transients in transformers has been discussed in this letter. An analytical formula that considers the nonuniformity of the longitudinal flux distribution has been proposed. According to the results, this formula can provide high accuracy for the complete frequency spectrum. On the other hand, it has been shown that when applying FEM, the method of energy can provide better results than the method of flux linkage since it considers the flux distribution inside the complete core geometry.

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