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Three principles of quantum gravity in the condensed matter approach

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ABSTRACT

Research on quantum gravity (QG) has historically relied on appeals to guiding principles. This essay frames three such principles within the context of the condensed matter approach to QG. I first identify two distinct versions of this approach, and then consider the extent to which the principles of asymptotic safety, relative locality, and holography are supported by these versions. The general hope is that a focus on distinct versions of a single approach may provide insight into the conceptual and foundational significance of these principles.

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1. Introduction

Guiding principles occur frequently in theory construction and justification. They appear in historical and conceptual accounts of both relativity theory (the principle of relativity, the equivalence principle, the geodesic principle, *etc.*), and quantum mechanics (the adiabatic hypothesis, the principle of mechanical transformability, the correspondence principle, the exclusion principle, *etc.*). Such principles play important conceptual roles in stages of theory development during which empirical testability is limited or non-existent. However, their significance is not just restricted to being stand-ins for empirical adequacy. Some authors have argued that guiding principles are a fundamental characteristic of certain types of fully developed theories.[1] Moreover, appeals to principles, when combined with appeals to empirical adequacy, can offer powerful arguments in the context of theory choice. For instance, an argument due to Steven Weinberg claims that quantum field theory is the way it is because it is the only way to reconcile the empirical evidence for the theory with two basic guiding principles.[2]

One area of current research in which guiding principles play a significant role in theory development is the field of quantum gravity (QG, hereafter). There is currently little contact between theoretical work in QG and empirical tests, as a recent review makes clear:

> Unfortunately, in spite of more than 70 years of theory work on the quantum-gravity problem, and a certain proliferation of theoretical frameworks being considered, there is only a small number of physical effects that have been considered within quantum-gravity theories. Moreover, most of these effects concern strong-gravity/large-curvature contexts, such as black-hole physics and big-bang physics, which are exciting at the level of conceptual analysis and development of formalism, but of course are not very promising for the actual (experimental) discovery of manifestations of non-classical properties of spacetime and/or gravity (Amelino-Camelia, 2008, p. 7).

The general hope is that at some point in the future, empirical testability will become more feasible; and even irrespective of this, the burgeoning field of quantum gravity phenomenology has made it clear that there are other, perhaps more indirect, methods of assessing the empirical status of various approaches to QG.[3] Historically, however, in the absence of immediate empirical testability, many approaches to QG have relied on appeals to guiding principles.

This essay is concerned with the extent to which a particular approach to QG, the condensed matter approach, satisfies three such principles: asymptotic safety, relative locality, and holography. In

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[1] The literature on Einstein's famous distinction between theories of principle and constructive theories is vast. See, *e.g.*, Brown & Pooley (2006) and references therein.

[2] This argument is summarized in Weinberg (1997) and analyzed in Bain (1999).

[3] Thanks to a referee for stressing this point.

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senses to be made more precise in Sections 3–5 below, these principles state the following:

Asymptotic safety: A theory of QG must scale towards an ultra-violet (UV) fixed point with a finite number of UV-irrelevant couplings. (Weinberg, 1979)

Relative locality: A theory of QG must entail that coincidence of events in spacetime ("locality") is relative to an observer's rest frame. (Amelino-Camelia, Freidel, Kowalski-Gllikman, & Smolin, 2011a; Amelino-Camelia, Freidel, Kowalski-Gllikman, & Smolin, 2011b)

Holography: A theory of QG must entail that the number of fundamental degrees of freedom in any region $\mathcal{O}$ of spacetime cannot exceed $A/4$, where A is the surface area of $\mathcal{O}$. (Bousso, 2002)

Asymptotic safety addresses a particular problem with reconciling general relativity (GR) and quantum field theory (QFT); namely, when GR is constructed as a QFT, it is non-renormalizable. In this context, this means that when one expands relevant quantities in GR in a perturbative power series expansion, there are terms that diverge at high energies and that cannot be regularized by renormalizing a finite number of parameters associated with the theory. Percacci (2009), p. 111 suggests that this has "…led to widespread pessimism about the possibility of constructing a fundamental QFT of gravity. Instead, we have become accustomed to thinking of general relativity (GR) as an effective field theory, which only gives an accurate description of gravitational physics at low energies." Weinberg (1979) suggested that this pessimism would be addressed if it could be shown that GR is asymptotically safe. In a sense to be made more precise below, an asymptotically safe theory is well-behaved at all energy scales and thus, perhaps, warrants the ascription of a fundamental theory. Viewing asymptotic safety as a guiding principle in the search for a theory of QG takes Weinberg's suggestion seriously: If GR does happen to possess a UV fixed point, our search is over; if it happens that GR does not possess a UV fixed point, then we should be looking for QFTs describing gravity that do.

Relative locality is motivated by taking seriously the non-linear composition law for velocities in special relativity (Amelino-Camelia et al., 2011a, p. 11). This law imposes an invariant scale, the speed of light c, on the theory (provided one upholds the principle of relativity), which serves to demarcate the scale at which effects due to the non-linearity become relevant. This scale then makes possible the mixing of spatial and temporal coordinates of an event under Lorentz transformations, and this entails the relativity of simultaneity (*i.e*, the relativity of coincidence of temporal coordinates). If one assumes a similar non-linearity in the composition of momenta, one is led to a mixing of spatiotemporal and 4-momentum coordinates of an event. In analogy with the relativity of simultaneity, this mixing entails a relativity of "locality" (*i.e.*, a relativity of coincidence of spatiotemporal coordinates). One motivation for assuming a non-linear composition law for momenta is that the associated scale may be identified with the Planck mass $M_p = \sqrt{\hbar/G}$ (where $\hbar$ is Planck's constant, G is the Newtonian gravitational constant, and the speed of light c is taken to be unity). One can thus argue that relative locality is an effect that probes the particular regime of QG characterized by the limit $\hbar \to 0$, $G \to 0$, holding M_p fixed. It turns out that this regime is empirically accessible (see Section 4, below). Moreover, relative locality can be encoded in non-trivial curvature in 4-momentum space, and this has suggested to its advocates that spacetime can be viewed as an emergent phenomenon, a common theme in many approaches to QG.

