



Whitehead's Theory of Gravity

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1. Introduction

In 1922 in *The Principle of Relativity*, Whitehead presented an alternative theory of gravitation in response to Einstein's general relativity. To the latter, he objected on philosophical grounds—specifically, that Einstein's notion of a variable spacetime geometry contingent on the presence of matter (a) confounds theories of measurement, and, more generally, (b) is unacceptable within the bounds of a reasonable epistemology. Whitehead offered instead a theory based within a comprehensive philosophy of nature. The formula¹ Whitehead adopted for the gravitational field has been described as involving both the flat metric $\eta_{\mu\nu}$ of Minkowski spacetime and a dynamic metric $g_{\mu\nu}$ dependent on the presence of source masses. The ontological relationship between the two must be fleshed out in the context of Whitehead's philosophy of nature. The relationship is of some importance, not only in casting Whitehead's theory within its proper metaphysical context *vis-à-vis* Einstein, but also in judging how the theory has fared empirically with respect to general relativity (GR hereafter). It makes the same predictions as GR with respect to the perihelion advance, the deflection of light rays and the gravitational red-shift; indeed, Eddington (1924) has shown that it is equivalent to the Schwarzschild solution of Einstein's field equations for the one-body problem. However, it also appears to predict an anisotropy in the locally measured gravitational constant γ that is in conflict

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¹ Whitehead (1922) gives four formulae, one of them being Einstein's field equations restricted to free space, all of which he maintains are consonant with his philosophy of nature. The fourth formula is the one that has been given the most attention in the literature and is the one presented in this paper.

with experimental data (Will, 1971). Just how the ontological status of the two metrics affects this result is a bit unclear in the literature. One of the aims of this paper is to attempt some clarification on this matter.

Before introducing Whitehead's formalism, I would like to set the philosophical stage, both in terms of Whitehead's ontology and in terms of his methodology. While this may appear tedious, I believe it is essential for a proper understanding of his theory, its reception and its subsequent fate. Sections 2 and 3 present a brief description of Whitehead's ontological and methodological commitments. Section 4 covers Whitehead's objections to GR and canvasses various responses that have appeared in the literature. Section 5 presents Whitehead's theory of gravity and Section 6 considers Will's (1971) claim of disconfirmation.

2. Philosophy of Nature

To understand the motivations for Whitehead's theory, one must first understand his critique of classical physics. This is given most succinctly in Whitehead (1925) in which he criticises scientific materialism and the effect it has had on science. The chief culprits are identified as the Aristotelian conception of independent, isolatable substance coupled with Cartesian dualism. This is worrisome, if only for the Humean skepticism it generates. Whitehead's solution is to question the fundamental claim that the immediate content of experience consists in perceptions of matter confined to instantaneous static moments. This is replaced with the claim that nature is experienced in temporal slabs of becoming. The effect of emphasising the former in theory construction is the bifurcation of nature into nature as sensed and nature as abstracted by science. In attempting to avoid this bifurcation, Whitehead's intent is not to do away with the modes of abstraction of science, but to revise them to account for what he claims is the more adequate metaphysics.² An adequate science must endeavour to acknowledge this by employing concepts that clarify resulting ontological commitments. This motivates Whitehead to replace the particle ontology of classical physics with an ontology based on dynamically related events.

Whitehead's philosophy of nature as given in (1922), then, is a doctrine of dynamic relatedness. The main problem he faces is to provide a means by which the relatedness exhibited in nature can be made known, the chief concern being that systems of complete relatedness face the quandary of requiring knowledge of everything before anything can be known. To avoid this, Whitehead emphasises both the continuity and the atomicity of nature. The three key ontological elements he introduces to accomplish this are *events*, *objects*, and the *physical field*. I now consider each of these in turn.

For Whitehead, the fundamental facts exhibited in nature are *events*. An event has the basic characteristic of extension in the sense of extending over, including and being included by other events. The essence of an event consists in its

² This essentially is a reversal of the Aristotelian categories: Becoming is given priority over Being.

relatedness. Nature is seen as '[...] a becomingness of events which are mutually significant of each other so as to form a systematic structure' (1922, p. 21). This systematic structure is necessary '[...] in order that we may know of nature as extending beyond isolated cases subjected to the direct examination of individual perception' (1922, p. 64). Again:

The structure [of the relations between events] is uniform because of the necessity for knowledge that there be a system of uniform relatedness, in terms of which the contingent relations of natural factors can be expressed. Otherwise, we can know nothing until we know everything (1922, p. 29).

Thus, uniformity is essential for knowledge, and this, as we shall see, is one of the key Whiteheadian gripes against Einstein.

An event passes and is gone. What endures to be recognised is what Whitehead refers to as an *object*. Objects are situated in events and serve as the basis for permanence and discreteness in nature. The relation between events and objects is a mutually supportive one: 'Without related objects, there can be no event' (1922, p. 26). Likewise, objects exist only in the sense of qualifying events. Whitehead maintains that, while the relations between events are necessarily uniform, the relations between objects are not:

It is not the case that the analysis of the adjectives of appearance [objects] attached to the events within a limited field of nature carries with it any certain knowledge of adjectives attached to other events in the rest of nature, or indeed of other such adjectives attached to those same events (1922, p. 64).

Whitehead's example is the perception of the colour green. Knowledge of green as an object presupposes the apprehension of times and spaces (events), but it does not require knowledge of green in relation to some other object — green in relation to a blade of grass, for example. The latter relation is contingent on the specific time and place in which the observation is made. The uniform significance of events assures us that knowledge is possible, but it does not inform us about the specific content of that knowledge.

Finally, the *physical field* of an event is '[...] the field of activity which regulates the transference of the objects situated in it to situations in subsequent events' (1961, p. 133). It is the sphere of influence due to the presence of the event, and acquires an atomic nature on behalf of the objects it governs. By means of an analysis of the physical field, laws of nature regulating the contingent relationships between objects are obtained.

Whitehead claims that the initial problem of relatedness has been met by the above account in two ways:

Insofar as nature is systematically related, it is a system of uniform relatedness; and in the second place, intelligibility is preserved amid the contingency of appearance by the breakdown of relatedness which is involved in atomicity (1922, p. 73).

Both conditions, uniformity and atomicity, can be seen to be derivative of events and objects, respectively. Again, the ontology described above is an attempt to avoid the bifurcation of nature. The stress is on what is directly perceivable and

on the most adequate formulation of this to account for certain key metaphysical assumptions (in particular, the fundamental status of becoming). Given this general discursive framework, Whitehead then proceeds to consider formulations of physical theories. These will involve particular mathematical formulations of the physical field for particular descriptions of physical situations.

3. Whitehead and Einstein on Methodology

There has been a tendency to associate the early Whitehead with positivist thought, given his emphasis on establishing a link between direct experience and theory.³ From the preceding, it is evident that this is certainly not accurate as an unqualified assertion. In this section, I shall try to debunk this myth a bit further by arguing that Whitehead's emphasis on direct experience is a bit more subtle than the logical positivists'. Whereas the latter regard the appeal to observation as the basis for justification in the context of theory construction and evaluation, Whitehead's emphasis sometimes takes a different turn. In the context of theory construction, it is an emphasis on the clarification of underlying ontological commitments and not on their justification. This is of import in addressing the various critiques of Whitehead's argument against general relativity, viz that the structure of spacetime must be uniform. Whitehead's position on this point becomes crucial in assessing the current empirical status of his theory of gravity.

The gist of this section will be a comparison of the methodologies employed by Whitehead and Einstein in the construction of their theories of motion and the subsequent extension of these to theories of gravity. This will set the stage for the discussion of Whitehead's critique of GR in Section 4.

3.1. *Einstein*

In his (1905), Einstein derives the Lorentz transformations via two postulates:

- (1) the laws of mechanics and electrodynamics are the same in all inertial reference frames; and

³North (1965) attributes to Whitehead the claim that it is necessary '[...] that each and every theoretical hypothesis should be rendered explicitly in terms of "the immediate facts of observation"' (p. 328). Thus, 'Whitehead was certainly no straight forward empiricist, but in this respect, he makes the same sort of mistake as one or two later positivists' (p. 327). He also attributes to Whitehead a complete separation of form and content (p. 329), which may be taken as implying a corresponding separation between theory and observation. (Friedman (1983) makes the claim that a neo-Kantian form/content distinction was the primary motivation for positivists like Carnap, Schlick and Reichenbach (see e.g. p. 7).) As to this distinction, it is evident from the brief description of Whitehead's early ontology above that the distinction between events and objects is certainly more complicated than a simple form/content reading.

Grünbaum (1953) likewise criticises Whitehead's attempt to found geometry in sense perception as positivistic. Of Whitehead's Method of Extensive Abstraction (see below), he declares that '[...] it fails to achieve its stated objective and that all such positivistic constructive attempts must share that failure' (p. 216).

- (2) the velocity of light is independent of the velocity of the source in all inertial reference frames.

The first postulate defines congruence relations for spatial measurements in different inertial frames and the second postulate allows for congruence relations for temporal measurements in different inertial frames. From these relations, the Lorentz transformations follow from simple algebra. (The additional assumption of the isotropy and homogeneity of space is also required.) A key ingredient in the derivation is Einstein's operational definition of simultaneity. By means of this definition, he obtains the second postulate from the more general result given in classical electrodynamics that the velocity of light is independent of the motion of its source.

In his *Autobiographical Notes*, Einstein distinguishes two criteria by which a theory should be judged. The first is that 'the theory must not contradict empirical facts' (1969, p. 21); i.e. it must be empirically adequate. The second involves an internal requirement — the theory must possess 'inner perfection'; i.e. it must possess internal logical coherency and simplicity. Simplicity for Einstein, while including a notion of economy of thought (among simple theories, that 'which most sharply delimits the quantities of systems in the abstract' (1969, p. 23) is taken as superior), is also concerned with the notion of unity: 'The problem here is not simply one of a kind of enumeration of the logically independent premisses [i.e. economy of thought], but that of a kind of reciprocal weighing of incommensurable qualities' (1969, p. 23). Theories may be simple in this Einsteinian sense and yet still possess a highly complex formulation.

Einstein makes a further distinction between *constructive theories* and *theories of principle*. Constructive theories 'attempt to build up a picture of the more complex phenomena out of the materials of a relatively simple formal scheme from which they start out'.⁴ They are based on hypothetical constructs and provide a description of particular phenomena via reduction to these constructs. Theories of principle, on the other hand, are based on observation: 'the elements which form their basis and starting-point are not hypothetically constructed but empirically discovered ones'.⁵ Such theories extrapolate from this basis to form general principles valid for the phenomena under question. The emphasis here is on theoretic structure — it is the incorporation of general principles into a logically coherent framework, one that possesses inner perfection, that results in a physical theory of this type. Einstein claims that special relativity is of this second type.⁶

Based on these considerations Einstein is motivated to seek a new kinematic theory due to the failure of classical Galilean kinematics to abide by his inner perfection criterion. (This failure is given by the fact that the laws of mechanics

⁴ Quoted in Schaffner (1974, p. 61).

⁵ *Ibid.*, p. 62.

⁶ *Ibid.*

are covariant under Galilean transformations but the laws of electrodynamics are not.) In keeping with his emphasis on theoretical integrity, Einstein's approach to the problem is through the covariance of Maxwell's equations — he seeks a theory in which 'the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good' (1905, p. 37). This emphasis on theoretical integrity can be seen as the origin of the light postulate: in classical electrodynamics, the velocity of light is independent of its source only in the aether frame. Einstein seeks a theory in which this holds for all (inertial) frames.

In this reading, special relativity has its origins in the high-level theoretical requirement of formal covariance. In keeping with the stipulation required by a theory of principle, it has firm observational foundations, chiefly in Einstein's operational definition of simultaneity.

3.2. Whitehead

In his (1919), Whitehead derives the Lorentz transformations from very general considerations of direct experience within the framework of his event ontology. His emphasis throughout is on the meaning of scientific concepts which, according to him, can only be given via recourse to direct experience. Requirements of theoretical integrity in the Einsteinian sense take second seat:

The philosophy of science is the endeavour to formulate the most general characteristics of things observed. These sought-for characters are not to be fancy characters of a fairy tale enacted behind the scenes. They must be observed characters of things observed (Whitehead, 1922, p. 5).

Accordingly, the meanings of space and time are not derivative of theory:

What we mean are physical facts expressible in terms of immediate perceptions; and it is incumbent on us to produce the perceptions of these facts as the meanings of our terms (Whitehead, 1919, p. 56).

These comments appear on the surface to lend credence to the association of Whitehead with positivist thought. However, I would argue that Whitehead's emphasis on direct experience is not an emphasis on what is or is not observable *per se*; rather, it is an emphasis on what is or is not *meaningful* within the context of his ontology. His stress is on the proper semantic formulation of scientific theories that makes clear what his ontological commitments are. His insistence on the link with direct experience is thus not an epistemic insistence — the meaning of scientific terms need not be based in experience in order for knowledge to be possible. (Again, that knowledge is possible and how it arises are claims that stem from the sort of ontology Whitehead adopts and this ontology is motivated by general metaphysical considerations.) Rather, it is a semantic insistence: the manner in which theories are formulated and the types

of concepts they employ should make clear what one's ontological commitments are.⁷

It is to establish a semantic (in the above sense) basis for the meanings of geometric and temporal concepts that Whitehead introduces the Method of Extensive Abstraction whereby geometric entities (points, lines, planes, volumes) are defined as the convergence properties of overlapping sets comprised ultimately of sense data.⁸ The construction of his kinematics involves employing the Method to clarify the extensive properties of 'time-systems'. Time-systems are constructed, via the Method, from what Whitehead refers to as durations — events of finite temporal and infinite spatial extent. What it means to be a duration is given directly in experience. Time-systems are families of 'parallel' durations; intuitively, the members of such families intersect in other durations and not in (temporally and spatially) finite events. By appealing to a second fundamental fact of nature, the relation of *cogredience*, Whitehead, in effect, establishes a correspondence between time-systems and inertial frames of reference.⁹

To determine the relations that hold between time-systems, Whitehead first establishes spatial congruence between time-systems via the Method of Extensive Abstraction. He then requires that relative velocities be equal and opposite, this also being directly given in experience. Congruence relations between time units then follow and the Lorentz transformations result. (Admittedly, this is a very brief sketch of what Whitehead takes an entire book to formulate. Hopefully, the essential elements have been conveyed.) In effect, Whitehead constructs Minkowski spacetime from a reductional relationist standpoint.¹⁰ It is relationist in the sense that Whitehead claims spacetime consists of the

⁷ That the positivist insistence on basing the meaning of scientific terms in direct experience is an epistemic insistence is inherent in the verifiability criterion of meaning. The meaning of a concept is to be cashed out in terms of how it is verified, and to verify something involves making implicit epistemological claims about it. In emphasising direct experience, Whitehead is not interested in how scientific terms are verified. He is just interested in whether or not their meanings are adequate.

⁸ The Method is described in Whitehead (1919, pp. 171–181). Again, it is not meant to construct geometry directly from sense perception. Rather, it is a mathematical model Whitehead uses to clarify certain relations given in sense perception; relations ultimately derived from the underlying metaphysics. Fitzgerald (1979, Chapter 4) offers a discussion on how to interpret the Method as such a model.

⁹ Cogredience is a relation holding between an event and a duration: an event is cogredient with a duration if it extends throughout the duration. An event cogredient with a duration is at rest with respect to the duration and to other events cogredient with the same duration. An event that is not cogredient with a given duration and its cogredient events is in motion with respect to them. Cf. Whitehead (1919, pp. 128–138) for details.

¹⁰ More precisely, what he effects is the construction of Euclidean four-space based on the Method of Extensive Abstraction. Minkowski spacetime is obtained by imposing a light cone structure at every point. This has two consequences: (1) multiple time-systems exist, and (2) the coordinates of time-systems are related by Lorentz transformations. Whitehead justifies the assumption of light cone structure, what he refers to as the 'multiplicity of time-systems', by appealing to empirical evidence.

relations between events; it is reductional in the sense that spacetime is constructed from the idealisations of these relations via the Method of Extensive Abstraction.¹¹

In semantically emphasising direct experience, Whitehead objects to Einstein's operational definition of simultaneity. Einstein is seen as providing two distinct meanings of simultaneity—an intuitive one for events occurring at a point and an operational one for distant events. For Whitehead, the meaning of simultaneity must be given in immediate perception. He refuses to base the meaning of simultaneity, and thereby the equations of kinematics, on any special characteristics of light:

[...] light signals are very important elements in our lives, but we still cannot but feel that the signal theory somewhat exaggerates their position. The very meaning of simultaneity is made to depend on them (Whitehead, 1919, p. 53).