Finally, the holographic principle was originally motivated by certain results in black hole thermodynamics, as Section 5 will explain. But it has since morphed into a general notion that the bulk properties of certain types of systems can be encoded in their edge states. This notion has been broadly applied in many approaches to QG in the form of a particular type of duality relation. For instance, much has been written on the AdS/CFT duality which describes a correspondence between a type of string theory in anti-de Sitter spacetime (the "bulk" theory), and a type of conformal gauge field theory on the boundary of anti-de Sitter spacetime (the "edge" theory).[4] This correspondence has, among other things, precipitated a recent resurgence in work on twistor theoretic approaches to QG; in particular, Witten (2004) demonstrated how scattering amplitudes in the boundary theory can be calculated via twistor methods. This might be viewed as an example of how a guiding principle can suggest overlaps between what initially may seem to be disparate fields (in this case, phenomenological particle physics, string theory, and twistor theory).

In an endeavor to assess these principles, I will focus on the condensed matter approach to QG. I'll begin by identifying two distinct versions of this approach and then consider how these versions satisfy the above principles. The general hope is that a focus on different versions of a single approach to QG may provide insight into the nature of these principles. There are many other guiding principles in the literature on QG (duality, minimal length, background independence, *etc.*). However the above three seem particularly relevant in the context of the condensed matter approach for the following reasons: First, being clear about the principle of asymptotic safety will require being clear about the notion of an effective field theory (EFT), which plays an essential role in the condensed matter approach. Moreover, in distinguishing between an EFT and an asymptotically safe theory, this principle raises the question, *What is a fundamental theory*? Second, being clear about the principle of relative locality will require being clear about the role that topological invariants play in both versions of the condensed matter approach, and how they relate to momentum space curvature. Moreover, this principle raises the question, *Is a state description of a physical system in terms of energy/momentum variables more fundamental than one in terms of spacetime variables?* and so questions the fundamentality of spacetime. Finally, being clear about the principle of holography will require being clear on how edge states of certain condensed matter systems may be said to encode essential properties of bulk states. This principle raises fundamental questions concerning the relations between information, entropy, and black hole thermodynamics.

Before I begin, perhaps a disclaimer is appropriate. The goal of this essay is rather modest. It is simply to describe the extent to which three particular guiding principles associated with research in quantum gravity are supported by one particular approach; namely, the condensed matter approach. The intent of the essay is not to argue in favor of the condensed matter approach, nor is it to argue in favor of the three principles above, as opposed to others. As will be seen below, connections between these three principles and the condensed matter approach have been made by some authors, however the implications of these connections have not been fleshed out in much detail. The primary goal of this essay is to provide some of these missing details.

2. The condensed matter approach: Two versions

The goal of the condensed matter approach to QG is to construct a low-energy effective field theory (EFT) of a condensate

[4] Teh (2013), pg. 3 identifies this as an example of a "holographic duality" relation, and views it as one of several types of duality relations in physics.

that mimics general relativity (GR) and the Standard Model. This is an approach to QG insofar as the latter attempts to reconcile GR with quantum theory. The reconciliation here takes the form of a common origin in the condensate, the low-energy excitations of which take the form of the gauge, matter, and metric fields of GR and the Standard Model.

EFTs play a fundamental role in this approach. In general, an EFT of a physical system is a description of the system at energies restricted to a given range. When a high-energy theory is known, an EFT is constructed by identifying and then eliminating high-energy degrees of freedom from it. One way to understand the nature of an EFT is by means of the concept of a renormalization group transformation. The intent is to analyze the behavior of a theory at different energy scales and the procedure involves the following three steps:[5] Given a high-energy theory encoded in an action,

$$S[\phi] = \sum_a g_a \mathcal{O}_a[\phi] \tag{1}$$

where g_a are coupling constants, and $\mathcal{O}_a$ are combinations of field variables ϕ and their first- and possibly higher-order derivatives,

(i) Decompose the field variables into a set to be eliminated $\{\phi_h\}$, and a complementary set $\{\phi_l\}$.[6]

(ii) Integrate over the high-energy degrees of freedom $\{\phi_h\}$. For non-trivial interactions, this requires a perturbative expansion in the low-energy degrees of freedom:

$$S'[\phi_l] = S_0[\phi_l] + \sum_a g'_a \mathcal{O}'_a[\phi_l] \tag{2}$$

which typically contains terms distinct from those in (1). For weak interactions, the expansion point S_0 can be taken to be the free action.

(iii) Absorb any changes in (2) into re-scalings of the couplings and fields to obtain a "renormalized" action

$$S[\phi] = \sum_a g'_a \mathcal{O}_a[\phi]. \tag{3}$$

These steps define a map $\tilde{R} : \mathbf{g} \mapsto \mathbf{g}'$ in the abstract parameter space of the theory that relates the initial "bare" coupling constants $\mathbf{g} = \{g_a\}$ to the renormalized ones $\mathbf{g}' = \{g'_a\}$, and successive iterations of this map generate a flow.[7] A fixed point $\mathbf{g}^*$ of a flow is a point that is invariant under further transformations: $\tilde{R}(\mathbf{g}^*) = \mathbf{g}^*$. At a fixed point, not only does the form of the action remain invariant, so do the values of its parameters. An *irrelevant coupling with respect to a fixed point* is a coupling that *decreases* towards the fixed point, whereas a *relevant coupling with respect to a fixed point* is a coupling that *increases* towards the fixed point.[8] The expansion point S_0 in (2) is an example of a fixed point, and the rescaling in Step (iii) is possible only if, in (2), there are a finite number of relevant terms with respect to S_0, and no irrelevant terms. Such a theory is referred to as renormalizable. A theory with irrelevant terms, on the other hand, is non-renormalizable. In such a theory, the contributions from high-energy degrees of freedom $\{\phi_h\}$ cannot be absorbed by re-scalings of the couplings and low-energy field variables.