Northrop (1941) interprets Whitehead's insistence on an intuitively given simultaneity relation as requiring that the simultaneity of spatially separated events be given directly in experience and rightly criticises this as incoherent. It is entirely obvious that events simultaneous to one inertial observer will not be simultaneous to another observer in the same inertial frame. However, in attributing to Whitehead this epistemic notion of intuitive simultaneity, he fails to take notice of Whitehead's semantic motivation. As Palter notes, Whitehead's objection to the use of light signals is not an objection to the method of *determining* the time-order between events (it is not an epistemic objection); rather, it is an objection against basing the *meaning* of simultaneity on the behaviour of light signals (it is a semantic objection; Palter, 1956, p. 131).¹²

Whitehead also notably does not accept a universal principle of relativity as a point of departure in theory construction:

It is not at all obvious that invariance of form in respect to all time-systems is a requisite in the complete expression of such laws; namely, the demand for relativistic equations is only of limited applicability (Whitehead, 1919, p. 161).

Where Einstein uses the principle to establish spatial congruence between inertial frames, Whitehead appeals to results obtained via the Method of Extensive Abstraction.

In developing theories of gravity, both Einstein and Whitehead begin with their respected 'special' theories. Einstein's initial motivation is to extend the principle of relativity to include acceleration (which may be viewed as an application of his inner perfection criterion). He does this via the principle of equivalence which initially allowed him to make the connection between acceleration and gravity, thereby bringing gravitation within the scope of the

¹¹ See Earman (1989, pp. 194–195) for a discussion of the feasibility of reductional relationism in the context of the absolute/relational debate in the philosophy of space and time.

¹² Notably, Grünbaum's critique of Whitehead refers approvingly to Northrop's interpretation. See e.g. Grünbaum (1962, p. 224; 1973, p. 345).

proposed theory. In the development of GR, Einstein relies to a large degree on results obtainable in special relativity which he then carries over into the new theory.¹³ Thus, in GR, the association of the metric with the gravitational potential is suggested by the form of the metric in the special relativistic gravitationless case. Also the association of geodesics with the world-lines of particles uninfluenced by external forces is suggested by the behaviour of such particles in the flat space of special relativity. Whitehead's theory of gravity is essentially an extension of his theory of kinematics to include gravitation, keeping all basic principles unchanged. Most notably, since his kinematics does not depend on a principle of relativity, he is not motivated to look for a generalised version.

4. Critique of Einstein: Spacetime must be Uniform

As was alluded to in Section 2, it is the uniformity condition that puts Whitehead at odds with Einstein. For Whitehead, the structure of spacetime must be uniform. In (1922), he gives two arguments in support of this claim. These may be categorised as dealing with epistemological concerns and with concerns about the possibility of measurement.¹⁴

4.1. Epistemological concerns

For Whitehead, space and time are abstractions from the extensive relations between events and not the objects qualifying them: 'Spatial [and temporal] relations between apparent bodies only arise mediately through their implications in events'.¹⁵ Whitehead claims therefore that the structure of spacetime is described by a uniform metric representative of the uniform relatedness between events.¹⁶ This argument for uniformity is based on the epistemology that derives from Whitehead's ontological commitments.

In talking about the possibility of knowledge of nature, Whitehead's discourse initially abandons the event/object taxonomy to centre on the more general notions of 'fact' and 'factors'. Nature is described as a combination of fact and factors. Factors are the limitations of fact, although the relation is not one of part-to-whole: it is the nature of fact to be differentiated into, but never exhausted by, factors. Whitehead then discerns two ways to know about factors: 'by relation' and 'by adjective'. The former yields knowledge of a factor by reason of its relations with other factors (e.g. knowing that there is an inside to a box without having to know what the qualitative aspects of the inside are: that

¹³ Cf. Norton (1985).

¹⁴ Broad (1923) makes this distinction.

¹⁵ Quoted in Broad (1923, p. 71).

¹⁶ This is not to say that spacetime must be flat. Whitehead is willing to consider the possibility of spacetimes of uniform curvature. See e.g. Whitehead (1922, p. v).

there is an inside is known by reason of the spatial relations it has with the outside of the box). The latter yields knowledge of a factor by reason of its qualities.¹⁷ The argument for uniformity then proceeds in two steps (as given in Chapter II of 1922).

Step I.

- (a) I cannot know F without knowing other factors to which it is related, where F is any given factor.
- (b) I do not have knowledge of all the factors related to F , else I would have knowledge of all the factors in nature. (This stems from the metaphysical assumption of relatedness.)
- (c) Thus, it is not the case that to have knowledge of F requires knowledge of all factors related to F .

The conclusion above motivates Whitehead to distinguish between a factor's *necessary* relations and its *contingent* relations. The necessary relations of F are those between F and *all* other factors that make F what it is; they comprise F 's identity as a factor. Contingent relations are those between F and other factors that could be different without affecting the identity of F . The argument for uniformity then continues as follows:

Step II.

- (d) I cannot know F without knowing its necessary relations.
- (e) There is only one way that I can have knowledge of F 's necessary relations and not have knowledge of every factor in nature. This is that what I know about F 's necessary relations from perceiving F in the limited region of nature in which it occurs extends throughout the whole of nature. In other words, the structure of necessary relations must be uniform such that knowing any part of the structure allows one to know the whole structure.
- (f) The final steps then are the assertions that knowledge of any given factor F is possible, and that spacetime consists in the necessary relations between factors. (This translates into the claim that spacetime is the relations between events, where perception of the necessary aspect of factors is perception of events.)

Three points are in order here.

(i) If (f) is undesirable, fault may readily be found with the definition of necessary relations; one might argue that the identity of a factor should not depend on all other factors in nature. However, in providing this definition, Whitehead is being consistent with his metaphysics of dynamical relatedness.

¹⁷ This ties in with Whitehead's event/object ontology. When we know a factor by relation, we know it as an event in nature dynamically related to other events. When we know a factor by adjective, we know it as an object which qualifies an event. The factor itself is the same: events and objects become two modes of describing it. The ontological priority of events stems from the metaphysical priority of Becoming *vis-à-vis* Being.

Any attempt to delineate factors out of their relatedness that does not depend on relations between all factors seems problematic in the context of a doctrine of relatedness.

(ii) Given that uniformity of spacetime structure is a necessary requirement in the context of relatedness, there is nothing in the above that picks out the *type* of structure we are going to require to be uniform. Whitehead was of the opinion that it should be metrical structure, as the succeeding argument demonstrates. However, we could just as well deem the essential properties of spacetime as topological and then require that topological structure be uniform in abundance with the above.¹⁸

(iii) It is possible to construe Whitehead's argument as a justification for induction, but some caution is advised here.¹⁹ In Step (e) above, the requirement that knowledge of a limited region of nature should extend to knowledge of all of nature might be associated with the statement 'induction should be possible'. Hence the argument can be reduced to 'if knowledge of *F* is possible, then induction should be possible' and 'if induction is possible, then spacetime must be uniform'. Indeed, in Whitehead (1925), uniformity is connected with the problem of induction. That a concept of the future exists, seems to imply that induction of some sort is possible. What makes it appear impossible are faulty ontological commitments, viz a commitment to discrete isolated particles with no inherent relations to past or future states. The solution seems to be simply that, if you are going to countenance the concept of induction, you are advised to adopt an ontology that is coherent with it.²⁰ It is clear then that Whitehead's argument is not so much a justification of induction as it is a clarification of the requirements necessary for the concept of induction to be meaningful.

4.2. Concerns over congruence and the possibility of measurement

In (1922, Chapter III) Whitehead essentially argues:

- (a) there is a unique meaning of congruence given in experience, and
- (b) the uniformity of spacetime structure is required to explain this.²¹

Elsewhere, Whitehead connects the uniformity of spacetime with the possibility of measurement. The claim is that measurement presupposes the notion of

¹⁸ This is suggested by Tanaka (1987, p. 51).

¹⁹ Broad (1923, p. 214) makes this construal.

²⁰ 'Induction presupposes metaphysics [...] you cannot have a rational justification for your appeal to history until your metaphysics has assured you that there is a history to appeal to; and likewise your conjectures as to the future presuppose some basis of knowledge that there is a future already subjected to some determinations' (1925, p. 44).

²¹ 'We have got to dismiss from our minds all considerations of number and measurement and quantity, and simply concentrate attention on what we mean by matching in length [...]. Our physical space therefore must already have a structure and the matching must refer to some qualifying class of qualities inherent in this structure' (1922, p. 51). Note also: 'Uniformity in change in directly perceived, and it follows that mankind perceives in nature factors from which a theory of [...] congruence can be formed' (1920, p. 137).

congruence. If not, then one is trapped in a vicious circle in defining measurement in terms of a notion of congruence and the latter in terms of the former. The implication is then made that notions of congruence (which are determined by what we mean by congruence) and hence measurement, cannot be obtained in a non-uniform spacetime:

Unless we start with some knowledge of a systematically related structure of space-time, we are dependent upon the contingent relations of bodies which we have not examined and cannot prejudice (1922, p. 59).

Again, for Whitehead, the implication is that uniformity is a requirement for measurement,

[...] for it must be remembered that measurement is essentially the comparison of operations which are performed under the same set of assigned conditions. If there is no possibility of assigned conditions applicable to different circumstances, there can be no measurement (1961, p. 134).²²

Note, however, that, given that measurement presupposes congruence, it is impossible in a non-uniform spacetime only if there is a unique standard of congruence. Alternative standards of congruence may provide the means to compensate for universal deforming forces associated with non-uniformity; i.e. any given meaning of congruence may be applicable in any given spacetime (of arbitrary geometry) with a suitably chosen standard of congruence. As Grünbaum points out,

[...] without a [unique] specification of the congruence criterion, the mere claim of spatial uniformity places no restrictions on the coincidence behavior of transported rods and does not rule out the variable geometry of Einstein's general theory of relativity (1973, p. 428).

Thus Whitehead's claim that measurement is impossible in non-uniform spacetimes requires the additional claim that the unique meaning of congruence given in experience picks out a unique standard of congruence.

Concerning (b): this is an inference to the best explanation. It implicitly assumes global uniformity. However, it would seem that local uniformity is sufficient for the explanation of (a).²³ Perhaps in conjunction with the first argument, a global requirement might be obtained, i.e. by the incorporation of an assumption of metaphysical relatedness. As indicated above, the resulting demand for uniformity does not explicitly pick out the specific structure that is to be designated as uniform. In conjunction with (a) and the uniqueness of a standard of congruence, however, the structure so indicated is metrical. What, then, is the status of (a), conjoined with the claim of a unique standard of congruence?

²² The intended target of this and the above passage is Einstein.

²³ This is Broad's (1923) assessment: 'I should have thought that the de facto agreement of our judgements of congruence, so far as it goes, required nothing more than an approximately uniform spatio-temporal structure within those regions of extension and duration which we have measured' (p. 217).

Grünbaum (1962, 1973) maintains that, while measurement does presuppose congruence, there is no unique standard of the latter given in experience — congruence relations are a matter of convention — hence (b) is superfluous. Grünbaum's critique is based on his ontologically-based conventionalism (in this context, his claim that congruence relations are conventional is due to the fact that spacetime is a continuum) which, he claims, is the antithesis of Whitehead's intuitionism. In Grünbaum (1962, p. 227), he argues that an intuitionist notion of psychological time can be reduced to the time of physical theory, and, since the latter is indifferent as to choice of congruence standard, the former is, too (and this carries over to spatial congruence as well). That such a reduction should not be made, Whitehead would argue from methodological grounds; doing so involves bifurcating the concept of time. More importantly, it seems that the same type of argument against Northrop's positivist reading of Whitehead's notion of simultaneity can be made against Grünbaum's intuitionist reading of Whitehead's notion of congruence.

Whitehead is not concerned with justifying that there is a unique standard of congruence; rather, he is concerned with the clarification of the ensuing requirements *given* that such a unique standard exists. Again, the given follows from metaphysical assumptions (compare with the first 'epistemological' argument in its form as a clarification of the requirements necessary for the concept of induction to be meaningful). What we mean by congruence is the uniform matching of geometric quantities; uniform in the sense that such matching can be applied in all regions of nature. Note that Whitehead is not arguing that different ways of applying this meaning of congruence do not exist; he agrees with the conventionalist that such possibilities do exist. It is the meaning of congruence that is not a matter of convention. And we had better adopt an ontology that is coherent with it.²⁴

Given that Whitehead requires uniformity of spacetime structure, how then shall we interpret this structure? In what sense does Whitehead consider spacetime to be real? Whitehead requires that the basis for uniformity — the structure of spacetime — be independent of the contingent relations between objects. Again, this is necessary (sufficient?) for a unique meaning of congruence and hence measurement. In his (1925), Whitehead associates spacetime with the notion of what he calls an 'ideally isolated system':

[...] the conception of an isolated system is not the conception of substantial independence from the remainder of things, but of freedom from causal contingent

²⁴ Perhaps it is not that constructive to pursue this defence of Whitehead further. I shall make one more attempt. The conflict between the conventionalist and Whitehead boils down to a choice between basing the meaning of uniformity on (a) the appearance of uniformity, or (b) the fact of uniformity. Given that uniformity has been established as a necessary fact previous to this, Whitehead opts for (b). His congruence argument then can be seen as a plausibility argument; again, it is the clarification of ontological commitments and not the justification of the same. In this context, ontology indicates a fact of uniformity and a unique standard of congruence clarifies this fact. However, even with this plausibility gloss, the fact of uniformity by itself does not pick out what it is that is uniform (i.e. metrical structure, topological structure etc.).

dependence upon detailed items within the rest of the universe. Further, this freedom from causal dependence is required only in respect to certain abstract characteristics which attach to the isolated system, and not in respect to the system in its full concreteness (1925, p. 46).

There are two questions concerning this notion of 'isolation' and the previous notion of 'independence': (1) can spacetime causally affect objects, and (2) can objects causally affect spacetime? The two arguments outlined above require that the answer to the latter be no. The answer to the former is a bit more tricky and is one of the issues surrounding Will's (1971) critique to Whitehead's theory of gravity. (1) will be taken up again in Section 6. For the present, I turn to a description of the theory in question.

In constructing his law of gravity, Whitehead seeks a mathematical description of the physical field of a given event in the context of gravitational influences. In accord with the above, the physical field cannot affect spatio-temporal relations since it shares in the contingent nature of the objects it regulates. Again, the question to be returned is, can the latter affect the former?

5. Whitehead's Formalism

At this point, it becomes necessary to give a formal exposition of Whitehead's theory of gravity. This will be followed by a historical assessment of its empirical status *vis-à-vis* general relativity.

In his (1922), Whitehead is concerned with formulating a Lorentz-invariant equation for the gravitational field to replace the classical Newtonian one. To this end, he considers in Minkowski spacetime how the worldline, or 'kinematic history', of a particle m is affected by the worldlines of all nearby particles M . He

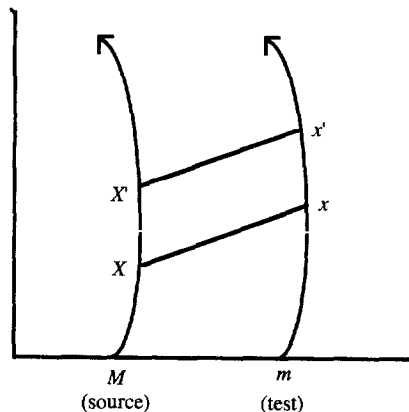


Fig. 1. Kinematic histories of m and M .

selects that part of M 's worldline, XX' , that is causally correlated with xx' on m 's worldline (see Fig. 1). Causal correlation is expressed by requiring points X and x to be separated by null vectors and similarly with X' and x' . Whitehead's equation for the gravitational field at the field point x then determines how an infinitesimal section of m 's kinematic history, denoted by dG_m^2 , is affected by infinitesimal sections of the kinematic histories of all bodies M , each denoted by dG_M^2 , on m 's past lightcone. The resulting modified kinematic route for m , denoted by dJ^2 , is given by (1922, p. 81)

$$dJ^2 = dG_m^2 - \frac{2}{c^2} \sum_M \Psi_M dG_M^2. \quad (1)$$

The tensor $J_{\mu\nu}$, given by $dJ^2 = J_{\mu\nu} dx^\mu dx^\nu$, is what Whitehead calls the gravitational 'potential impetus' acting on m at x due to M at X . In (1), $dG_m^2 = G_{\mu\nu} dx^\mu dx^\nu$, and $dG_M^2 = G_{\mu\nu} dX^\mu dX^\nu$, where $G_{\mu\nu}$ is the Minkowski metric with signature $(-1, -1, -1, 1)$.²⁵ The quantity $\Psi_M = \gamma M/w$, where γ is the Newtonian gravitational constant and w is the Lorentz-invariant distance between the worldlines of m and M (w is the inner product of the velocity of M at X with the null vector at X separating M and m). In a straightforward manner, Ψ_M is the Lorentz-invariant equivalent of the Newtonian gravitational potential and Whitehead's law (1) is essentially a 'relativistic' modification of Newton's law implemented by introducing a retarding effect on the speed of gravitational propagation.²⁶ (Of course Whitehead would not describe his modification as 'relativistic'.) Finally, the paths of particles are determined by the variational principle, $\delta \int M dJ = 0$, and the paths of light rays are determined by $dJ^2 = 0$.