One can stop at Step (ii) in this process and consider the action (2) as defining an EFT. Such an EFT can thus be either renormalizable or non-renormalizable. Examples of renormalizable EFTs include (2+1)-dim QED as an EFT of a superfluid Helium 4 film (Zhang, 2002), and the sector of the Standard Model below electroweak symmetry breaking as an EFT of superfluid Helium 3-A (Volovik, 2003, pp. 114–115). Typically non-renormalizable EFTs contain an infinite number of irrelevant terms, but one can show that these terms are suppressed at low energies (by inverse powers of the relevant cutoff). Moreover, for (3+1)-dim spacetimes, such EFTs contain only a finite number of relevant terms; thus they will effectively only depend on the high-energy theory through a finite number of parameters (Polchinski, 1993, p. 3). An example of a non-renormalizable EFT is Donoghue (1995) EFT for general relativity.

Associated with a fixed point $\mathbf{g}^*$ is the concept of a universality class. This is an equivalence class of theories that all possess the same behavior in the neighborhood of $\mathbf{g}^*$ (as characterized by the same relevant terms with respect to $\mathbf{g}^*$), but differ in their behavior away from $\mathbf{g}^*$ (*i.e.*, in their irrelevant terms with respect to $\mathbf{g}^*$). All such theories thus have the same low-energy/macroscopic behavior characterized by the fixed point, but may have different high-energy/microscopic characteristics.

In the condensed matter context, fixed points and universality classes are associated with spontaneously broken symmetries, and internal orders characterized by symmetries. Under the Landau–Ginzberg theory of phase changes (and its extension by renormalization group (RG) techniques), a fixed point corresponds to a phase transition at which the symmetry characterizing the internal order of the given physical system is spontaneously broken. The universality class associated with such a fixed point consists of theories that describe microscopically distinct physical systems (*i.e.*, systems that differ in their short-distance/high-energy degrees of freedom) that all exhibit the same macroscopic low-energy phase transition behavior.

These considerations suggest that there are two distinct versions of the condensed matter approach to quantum gravity, depending on the nature of the condensate one chooses from which to recover GR and the Standard Model. One version focuses on condensates characterized by spontaneously broken symmetries and universality. The goal of this version is to construct EFTs of an appropriately identified condensate that belong to the same universality class of (relevant sectors of) the Standard Model, with the hope that GR can likewise be recovered.[9] An example of this version is Volovik's (2003), pp. 114–115 construction of an EFT for superfluid Helium 3-A, and the demonstration that it belongs to the same universality class as the massless sector of the Standard Model above electroweak symmetry breaking (see Bain, 2012, pp. 3–4, for a brief discussion of this example).

A second version of the condensed matter approach focuses on condensates for which universality (defined in terms of a fixed point of an RG flow) does not apply, and internal order cannot be characterized by symmetry. Such condensates are rather characterized by what Wen (2004), p. 341 refers to as topological order.[10] The primary example of this type of condensate is a fractional quantum Hall liquid, although, according to Wen, topological

[5] The following is based on the exposition in Altland & Simons (2010), pp. 429–432.

[6] This identification can proceed in a number of distinct ways (Altland & Simons, 2010, p. 430), including the following: One can impose a high-energy cutoff Λ and identify the $\{\phi_h\}$ as possessing momenta $\mathbf{p}$ within a finite range $\Lambda/b \le |\mathbf{p}| < \Lambda$, for $b > 1$, in which case the divergent integrals that typically appear in Step (ii) are rendered finite. Alternatively, one can identify the $\{\phi_h\}$ as possessing momenta greater than a given low-energy cutoff, in which case the divergent integrals that appear in Step (ii) are typically handled by dimensional regularization.

[7] The flow is described by the Gell–Mann-Low equation $dg/d\ell = R(\mathbf{g})$, where $\ell = \ln b, R(\mathbf{g}) = \lim_{\ell \to 0} \ell^{-1}(\tilde{R}(\mathbf{g}) - \mathbf{g})$.

[8] These definitions can be made rigorous by an analysis of the eigenvalues associated with a linearization of the map $R(\mathbf{g})$ about a given fixed point (Altland & Simons, 2010, pp 434–435).

[9] This hope is tempered by the fact that effective metrics can be constructed in such models and that curvature can be represented by low-energy perturbations of these metrics. The acoustic spacetime program (Barcelo, Liberati, & Visser, 2011) focuses on this aspect of these models, as opposed to universality.

[10] Note that Wen (2004), pg. 408 does associate the concept of topological order with a notion of universality; however, this notion is not the same as the one informed by RG analyses of fixed points associated with spontaneous symmetry breaking. In any event, the distinction I'd draw between these two versions of the condensed matter approach is that the first explicitly employs universality as a guiding principle in model construction, whereas the second does not.

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orders characterize any condensate with ground states that possess a finite energy gap.[11] An example of the condensed matter approach to QG based on this type of condensate is Zhang and Hu's (2001) construction of an EFT for the edge of a 4-dimensional fractional quantum Hall liquid. This EFT describes $(3+1)$-dim zero-rest-mass fields, and the hope is that GR and the Standard Model can be reconstructed from such fields, perhaps by employing twistor theory (Bain, 2012, p. 4, provides brief discussion).

3. Asymptotic safety

I'd now like to consider the principle of asymptotic safety. Recall that this requires that a theory of QG must scale towards an ultraviolet (UV) fixed point with a finite number of UV-irrelevant couplings. An *ultraviolet* (*UV*) *fixed point* is a fixed point associated with high-energies, whereas an *infra-red* (*IR*) *fixed point* is a fixed point associated with low-energies. Thus a UV-irrelevant coupling decreases at high energies, whereas an IR-irrelevant coupling decreases at low energies. Similarly, a UV-relevant coupling increases at high energies, whereas an IR-relevant coupling increases at low energies.

These distinctions allow one to characterize theories in the following way (see Table 1). A *renormalizable theory* is associated with an IR fixed point $\mathbf{g}_{IR}{}^*$ and possesses no IR-irrelevant, and a finite number of IR-relevant couplings, whereas a *non-renormalizable theory* is associated with an IR fixed point and possesses an infinite number of IR-irrelevant and a finite number of IR-relevant couplings. In this context, Weinberg (1979) defined an *asymptotically safe theory* (AST) as a theory associated with a *UV* fixed point $\mathbf{g}_{UV}{}^*$ and possessing a finite number of UV-irrelevant couplings and a (potentially) infinite number of UV-relevant couplings. An AST is essentially the UV mirror-image of a non-renormalizable theory.

As noted in Section 2, an example of a non-renormalizable theory is GR formulated as a quantum field theory: it has an infinite number of IR-irrelevant couplings that supposedly blow up at high-energies. An example of an AST is quantum chromodynamics (QCD). The UV fixed point of QCD is the free theory: the strong force goes to zero at high energies (thus, not only is QCD asymptotically safe, it is also asymptotically *free*). Weinberg (1979) originally suggested that the formulation of GR as a quantum field theory might be another example of an AST. If it has a UV fixed point (not necessarily a free-theory Gaussian fixed point), its IR-irrelevant couplings would be tamed, and the theory would be well-behaved at all scales. This suggestion has spawned a research programme that attempts to identify UV fixed points of GR, hence the associated guiding principle of asymptotic safety (see, *e.g.*, Percacci, 2009).

An initial assessment of this principle in the context of the condensed matter approach might begin with the following claim:

The EFTs in both versions of the condensed matter approach should aspire to be ASTs.

This claim seems reasonable to the extent that both versions attempt to reproduce the QCD sector of the Standard Model (and, potentially, the GR sector of QG). On the other hand, this would seem to mean that the EFTs in both versions should aspire to be associated with two fixed points: An IR fixed point defined with respect to the "high-energy" theory of the condensate (defining the expansion point in (2)), and a UV fixed point associated with the QCD and GR sectors of QG.

One potential worry here is whether it's consistent to consider an EFT as an AST. Under Weinberg's interpretation, an AST is a fundamental theory to all orders, insofar as it is supposed to get the fundamental degrees of freedom right: If GR is an AST, then "… the appropriate degrees of freedom at all energies are the metric and matter fields…" (Weinberg, 2009, p. 17). An EFT, on the other hand, is typically not taken to be fundamental in this sense. It's typically interpreted as restricted to a given energy range, beyond which new physics is supposed to arise (or, minimally, beyond which one should remain agnostic). Indeed, under a literal interpretation of the condensed matter approach, the fundamental degrees of freedom are those of the condensate, and the degrees of freedom associated with GR and the Standard Model are simply low-energy approximations of the former.

On the other hand, the relation between an EFT and a high-energy theory need not be interpreted as one of approximation. If one can argue that an EFT is autonomous, in an appropriate sense, from its high-energy theory, one need not view the latter as fundamental and the former as less so. For instance, if one can describe the relation between an EFT and a high-energy theory as one of emergence (in some sense), then, at least conceptually, it may be consistent to claim that an AST can emerge in the form of an EFT of a fundamental condensate. Thus whether or not it's consistent to consider an EFT as an AST will depend, in particular, on how the relation between an EFT and a high-energy theory is cashed out.

As an example, suppose the relation between an EFT (2) and a high-energy theory (1) can be characterized by the following properties:

(a) *Failure of law-like deducibility*. The phenomena described by an EFT are not deducible consequences of the laws of a high-energy theory.
(b) *Ontological distinctness*. The degrees of freedom of an EFT characterize physical systems that are ontologically distinct from physical systems characterized by the degrees of freedom of a high-energy theory.
(c) *Ontological dependence*. Physical systems described by an EFT are ontologically dependent on physical systems described by a high-energy theory.

Property (a) understands the laws of a theory encoded in an action to be its Euler–Lagrange equations of motion, and is thus suggested by the formal distinctions between the EFT (2) and the high-energy theory (1), and their corresponding Euler-Lagrange equations of motion. In the case of property (b), this suggests that the degrees of freedom of an EFT are dynamically distinct from those of a high-energy theory (in the sense of satisfying different dynamical laws); moreover, the former are typically encoded in field variables that are formally distinct from those that encode the latter; *i.e.*, different field variables, ϕ_l, ϕ, appear respectively in the actions of an EFT (2) and a high-energy theory (1).[12] On the other hand, the fact that the degrees of freedom of the former can be identified as the low-energy degrees of freedom of the latter suggests property (c): the physical systems described by an EFT do not completely "float free" of the physical systems described by a high-energy theory.

One way to connect these properties of the relation between an EFT and a high-energy theory to a concept of emergence is to conceive of the latter as embodying both a notion of *novelty* (in the sense that emergent properties should not be deducible from fundamental properties), and a notion of *microphysicalism* (in the sense that the emergent system should ultimately be composed of

[11] Wen (2004), pg. 341. In addition to fractional quantum Hall liquids, Wen lists the following condensates as possessing topological orders: chiral spin liquids, anyon superfluids, short-ranged resonating valence bound states of spin systems, and superconducting states with dynamical electromagnetic interactions.

[12] This is not the case when the identification in Step (i) is done in terms of a high-momenta/low-momenta splitting, in which case ϕ_l are the same functions as ϕ, just restricted to a given range of momentum. In general, however, the identification of the low-energy degrees of freedom need not follow this procedure.

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Table 1
Theory types.

Renormalizable theory	$S'[\phi] = S_{\mathbf{g}\mathrm{IR}^*}[\phi] + \sum_a g'_a \mathcal{O}'_a[\phi]$	No # IR-irrelevant couplings Finite # IR-relevant couplings
Non-renormalizable theory	$S''[\phi] = S_{\mathbf{g}\mathrm{IR}^*}[\phi] + \sum_a g''_a \mathcal{O}''_a[\phi]$	Infinite # IR-irrelevant couplings Finite # IR-relevant couplings
Asymptotically safe theory	$S'''[\phi] = S_{\mathbf{g}\mathrm{UV}^*}[\phi] + \sum_a g'''_a \mathcal{O}'''_a[\phi]$	Finite # UV-irrelevant couplings Infinite # UV-relevant couplings

Table 2
Theories and their single particle phase spaces.

Γ = phase space (x^μ, p_μ)	$\mathscr{M}$ = configuration space (x^μ)	$\mathcal{P}$ = momentum space (p_μ)
$\Gamma^{SR} = \mathscr{M} \times \mathcal{P}$	Flat	Flat
$\Gamma^{GR} = T\mathscr{M}$	Curved	Flat
$\Gamma^{RL} = T\mathcal{P}$	Flat	Curved

microphysical systems that comprise the fundamental system). One might then attempt to argue that properties (a) and (b) underwrite novelty, whereas property (c) underwrites microphysicalism (see, *e.g.*, Bain, 2012).