An interesting methodological question is the following: to what extent do process ideas pick out Equation (1) over other Lorentz-invariant gravitational theories? (I thank an anonymous referee for posing this question.) Whitehead lists three general requirements that a law of gravitation should satisfy:

- (i) to have no arbitrary reference to any one particular time-system, and (ii) to give the Newtonian term of the inverse square law, and (iii) to yield the small corrections which explain various residual results which cannot be deduced as effects of the main Newtonian law (1922, pp. 84–85).

Requirement (i) is the requirement of Lorentz-invariance. Arguably, only requirement (i) is motivated by Whitehead's philosophy of nature; specifically, by the requirement of the uniform significance of events (although recall that this requirement does not necessarily entail a uniform spacetime metric). It is possible to read Whitehead as claiming that equation (1) is the only particle

²⁵ For purposes of immediate exposition. Whitehead does not refer to $G_{\mu\nu}$ as the Minkowski metric. Doing so arguably involves subscribing to a geometrical interpretation of the formula (1), which Whitehead would object to.

²⁶ Schild (1956, p. 208) describes the difference between Whitehead and GR as analogous to the difference between the Lienard–Wiechert retarded potential particle formulation of classical electrodynamics and the Maxwell field formulation.

action-at-a-distance theory that satisfies (i)–(iii). Whitehead (1922, p. 82) briefly mentions that Equation (1) can be arrived at by first considering the four-dimensional wave equation $\nabla^2\psi - 1/c^2 (\partial^2/\partial t^2)\psi = 0$ as the only Lorentz-invariant linear differential equation of second order, and then deducing $\psi = \sum \gamma M/w$ as the only Lorentz-invariant solution to it for a ‘point-wise discontinuity’ that reproduces the Newtonian law.²⁷

However, Schild (1956) used the Schwarzschild metric of GR to generate an infinite number of gravitational theories of the ‘Whitehead-type’ (i.e. retarded Lorentz-invariant action-at-a-distance theories in Minkowski spacetime) that are in complete agreement with GR on the classical tests.²⁸ Hence, it is not the case that Equation (1) is the unique Lorentz-invariant retarded action-at-a-distance particle theory of gravitation that reproduces both the Newtonian limit and the classical tests of GR. Note that Whitehead also considers alternative gravitational laws that satisfy his three requirements but that are not, implicitly, particle action-at-a-distance theories. As indicated above, he lists three alternatives, the first being the vacuum field equations of GR. The other two have not been given much attention in the literature (see Russell (1988) for an initial investigation). Hence, Whitehead certainly did not mean for process ideas to uniquely pick out Equation (1).

In 1924, Eddington demonstrated that, for the one-body problem, Whitehead’s dJ^2 is exactly equal to the Schwarzschild line element ds^2 given in general relativity. In spherical coordinates, the Schwarzschild line element is given by

$$ds^2 = - (1 - 2M/r)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + (1 - 2M/r)d\hat{t}^2, \quad (2)$$

for a spherically symmetric body of mass M and radius r . Eddington introduced the coordinate transformation

$$t = \hat{t} - 2M \ln(r - M). \quad (3)$$

Substituting into (2) and simplifying yields:

$$ds^2 = [-dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + dt^2] - (2M/r)(dt - dr)^2. \quad (4)$$

The first term on the right gives the element dG_m^2 in spherical coordinates. Corresponding to this is the causally correlated element dG_M^2 . For the one-body

²⁷ Harris (1981) lends this claim some plausibility. He demonstrates by construction that there are an infinite number of particle action-at-a-distance gravitational theories satisfying (1) Lorentz invariance, (2) gravitational effects propagate at the speed of light, and (3) the low velocity, weak field limit is Newtonian gravitation. With charity, these amount to Whitehead’s first two requirements. Harris demonstrates explicitly that for tensor theories of rank 2 or less, only Whitehead’s gives the correct perihelion advance for planetary orbits. (Harris does not consider the other classical tests, nor does he consider tensor theories of rank 3 or higher.)

²⁸ All such theories predict a non-zero secular acceleration for the centre of mass in the two-body problem (this had already been predicted by Clarke in 1955 for Whitehead’s original theory; see text below). Schild (1956) interpreted this as a violation of conservation of angular/linear momentum. Schild (1963) proposed a variant of Whitehead’s theory that uses both retarded and advanced gravitational potentials (in analogy to the Wheeler–Feynman particle action-at-a-distance formulation of classical electrodynamics) and which agrees with GR in predicting a zero secular acceleration.

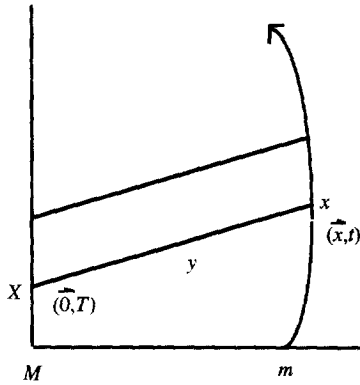


Fig. 2. Kinematic histories of m and M in the rest frame of M .

problem, M may be considered at rest relative to m . From Fig. 2, using the Minkowski metric with signature $(-1, -1, -1, 1)$, we have:

$$dG_M^2 = dT^2, \tag{5a}$$

$$y^\mu y_\mu = (x^\mu - X^\mu)(x_\mu - X_\mu) = -(x^1)^2 - (x^2)^2 - (x^3)^2 + (t - T)^2 = 0. \tag{5b}$$

Thus,

$$t - T = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2} = r, \tag{6}$$

and

$$dG_M^2 = (dt - dr)^2. \tag{7}$$

Also, from the definition of the Lorentz-invariant distance, w :

$$w = (dX/dG_M)^\mu y_\mu = (0, 0, 0, 1) \cdot (x^a, r) = r, \quad a = 1, 2, 3. \tag{8}$$

Thus (4) becomes

$$ds^2 = dG_m^2 - 2\Psi_M dG_M^2, \tag{9}$$

which is identically Whitehead's expression for dJ^2 restricted to a single particle M (Eddington uses units in which $c = 2\gamma = 1$).

Thus the equations of motion for particles and light rays in the one-body problem are identical in both theories. Since two of the classical tests of general relativity — the deflection of light rays in the presence of a gravitational source and the perihelion advance of Mercury — are based on predictions derived from the Schwarzschild solution, it follows that Whitehead's theory makes the same predictions.²⁹ The third test — the red-shift of atomic spectra in the presence of

²⁹ Whitehead derives these results in (1922, pp. 105, 111).

a gravitational source—requires a particular interpretation of the symbols used in the metric, specifically, the time coordinate. Thus divergences in prediction may occur insofar as it is a specific transformation of the time coordinate that allows both theories to be formulated identically. Indeed, Whitehead derived a predicted red-shift that differs from the general relativity prediction by a factor of $7/6$ (Whitehead, 1922, p. 116).³⁰ While this factor appears fairly insignificant, it still amounts to a difference. However, the empirical data on the red-shift at the time of Whitehead's publication were scratchy at best and their confirmatory role in support of general relativity is debatable.³¹ Data in support of the GR predicted value were only obtained to satisfaction relatively recently in the Pound–Rebka–Snider experiments of 1960–1965 (Will, 1987, p. 92). In any event, Whitehead's prediction is based on a simplified model of the atom that employs outmoded quantum considerations (Synge, 1951, p. 46; Palter, 1960, p. 206), and Synge demonstrated that, under a metric interpretation (see below), the Whitehead theory yields the same prediction as GR (Synge, 1951, p. 46).

It should be mentioned that the first two tests also cannot be said to be instances of uncontroversial confirmation of GR. The results of the eclipse expedition of 1919, apparently confirming the light deflection prediction, had only a 30% accuracy, and the 1966 detection of a possible solar oblateness which would contribute to the perihelion advance of orbiting planets has challenged the accuracy of the GR prediction (Will, 1987, pp. 111, 104).