4. Relative locality

I'd like to move on to the principle of relative locality. This requires that a theory of QG must entail that coincidence of events in spacetime is relative to an observer's rest frame. Recall that one way to motivate this is by allowing a non-linear law of composition for 4-momenta, in analogy with the non-linear law of composition of velocities in special relativity. This non-linearity can be encoded in a non-flat affine connection on 4-momentum space (Amelino-Camelia et al., 2011a, pp. 3–4).

To understand this, consider the single particle phase spaces of special and general relativity (see Table 2, after Amelino-Camelia et al., 2011b, p. 2549). In both cases, the canonical variables that coordinatize this phase space are the positions and 4-momenta (x^μ, p_μ) of the particle.[13] The x^μ encode the possible positions of the particle, and form the particle's configuration space $\mathscr{M}$, while the p_μ are the canonically conjugate variables that encode the possible 4-momenta of the particle, and coordinatize the particle's 4-momentum space $\mathcal{P}$. The phase space can be geometrically represented by the cotangent bundle $T^*\mathscr{M}$ over $\mathscr{M}$, where, for a single particle, the latter is identifiable with a Lorentzian manifold $(M, g_{\mu\nu})$, where $g_{\mu\nu}$ is a solution to the Einstein equations. In the case of special relativity, $g_{\mu\nu}$ is the Minkowski metric, and $\mathscr{M}$ is flat, thus the cotangent bundle is given by the Cartesian product $\mathscr{M} \times \mathcal{P}$. In general relativity, $\mathscr{M}$ is allowed to have nontrivial curvature, thus $T^*\mathscr{M}$ will be a nontrivial bundle space. In both cases, however, $\mathcal{P}$ is assumed to be flat.

In contrast, the momentum space of a theory that satisfies relative locality is allowed to be curved. In particular, Amelino-Camelia et al. (2011a), p. 5 show that non-linearity in the momentum composition law can be encoded in the curvature of $\mathcal{P}$. They demonstrate how the dynamics of a single particle can be encoded in an action defined on the cotangent bundle over $\mathcal{P}$. In this case, the phase space for such a particle is given by $T^*\mathcal{P}$, with canonical variables given by (p_μ, x^μ). Here the p_μ act as the canonical "position" variables, while the x^μ act as their canonically conjugate "momentum" variables (the x^μ coordinatize the cotangent spaces $T_p\mathcal{P}$ over $\mathcal{P}$). Thus in such a theory, there's a separate conjugate variable space (*i.e.*, spacetime) $\mathscr{M}_p$ for each point $p \in \mathcal{P}$. And if $\mathcal{P}$ is curved, then the $\mathscr{M}_p$'s will differ from point to point. In particular, suppose two particles have phase space coordinates (p_μ, x^μ), (q_μ, y^μ), where $p_\mu \neq q_\mu$. To compare their spacetime coordinates, one needs to map the point $x^\mu \in \mathscr{M}_p$ to the point $y^\mu \in \mathscr{M}_q$ by means of parallel transport in $T\mathcal{P}$. One then finds $x^\mu = y^\mu$ (*i.e.*, the particles coincide) just when either $x^\mu = y^\mu = 0$, or the connection vanishes. The former condition holds in the rest frame of an observer very close to the particles, and the latter condition entails 4-momentum space is flat (Amelino-Camelia et al., 2011b, p. 2552). Thus two events that coincide in spacetime for a local observer may not coincide in spacetime for distant observers.[14]

One motivation for taking $\mathcal{P}$-space curvature seriously is that it entails non-commutativity of spacetime coordinates, and various approaches to QG employ non-commutative geometry. Moreover, advocates of relative locality have suggested that $\mathcal{P}$-space curvature has observable effects that are detectable by current technology. One can show that the deviation $\Delta x = y^\mu - x^\mu$ due to relative locality between the spacetime coordinates x^μ and y^μ of two events, A, B is of the order $\Delta x \sim x$—E/M_p, where E is the energy associated with event A; thus

> We see from this formula that the smallness of $M_p{}^{-1}$ can be compensated by a large distance x, so that over astrophysical distances values of Δx which are consequences of relative-locality effects take macroscopic values. (Amelino-Camelia et al., 2011b, p. 2552.)

Amelino-Camelia and Smolin (2009) have shown that the time of arrival of cosmic gamma-ray bursts as measured by the Fermi telescope may display this effect.

How does relative locality relate to the condensed matter approach? According to its advocates,

> …just as some condensed matter or fluid systems provide analogues for relativity and gravity, it may be that condensed matter systems with curved momentum spaces may give us analogues to the physics of relative locality. (Amelino-Camelia et al., 2011a, pp. 12.)

I'd now like to consider how this might be made a bit more precise in the context of the two versions of the condensed matter approach. It turns out that both versions encode aspects of their EFTs in aspects of $\mathcal{P}$-space *topology*, and these topological aspects can then be related to $\mathcal{P}$-space curvature. An example of such a relation is the Gauss–Bonnet–Chern theorem, which relates an aspect of the topology of a given parameter space to an aspect of

[13] In Amelino-Camelia et al.'s presentation, gravitational degrees of freedom are ignored.

[14] Thus relative locality is entailed by $\mathcal{P}$-space curvature. However relative locality, assumedly, does not entail $\mathcal{P}$-space curvature, insofar as one can conceive of other ways besides the latter to encode the former.

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its geometry:

$$2(1-g) = 1/(2\pi)\int_S K dA \quad (4)$$

where the integral is over a surface S without boundary, K is the local curvature of S, and the integer g is the number of handles characterizing the topology of S (Avron, Osadchy, & Seller, 2003, p. 40). Intuitively, one can identify analogues of (4) in both versions of the condensed matter approach.

The first version fleshes this out in the following three steps (after Volovik, 2003):

1. One first encodes low-energy dynamics in the form of a (single-particle, retarded or advanced) Green's function on $\mathcal{P}$-space:

   $$\mathcal{G}(p_0, \mathbf{p}) = [ip_0 - \mathscr{H}(\mathbf{p})]^{-1} \quad (5)$$

   where $\mathscr{H}(\mathbf{p})$ is the condensate Hamiltonian. For superfluid Helium 3-A, low-energy excitations correspond to poles in the Green's function, which are represented by points in $\mathcal{P}$-space (referred to as "Fermi points").
2. One then demonstrates that the low-energy dynamics is stable under perturbations. Mathematically, one can construct a topological invariant,

   $$N_3 = (1/24\pi^2)\varepsilon_{\mu\nu\lambda\gamma}\mathrm{Tr}\int_\Sigma dS^\gamma\, \mathcal{G}\partial_{p_\mu}\mathcal{G}^{-1}\mathcal{G}\partial_{p_\nu}\mathcal{G}^{-1}\mathcal{G}\partial_{p_\lambda}\mathcal{G}^{-1} \quad (6)$$

   given by the integral in $\mathcal{P}$-space over a surface Σ surrounding the Fermi points, where the integrand depends on the Green's function and derivatives of its inverse (Volovik, 2003, p. 97). This defines a nontrivial winding number of the map from Σ to the space of Green's function matrices. N_3 is thus invariant under continuous deformations of the Green's function, which entails that it is invariant under low-energy perturbations of the Hamiltonian. This indicates that N_3 defines a fixed point/universality class.
3. Finally, one can relate the $\mathcal{P}$-space topological invariant N_3 to $\mathcal{P}$-space curvature. Intuitively, N_3 encodes topology, while the integral on the RHS of (6) encodes $\mathcal{P}$-space geometry (as in the Gauss–Bonnet–Chern theorem). More specifically, one can show that, in the case of the integer quantum Hall effect, the quantized Hall conductance is given by a topological invariant that can be obtained from N_3 *via* dimensional reduction (Volovik, 2003, pp. 136, 269); and it's been shown that the Hall conductance can be encoded in the adiabatic curvature of the relevant parameter space (Thouless, Kohmoto, Nightingale, & den Nijs, 1982). This suggests that the integral expression that defines N_3 in (6) also encodes parameter space (*i.e.*, $\mathcal{P}$-space) curvature.

Similar steps can also be identified in the fractional quantum Hall example of the second version of the condensed matter approach:

1. One first encodes the *internal order* of the condensate in its ground state degeneracy (GSD). One can show that two distinctly ordered fractional quantum Hall states can have the same symmetries but different GSD (Wen, 2004, p. 342). Thus the internal order of fractional quantum Hall states cannot be characterized by symmetry, but can be (partially) characterized by GSD.
2. One can then demonstrate that the GSD of fractional quantum Hall states depends on topology, and is robust under arbitrary perturbations, which indicates it's encoded in a topological invariant (Wen & Niu, 1990, p. 9378).
3. Finally, one can relate GSD to $\mathcal{P}$-space curvature in the following way (Wen, 1990): Fractional quantum Hall states can be classified by matrices K and described by an effective topological quantum field theory, where the determinant of the K matrix encodes the GSD of a given fractional quantum Hall liquid. Wen then showed that K can be encoded in the Berry phase characterizing adiabatic deformations of the fractional quantum Hall Hamiltonian, where the Berry phase is an ingredient in the definition of the adiabatic (*i.e.*, parameter space) curvature.

Thus, charitably, both versions of the condensed matter approach to QG may be said to satisfy the principle of relative locality, to the extent that both can be associated with curved momentum spaces. In the following two subsections, I'd like to consider what, if any, questions of fundamentality this raises.

4.1. Relative locality and phase space realism

Advocates of relative locality suggest that it entails that what is fundamentally real is neither spacetime nor momentum space, but phase space:

> We do not live in spacetime. We live in Hilbert space, and the classical approximation to that is that we live in phase space. (Amelino-Camelia et al., 2011a, p. 12.)

> If… momentum space is curved, spacetime is just as observer dependent as space, and the invariant arena for classical physics is phase space. (Amelino-Camelia et al., 2011b, p. 2553.)

There seem to be two motivations for this view, call it *phase space realism*, in Amelino-Camelia et al. (2011a, 2011b). The first is based on the claim that spacetime is not fundamental, and the second is based on an analogy between the relativity of simultaneity, on the one hand, and the relativity of locality, on the other. The first motivation is underwritten by two types of argument. One type argues that descriptions of physical systems in terms of their energies and momenta are more fundamental than descriptions in terms of their spatiotemporal properties:

> Our most fundamental measurements are the energies and angles of the quanta we emit or absorb, and the times of those events. Judging by what we observe, we live in energy-momentum space, not in spacetime. (Amelino-Camelia et al., 2011a, p. 1.)
>
> What we really see in our telescopes and particle detectors are quanta arriving at different angles with different momenta and energies. Those observations allow us to infer the existence of a universal and energy-independent description of physics in a spacetime only if momentum space has a trivial, flat geometry. (Amelino-Camelia et al., 2011b, pp. 2552–2553.)

The suggestion here is that our impression of the fundamentality of spacetime is based, ultimately on operational procedures by means of which we come to have knowledge of spatiotemporal properties. For instance, Einstein's operational definition of simultaneity is based, in part, on the exchange of photons, and it implicitly assumes that the energies of such photons do not affect the outcome of the operational procedure. This assumption is put into question if one introduces a fundamental energy/momentum scale, such as the one associated with relative locality. In particular, the claim is that the spacetime structure we observe via our physical probes (for instance, observations that suggest locality is absolute) is a reflection of the low-energy nature of these probes with respect to the scale. This appeal to experience for the non-fundamentality of spacetime is further buttressed by another type of argument that claims that spacetime emerges from the dynamical interactions of particles in momentum space:

> We take the point of view that spacetime is an auxiliary concept which emerges when we seek to define dynamics in momentum space. (Amelino-Camelia et al., 2011a, p. 5.)

This latter claim is underwritten in Amelino-Camelia et al. (2011a) by an analysis of a relativistic multi-particle action constructed on the phase space $\Gamma = T^*\mathcal{P}$. Amelino-Camelia et al. (2011a), p. 7 note that "…there is neither an invariant projection from Γ to a spacetime $\mathcal{M}$, nor is there defined any invariant spacetime metric. Yet this structure is sufficient to describe the dynamics of a free particle."