For the period from 1922–1951, then, a historical case against Whitehead in favour of GR strictly on grounds of empirical adequacy could be made, but such a case would be shaky indeed. It would have to assume that red-shift data at the time were accurate to the satisfaction of the scientific community and it would have to assume that discriminating scientists ignored the possibility that an inadequate interpretation of the physical model on the part of Whitehead was to blame for the discrepancy in predictions.

In addition to the three classical tests, there are predictions that both theories make that, while not at present empirically confirmable, may be used as further comparison. In 1960, Whitrow and Morduch constructed a compendium of such results. What follows is taken in part from them. Two further predictions for the one-body problem were derived from GR by Lense and Thirring in 1918. These are an additional advance in perihelion due to an axial rotation of the central body and a precession of the normal to the plane of orbit of a particle about the axis of rotation of the central body. Identical predictions were derived by Rayner in 1955 using Whitehead's theory. In the case of the two-body problem, Whitehead's law yields exact solutions, whereas the non-linearity of the GR equations requires approximation schemata. Furthermore, and more importantly, Whitehead's law (1) is a linear superposition of one-body solutions

³⁰ Apparently Whitehead later redid the calculation and came up with a different factor of $13/6$, which he wrote in the margins of his copy of *The Principle of Relativity* (see Palter 1960, footnote on p. 206).

³¹ Cf. Earman and Glymour (1980).

that omits the self-field of the test particle; hence for the many-body problem, divergences in predictions with GR are to be expected.³² In GR, the advance of the periastron of the orbit of one body with respect to the other is given by the same formula as the perihelion advance in the one-body problem, but with m now denoting the sum of the masses of the systems (Whitrow and Morduch, 1965, p. 2). Whitrow and Morduch cite work done in 1959 as an example (although it is unclear if this citation refers to historical precedence). Also cited is work done by Eddington and Clarke in 1938 in which they derive the GR prediction that the motion of the centre of mass is zero. Clarke in 1955 applied Whitehead's theory to this problem and derived a non-zero result. Clarke concluded that observations of binary stars could provide a basis for selection between the theories.

Thus, from its inception in 1922 at least until 1971, Whitehead's theory was as empirically adequate as GR. Between 1955 and 1971 there was at least one in-principle means of empirically distinguishing between the two theories, viz data on binary star systems, but this was not available to the scientific community.

6. Has Whitehead's Theory been Disconfirmed?

In 1951, Sygne reformulated Whitehead's equation for the gravitational field and gave it a two-metric, global flat background interpretation.

Sygne's approach is to determine the element of proper time $P'Q'$ that depends only on the events P and Q and the worldline L' (see Fig. 3). This element, ds'^2 , is then inserted into Sygne's formulation of Whitehead's law:

$$g_{\mu\nu} dx_\mu dx_\nu = dx_\nu dx_\nu + (Mk/w) ds'^2, \quad \mu, \nu = 0, 1, 2, 3, \quad (10)$$

where w is the invariant distance between P' and P and is given by $-\zeta_\nu \lambda'_\nu$ (where λ'_ν is the unit tangent to L' at P'), with $\zeta_\nu \zeta_\nu = 0$, and $k = 2\gamma/c^2$. Sygne's result is³³

$$g_{\mu\nu} = \delta_{\mu\nu} + \sum \frac{Mk}{w^3} \zeta_\mu \zeta_\nu. \quad (11)$$

(As with Whitehead's formula (1), the summation is over all mass sources M on the past light cone of P .) Whitehead's original formula, (1), can be shown to be equivalent to (11) by the substitution of the identity

$$G_{\mu\nu}^{(x)} dp_\mu dp_\nu = \frac{1}{w^2} (x_\mu - p_\mu)(x_\nu - p_\nu) dx_\mu dx_\nu. \quad (12)$$

³² I thank an anonymous referee for making this point explicit.

³³ Sygne uses the 'ict' convention in which time coordinates are multiplied by a factor of ic : $x_0 = ict$. This allows him to write the Minkowski line element as $ds^2 = dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 = \delta_{\mu\nu} dx_\mu dx_\nu$. Hence, in (11), the δ term $\delta_{\mu\nu}$ corresponds to the components of the Minkowski metric $\eta_{\mu\nu}$.

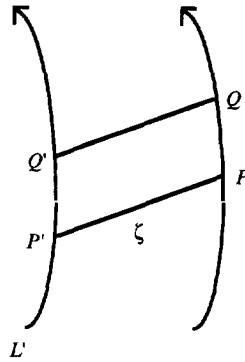


Fig. 3. Diagram for Synge's (1951) formulation of Whitehead's theory.

The result is (Palter, 1960, p. 203)

$$\|J_{\mu\nu}^{(x)}\| = \|G_{\mu\nu}^{(x)}\| - \frac{2}{c^2} \sum_M \frac{\gamma M}{w^3} \|x_\mu - p_\mu\| \|x_\nu - p_\nu\|.^{34} \quad (13)$$

Equation (11) can be formally compared with Einstein's field equations. The need to solve ten non-linear partial differential equations is absent in Whitehead's formula which omits the self-field of the test particle as in Newtonian mechanics. Synge describes the Whiteheadian gravitational field as 'displayed against a flat four-space with Euclidean topology' and speaks of the 'Minkowskian background' (1951, pp. 10, 4). He makes it generally clear that he does not attempt to reconstruct Whitehead's philosophy: 'The account of Whitehead's theory given in these lectures is emphatically one in which the philosophy is discarded and attention directed to the essential formulae' (1951, p. 2). This, one might argue, does severe injustice to Whitehead's motivation in constructing his formula. As just one example, Synge's interpretation implies that, for Whitehead, spacetime is absolute in the sense of existing prior to objects. All aspects of Whitehead's avowed relationalism are thus lost. Synge's equation (11), without further qualification, also implies that for Whitehead the metric of spacetime is to be associated with the gravitational field. Clearly, in one reading of spacetime, Whitehead does not make this claim: spacetime, as the relations between events, must be uniform and cannot be affected by objects. Whether or not qualifications can be made to make the metrical interpretation of (11) viable will be addressed below.

³⁴ The notation is Whitehead's. $\|J_{\mu\nu}^{(x)}\|$ is the tensor $J_{\mu\nu}$ in the coordinate chart $\langle x_i \rangle$. Here and in the formula immediately above, the point x corresponds to point P in Figure 3 and the point p corresponds to point P' .

The questions that Sygne's interpretation raise become important in discussing the only claim of empirical disconfirmation that has been made against Whitehead's theory. This is given in Will (1971) and to this I shall now turn.

In his (1971), Will claims to have provided 'the first accurate experimental evidence ruling out [Whitehead's] theory' (p. 145). Whitehead's theory is seen to predict an anisotropy in the locally measured gravitational constant γ , which is ~ 200 times the limit set by empirical data. Will's analysis is done within the context of his Parametrised Post Newtonian (PPN) formalism and to this I turn first.

The PPN formalism is a theoretical framework constructed to compare metric theories of gravity and test them against empirical data. Metric theories must satisfy three characteristics: (i) spacetime is endowed with a metric $g_{\mu\nu}$, (ii) the worldlines of test bodies are geodesics of that metric, and (iii) in local freely falling frames (Lorentz frames) the non-gravitational laws of physics are those of special relativity. As a result of (iii), all metric theories will reduce to Newtonian gravitation in the weak field, low velocity limit. In order to account for post-Newtonian effects (such as the additional perihelion shift of Mercury), viable theories must contain post-Newtonian terms. The PPN formalism is based on the most general form the post-Newtonian approximation of the metric of a metric theory can take. This form, the PPN metric, contains a number of arbitrary variables, the PPN coefficients. Competing theories can then be compared by taking their post-Newtonian approximations, comparing these with the generalised PPN metric, and reading off the values of the PPN coefficients associated with each theory. Definite limits on the values for some of the coefficients may be obtained by comparing predictions derived from the PPN metric with empirical data.³⁵

The gravitational constant γ can be measured locally by means of Cavendish experiments in which the relative acceleration of two bodies is measured as a function of their masses and distance of separation. γ is then identified according to Newton's law of gravity for the two bodies. In his (1971), Will considers the Earth and a gravimeter on its surface as the two masses in a Cavendish experiment. Any fluctuations in γ will then appear as Earth tides, experimental measurements of which are available. Will's analysis, then, proceeds in three steps.