These considerations may add weight to the claim that spacetime is less fundamental than momentum space, but it's not entirely clear how the inference from this claim to the fundamentality of phase space is to be made. This is where the second motivation for phase space realism becomes important. This seems to be based on an analogy between the relativity of locality and the relativity of simultaneity. Relative simultaneity can be explained by appeal to an invariant spatiotemporal interval that can be decomposed into separate spatial and temporal intervals, but in a non-invariant, observer-dependent way. Analogously, relative locality can be explained by appeal to an invariant phase space interval that can be decomposed into separate spatiotemporal and energy-momentum intervals, but in an observer-dependent way:

> Physics takes place in phase space and there is no invariant global projection that gives a description of processes in spacetime. From their measurements local observers can construct descriptions of particles moving and interacting in a spacetime, but different observers construct different spacetimes, which are observer-dependent slices of phase space. (Amelino-Camelia et al., 2011a, p. 2.)

There are at least two concerns with this motivation.

(a) First, we may grant that spatiotemporal intervals are relative, whereas phase space intervals are absolute; but does it necessarily follow that phase space descriptions are more fundamental than spatiotemporal descriptions? That spatial intervals and temporal intervals are relative in special relativity, whereas spatiotemporal intervals are not, does not, by itself, entail that space and time are less fundamental than spacetime in special relativity.[15] In general, the distinction between an absolute property and a relative property doesn't necessarily map onto the distinction between a fundamental property and a non-fundamental property.

(b) Second, one might question the analogy between the relativity of simultaneity and the relativity of locality in the following way. The relativity of simultaneity involves the relativity of the temporal coincidence of two events with respect to inertial observers. In particular, the distance that separates these events is not essential to the effect (in principle observers in different inertial frames will disagree on whether two events A, B are simultaneous, regardless of the distance that separates A and B). The relativity of locality, on the other hand, essentially involves the observer-dependence of spacetime coincidences of events with respect to distances. Whether or not two observers will judge A and B to be coincident in spacetime, according to relative locality, essentially depends on how far away from A and B, the observers are (as well as the energies and momenta of A and B). The relativity of locality thus seems to involve a distinction between a global understanding of spacetime coincidences (*i.e.*, whether such coincidences are global observables) *versus* a local understanding of spacetime coincidences (*i.e.*, whether such coincidences are local observables).[16] Put simply, whereas the relativity of simultaneity suggests a distinction between absolute and relative quantities, the relativity of locality suggests a distinction between global and local quantities.

Thus while taking relative locality seriously arguably supports what might be called $\mathcal{P}$-space fundamentalism, the inference to phase space realism seems a bit less secure.

Note that phase space realism is similar to the views of wavefunction realists (see, *e.g.*, Ney & Albert, 2013; Wallace & Timpson, 2010). The latter claim that "…the quantum state, if understood physically at all, should be understood in terms of its configuration space representation; that is, as a complex-valued field on 3*N*-dimensional space, for an *N*-particle quantum theory", thus, "[i]f wave-function realism is correct…, the world is really 3*N*-dimensional at its most fundamental level, and our 3-dimensional world is in some sense emergent from it" (Wallace & Timpson, 2010, p. 704). However, in comparing Amelino-Camelia et al.'s phase space realism with wavefunction realism, the following points should be kept in mind. First, wavefunction realism is based on a literal interpretation of the quantum mechanical configuration space formalism, and this should be made distinct from the more general quantum mechanical Hilbert space formalism; in particular, the configuration space formalism is based on a choice of the position basis in Hilbert space. Second, both of these quantum formalisms should be made distinct from the classical mechanical phase space formalism, which is the basis for the phase space realism of advocates of relative locality. The classical analog of wavefunction realism, evidently, would be classical configuration space realism (as opposed to phase space realism). For a single particle, this amounts to spacetime realism (in some sense, given that the object of wavefunction realism is relativistic quantum mechanics); whereas for N particles, this is realism with respect to a 4*N*-dimensional spacetime. The quantum analog of phase space realism, apparently, is Hilbert space realism (Amelino-Camelia et al., 2011a, p. 12.); although how this is to be cashed out is left unclear; *i.e.*, does it amount to realism with respect a particular representation of Hilbert space, or realism with respect to more abstract structural features of Hilbert space? The latter view suggests Wallace & Timpson's (2010) spacetime state realism, which they allow to take the form of realism with respect to the elements of a quasi-local C^*-algebra, indexed to a finite region of spacetime (p. 712). However, the central role that spacetime plays in this view may not be appropriate for advocates of relative locality.

Certainly, more could be said about the relation between the phase space realist interpretation of relative locality and wavefunction, or spacetime state, realism. For the purposes of this essay, I would like to move on to a discussion of how phase space realism might be understood in the condensed matter context.

[15] The dynamical constructive interpretation of special relativity, for instance, claims that the invariant structure of Minkowski spacetime is not fundamental, but rather derivative, being an expression of the Lorentz-invariance of more fundamental dynamical laws (see, *e.g.*, Brown & Pooley, 2006).

[16] This is also suggested by general relativity, in which energy-momentum is at best representable as a quasi-local observable. Thus if relative locality is to be motivated by a reciprocal relation between spatiotemporal properties and energy-momentum properties, general relativity suggests the relevant distinction is not between absolute and relative quantities, but between global and (quasi-) local quantities.

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4.2. Phase space realism and the condensed matter approach

Phase space realism is supposed to apply to theories that exhibit curved momentum space, and hence relative locality. Recall from the introduction that these are theories in the QG regime characterized by taking the limit $\hbar \to 0$, $G \to 0$, while holding M_p fixed; call this the *relative locality limit*. Thus suppose we have a hypothetical theory of QG in the form of a theory, call it T, of a condensate that reproduces GR and the Standard Model in its low-energy sector. In order for phase space realism to be applicable, assumedly, one would first have to establish the existence of the relative locality limit of T, and then identify the resulting relative locality regime of T with the low-energy sector of T that exhibits curved momentum space structure. Of course this task is difficult to assess without an explicit form of T; however, one can at least argue that the relative locality limit should be meaningful in this context. In particular, the parameters $\hbar$, G, M_p, should all be well-defined in T: $\hbar$ and G will appear in the low-energy EFT, constructed from T, that describes GR and the Standard Model, and will be composites of the fundamental parameters that describe the condensate. M_p will appear as the inverse of the atomic spacing between condensate atoms (*i.e.*, such spacing, given by the Planck length $l_p = 1/M_p$, represents the cut-off between the EFT that describes GR and the Standard Model, and the "trans-Planckian" theory of the condensate). Arguably, then, it should make sense to ask: (i) What would a phase space realist interpretation of the low-energy sector of T look like; and (ii) In the low-energy sector of T, in what sense are momentum space descriptions more fundamental than spacetime descriptions?