- (a) The force between the Earth and a gravimeter is calculated using the PPN metric.
- (b) The post-Newtonian approximation for Whitehead's theory is calculated and Whiteheadian values for the PPN coefficients are obtained.
- (c) The values in (b) are substituted into (a) and the value for γ predicted by the PPN version of the Whitehead metric is read off.

Whitehead's value is a function of the mass of the rest of the galaxy. Will approximates this as concentrated in the galactic centre and then calculates the

³⁵ See Will (1993, p. 22 and Chapter 4) for greater detail.

amplitude of anisotropy due to the rotation of the Earth/gravimeter system with respect to the galactic centre. This amplitude is ~ 200 times greater than the value inferred from Earth tide data.

Will's description of Whitehead's formalism follows Synge's lead: Whitehead's theory is described as a Lorentz-invariant action-at-a-distance metric theory; the metric $g_{\mu\nu}$ determines the geodesics of free-falling bodies, and the laws of non-gravitational physics take on their special-relativistic forms in the local inertial frames of $g_{\mu\nu}$. For an arbitrary field point x^μ , let the configuration of mass sources M_i on the past light cone of x^μ each have four-position vector x_i^μ . Then the metric $g_{\mu\nu}$ at x^μ is given by:

$$g_{\mu\nu} = \eta_{\mu\nu} - 2 \sum_i \frac{M_i}{w_i^3} y_i^\mu y_i^\nu, \quad (14)$$

where $y_i^\mu = x^\mu - x_i^\mu$, $\eta_{\mu\nu} y_i^\mu y_i^\nu = 0$, and $w_i = \eta_{\mu\nu} y_i^\mu dx_i^\nu / d\sigma$, where $d\sigma^2 = \eta_{\mu\nu} dx^\mu dx^\nu$ (see Fig. 4).³⁶

To obtain the post-Newtonian limit of the Whitehead metric, Will evaluates all quantities in (14) on a constant-time hypersurface Σ instead of along the past light cone Σ^- . This involves expanding the pertinent quantities in Taylor series about the incremental difference in proper time between the position of the i th source mass on the past light cone and its position on the hypersurface containing the position of the test mass. The results are:

$$y_i^a = r_i^a + r_i v_i^a + (\eta_{\mu\nu} v_i^\mu r_i^\nu) v_i^a - \frac{1}{2} r_i^2 \frac{dv_i^a}{dt}, \quad a = 1, 2, 3, \quad (15)$$

$$y_i^0 = r_i \left[1 + \frac{\eta_{\mu\nu} v_i^\mu r_i^\nu}{r_i} + \frac{1}{2} \left(\frac{\eta_{\mu\nu} v_i^\mu r_i^\nu}{r_i} \right)^2 - \frac{1}{2} \eta_{\mu\nu} r_i^\mu \frac{dv_i^\nu}{dt} \right], \quad (16)$$

$$w_i = r_i \left[1 + \frac{1}{2} \left(\frac{\eta_{\mu\nu} v_i^\mu r_i^\nu}{r_i} \right)^2 + \frac{1}{2} \eta_{\mu\nu} r_i^\mu \frac{dv_i^\nu}{dt} \right], \quad (17)$$

where $r_i^\mu = x^\mu - x_i^\mu$, $r_i = (\eta_{\mu\nu} (x^\mu - x_i^\mu) (x^\nu - x_i^\nu))^{1/2}$, and $v_i^\mu = dx_i^\mu / dt$. Substituting the above into (14) and making the coordinate transformation

$$x'^a = x^a + \sum_i \frac{M_i r_i^a}{r_i}, \quad x'^0 = x^0 - 2 \sum_i M_i \ln r_i + \frac{5}{2} \frac{\partial}{\partial t} \sum_i M_i r_i, \quad (18)$$

Will puts the post-Newtonian Whitehead metric into the standard PPN form and then is able to read off the values of the parameters.³⁷

Will's result has been recast in the following manner. In Whitehead's theory,

the attraction between two given bodies, which measures the value of γ , is a function not only of the masses and the separation involved, but also of the prior geometry (Ariel, 1974, p. 286).

³⁶ Will works in units for which $c = \gamma = 1$. I have slightly modified his notation for the sake of continuity of exposition.

³⁷ The transformation (18) is the standard PPN gauge (in which the PPN metric takes a simple form (Will, 1993, p. 96) with the addition in the x^0 transformation of the Eddington term $-2 \sum_i M_i \ln r_i$.

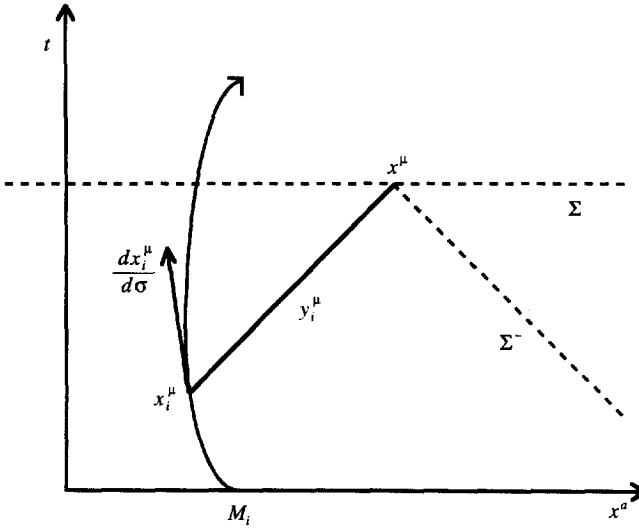


Fig. 4. Diagram for Will's (1971) formulation of Whitehead's theory.

Indeed, Will's interpretation of his result is as follows:

Since the Whitehead metric must be calculated in a global Lorentz coordinate system of $\eta_{\mu\nu}$ [...] the field due to the Galaxy cannot be removed by transformation to a local inertial frame surrounding the solar system (as one would do in general relativity). Hence the galactic gravitational field must appear in equation (38) [the Whitehead-predicted value for γ] (1971, p. 149).

How are we to interpret Will's article? Will builds on the work of Synge, which leads to two main assumptions: (1) Whitehead's theory is a metric theory of gravity, specifically, it satisfies characteristic (ii) of metric theories; (2) Whitehead's theory involves a 'prior geometry'. As a result of (1), the PPN formalism can be applied in analysing Whitehead. The above quotes indicate that the claim of disconfirmation is a result of (2). The questions I will address below are: (a) are the above assumptions legitimate, and (b) if not, how are Will's results affected?

Is it legitimate to interpret Whitehead's theory as a metric theory? First of all, it had better be for further discussion to be possible: there is a consensus among theoreticians that only metric theories of gravity are viable. Red-shift and Eötvös-type experiments indicate that the conjunction of a uniform spacetime metric and characteristic (ii) is ruled out.³⁸ The compatibility of Whitehead's

³⁸ Schild (1963, pp. 101–104) provides one form of the argument. Red-shift data indicate that time flows at different rates in different regions of a static gravitational field. To explain this, flat (or uniform) spacetime theories must postulate the existence of differential forces due to the gravitational field that affect the internal workings of clocks. This violates the weak equivalence principle (viz the world line of a test body in a gravitational field is independent of its composition and internal structure), which Eötvös-type experiments confirm.

metric and the Schwarzschild metric indicate (1), as well as Whitehead's own derivation of a red-shift prediction. However, how is this to be reconciled with Whitehead's critique of non-uniform spacetimes and his arguments for uniformity?

Misner *et al.* (1971) (hereafter referred to as MTW) offer the following solution. They claim that Whitehead's theory requires that gravitational influences propagate along the null geodesics of Minkowski spacetime while objects travel along the geodesics of a variable spacetime given by the metric $g_{\mu\nu}$ (p. 430).³⁹ In this sense, $\eta_{\mu\nu}$ is defined everywhere and Will's claim that Whitehead's theory incorporates well-defined global Lorentz coordinates goes through. But is it apparent that this is Whitehead's take of the situation? This might appear to be what he indicates by the causal correlation between the kinematic histories of a test mass and all masses on its past light cone. It would then appear that the first question at the end of Section 4, viz 'can spacetime causally affect objects?', is 'yes'. However, Whitehead's relational construal of spacetime on first sight appears incompatible with the notion of an independent entity causally affecting objects. This brings us to the question of prior geometry.

MTW offer the following:

By 'prior geometry' one means any aspect of the geometry of spacetime that is fixed immutably, ie, that cannot be changed by changing the distribution of gravitating sources (p. 429).

It is useful to refer to geometric objects that encode the properties associated with prior geometry as absolute objects, and geometric objects that are functions of matter variables as dynamical objects.⁴⁰ MTW's claim, then, is that $\eta_{\mu\nu}$ is an absolute object in Whitehead's theory.

In one sense, reading $\eta_{\mu\nu}$ as an absolute object appears inaccurate. Whitehead considers spacetime to be derived from the relations between events in process. There are no static events from which to abstract a prior geometry; rather, geometry arises out of the dynamic interactions between events. In this vein, Fowler (1972) insists that a prior geometry, in the sense of existing prior to its occupants, is out of place in Whitehead's philosophy of nature. Fowler also claims that Whitehead so separates geometry from physics that talk of causal connections between the two is precluded. At the onset, Whitehead declares:

It is inherent in my theory to maintain the old division between physics and geometry. Physics is the science of the contingent relations of nature and geometry expresses its uniform relatedness (1922, p. v).

In accord with this division, Fowler reasons that gravitational forces in Whitehead's theory should be interpreted as propagating along the complete metric

³⁹ Here they claim that experimental evidence in the form of interactions between ordinary matter and gravitational waves could distinguish between GR and Whitehead's theory. For Whitehead, gravitational waves propagate along geodesics of $\eta_{\mu\nu}$ and should be unaffected by the presence of intervening matter.

⁴⁰ See e.g. Earman (1989, p. 45).

$g_{\mu\nu}$, and the theory as a whole should not be considered as a two-metric theory incorporating prior geometry. Thus Whitehead's formula (1) may be considered a metric theory where $\eta_{\mu\nu}$ is just a computational device used in the derivation of $g_{\mu\nu}$. It may then appear questionable whether the theory so interpreted requires the existence of global Lorentz coordinates.

Two points should be made here. The first is that it would appear obvious that Whitehead's insistence on the global uniformity of spacetime structure is consonant with MTW's definition of prior geometry. The important qualification to make here is that it is not a static uniformity of the type which could be associated with an independently existing substantial spacetime. Rather, it is a dynamic uniformity: it consists of the necessary constraints the dynamic relations between events must possess. The conception of spacetime structure that arises may be called 'absolute relationism', for lack of a better term.

The second point is that Will's analysis depends only on the fact that Whitehead's theory requires the existence of a global Lorentz chart; it does not depend on a realist interpretation of this chart in terms of a substantial spacetime. To explain why, it is best to look at Will's own notion of prior geometry. In a metric theory of gravity, on Will's account, there may be additional gravitational fields besides the spacetime metric that play a role in determining the manner in which the metric couples to matter. (In all metric theories, however, it is only the metric that, once determined, couples to matter.) Will (1993, p. 79) thus distinguishes between two types of metric theory: 'purely dynamical' and 'prior geometric'. In a purely dynamical metric theory, each gravitational field is coupled to at least one of the other fields in the theory (in GR, there is only one dynamical gravitational field, viz the metric). In a prior geometric metric theory, absolute elements can be identified that are given a priori and are independent of the nature and evolution of the other fields in the theory. One result of this distinction is the following: for purely dynamical theories (like GR), one can always transform to coordinates in which the metric $g_{\mu\nu}$ takes on asymptotically vanishing values for local gravitational experiments (like Cavendish experiments). For prior geometric metric theories, one cannot always do this. In particular, for Whitehead's theory, the background solutions for $g_{\mu\nu}$ and $\eta_{\mu\nu}$ will, in general, be different. Hence Will's predicted value for the gravitational constant γ does depend on the fact that in Whitehead's theory, one cannot always transform to a coordinate system in which the spacetime metric takes on asymptotically vanishing values, thereby allowing one to disregard matter distributions at large distances from the experiment. Will's result does not, however, depend on a realist interpretation, such as MTW's, of Whitehead's metric $\eta_{\mu\nu}$.

Indeed, Will views Whitehead's theory as a prior geometric metric theory and explicitly describes the uniform metric $\eta_{\mu\nu}$ as unobservable and used only in the calculation of the metric $g_{\mu\nu}$ (1971, p. 153). Explicitly, then, he adopts an instrumentalist interpretation of $\eta_{\mu\nu}$. This indicates that Fowler's reading of Will's result is in error. Note, too, that Whitehead explicitly claims that one advantage his theory has over Einstein's is that, in his theory, global

Lorentz-invariant inertial frames can be defined (his 'permanent spaces'). These can be used to unambiguously define absolute rotation, a concept Whitehead believes is inexplicable in GR (Whitehead, 1922, pp. 87–88).⁴¹ Russell (1988, p. 182) is sensitive to this aspect of Will's analysis and claims it is the summation term in Whitehead's formula that is at the base of Will's result. Given Will's distinction between dynamical metric theories and prior geometric metric theories, and Whitehead's own remarks concerning global Lorentz frames (see also 1922, p. 85), I think that (a) Whitehead's theory can legitimately be interpreted as a prior geometric metric theory of gravity (with the qualification that by 'absolute' is not meant 'substantial'); and (b) the problem it faces in light of Will's result rests firmly with Whitehead's commitment to a uniform spacetime *together with* the particular form of Equation (1). Note that it is still open for a Whiteheadian to construct a viable prior geometric metric theory of gravity that allows for both the uniform significance of events and the contingency of appearances.⁴²

7. Conclusion

In summary, Whitehead's theory describes gravity as a retarded action-at-a-distance force. The trajectories of gravitational sources (ponderable masses) are determined by a dynamic metric $g_{\mu\nu}$ derived by modifying a flat Lorentzian metric $\eta_{\mu\nu}$ with a second-rank tensor field given by a Lorentz-invariant extension of Newton's law of gravity. On Whitehead's preferred ontological interpretation, the flat Lorentzian metric describes the uniform dynamic relatedness of events in process. It is absolute insofar as this dynamic relatedness is uniform and exists in causal independence of matter fields in order for knowledge and induction to be possible. It is dynamic insofar as it describes not a fixed eternally existing set of substantial spacetime points, but rather an ever-changing flux of inter-related events. Furthermore, the flat metric $\eta_{\mu\nu}$ is interpreted as the metric of spacetime, and not the dynamic metric $g_{\mu\nu}$. However, all non-gravitational fields couple to $g_{\mu\nu}$ and only to $g_{\mu\nu}$; hence $g_{\mu\nu}$ satisfies the requirements for the spacetime metric appearing in metric theories of gravity. Whitehead's theory can thus be considered a prior geometric metric theory of gravity with the above qualifications.

It was argued above that it is Whitehead's insistence on the uniformity of spacetime coupled with the specific form (1) of his gravitational law that is at odds with empirical data. Notably, in (1922), Whitehead expressly offers (1) as

⁴¹ Note that this is erroneous. In both special and general relativity there is a well-defined notion of absolute rotation. See e.g. Earman (1989, pp. 97–98).

⁴² While plausibility arguments exist that indicate that only purely dynamical metric theories of gravity are viable (and, in particular, only a purely dynamical metric theory that admits just one gravitational field), no rigorous proof of this claim has yet been constructed (see Will, 1993, pp. 82–83).

a hypothesis open to empirical disconfirmation and is willing to consider other laws consonant with his philosophy of nature; in particular, with his spacetime uniformity requirement.

It was also argued above that this spacetime uniformity requirement, which is also the basis of Whitehead's critique of Einstein, does not specify what geometric structure must fulfil it. Consonant with the first argument presented in Whitehead (1922), topological structure, as opposed to Whitehead's choice of metrical structure, may be considered as fulfilling it. Furthermore, Whitehead's second argument in (1922) based on an intuitive unique meaning of congruence may be seen not so much as a requirement for a unique physical standard of congruence given in direct experience, but as a plausibility argument. Given the fact of ontological uniformity, it is plausible to assume a unique standard of congruence; however, that one exists may not empirically be the case.

I have thus argued that Whitehead's arguments that are meant to grant $\eta_{\mu\nu}$ epistemological status are flawed. It still remains feasible within Whitehead's ontology to adopt other gravitational laws that do not place significance on uniform spacetime metrics. It also still remains feasible to drop Whitehead's epistemological arguments, maintain the fundamental status of a uniform metric by some other means, and construct an alternative prior geometric metric theory of gravity. In any event, I hope to have demonstrated that an informed critique of Whitehead's theory can only come with an understanding of the ontological commitments that went into its construction, commitments that cannot be simply read off the manner in which the theory was formulated.

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