Under a literal interpretation of T, reality consists of a fundamental condensate, the low-energy excitations of which constitute the phenomena described by GR and the Standard Model. These same phenomena can be encoded in curved momentum space and, as argued above, can be said to exhibit relative locality. A phase space realist interpretation of these phenomena would assumedly claim that neither their spatiotemporal properties, nor their energy/momentum properties are fundamental. Rather, what is fundamental are their spatiotemporal-energy/momentum properties, and how these are split into separate spatiotemporal and energy/momentum properties is observer-dependent. Beyond this, it's not entirely clear what more might be said with respect to phase space realism. Perhaps, one might attempt to argue that the invariant reality represented by phase space is underwritten ontologically by the condensate.[17] In any event, I suggested above that while Amelino-Camelia et al.'s arguments for the fundamentality of $\mathcal{P}$-space state descriptions over spacetime state descriptions bore weight, their inference from $\mathcal{P}$-space fundamentality to phase space realism was more problematic. Thus perhaps it would be more productive to consider how $\mathcal{P}$-space fundamentalism might be understood in the condensed matter context.

A $\mathcal{P}$-space fundamentalist would, assumedly, claim that $\mathcal{P}$-space state descriptions of the low-energy, relative locality phenomena of the condensate are more fundamental than their configuration space ($\mathscr{M}$-space) state descriptions; and, in particular, that the relativistic spacetime associated with these phenomena emerges from the dynamics of their $\mathcal{P}$-space state descriptions. The idea that relativistic spacetime structure emerges in the low-energy sector of a condensate could be supported in one of (at least) two ways. First, one might appropriate a concept of emergence suitable for EFTs, perhaps the one suggested in Section 3 above (see, also, Bain, 2012). Alternatively, assuming that the relevant low-energy sector is associated with the relative locality limit, one might attempt to base an appropriate concept of emergence on the taking of this limit (see, *e.g.*, Bouatta & Butterfield, 2012 for how this can be done in the context of gauge theories). On the other hand, one would still need to articulate how this emergence of spacetime is from the dynamics on an underlying momentum space; although, in this case, physical intuitions seem a bit more plausible than in the case of phase space realism. One can imagine that the condensate is characterized not in terms of fundamental spatiotemporal properties, but rather in terms of fundamental energy/momentum properties, and the relevant dynamics is just that from which the EFT that describes the relevant low-energy sector is constructed.

[17] A structural realist interpretation of phase space may be one direction phase space realists might consider. For instance, North (2009) claims that in classical mechanics, phase space structure is fundamental, insofar as it is the structure that is minimally necessary to encode the symmetries of the dynamical laws of motion. On the other hand, North (2009), p. 29 suggests that "…phase space is as much a part of the representational content of classical mechanics as the theory's spacetime is", which does not seem appropriate for advocates of relative locality.

5. Holography

The last principle I'd like to consider is holography. This requires that a theory of QG must entail that the number of fundamental degrees of freedom N in any region of spacetime cannot exceed a quarter of the region's surface area (Bousso, 2002, pp. 838, 859). Informally, this is sometimes taken to mean that the information encoded in a physical system is contained not in its volume, but in its boundary. The first version of the condensed matter approach doesn't refer to such things. However, in the second version, one can show that the edge states of a fractional quantum Hall liquid (partially) encode the internal order exhibited by the bulk states, and Wen has explicitly related this to the holographic principle:

> This phenomenon of two-dimensional topological orders being encoded in one-dimensional edge states shares some similarities with the holomorphic [*sic*] principle in superstring theory and quantum gravity… (Wen, 2004, p. 347.)

How seriously should we take this? The holographic principle is based on two steps, as described by Bousso (2002). The first step identifies the number of fundamental degrees of freedom N of a given system with the natural logarithm of the number of its states $\mathcal{N}$,

$$N = \ln \mathcal{N} \tag{7}$$

and this is taken to represent the system's Boltzmann entropy S_B (Bousso, 2002, pp. 835–836). More precisely, Bousso (2002), p. 835 defines "…the *number of degrees of freedom* of a quantum-mechanical system N to be the logarithm of the dimension $\mathcal{N}$ of its Hilbert space $\mathscr{H}$: $N = \ln\mathcal{N} = \ln \dim(\mathscr{H})$", and this is suggested by the fact that "[t]he number of degrees of freedom is equal (up to a factor of ln2) to the number of bits of information needed to characterize a state". Bousso (p. 836) then identifies the number of states $\mathcal{N}$ with e^S, where S is the Boltzmann entropy (*i.e.*, in Bousso's words, the "statistical interpretation" of thermodynamic entropy).[18]

The second step makes an appeal to various entropy bounds. One example is Susskind's (1995) spherical entropy bound, which states that the entropy $S(\mathcal{O})$ associated with a spherical region $\mathcal{O}$ of spacetime of radius R cannot exceed $A/4$, where A is the surface area of a black hole with the same radius. This, and similar entropy

[18] t Hooft (1993), p. 4 characterizes these relations in the following way: "In any quantum theory there is a 'third law of thermodynamics' relating the entropy to the total number of degrees of freedom: the dimension of the vector space describing all possible states our system can be in is the exponent of the entropy."

bounds, is ultimately motivated by Bekenstein's (1973) Generalized Second Law (GSL) of thermodynamics, which requires that in the vicinity of a black hole, $\Delta S_{bh}+\Delta S\geq 0$, where ΔS_{bh} is the change in entropy of the black hole and ΔS is the change in entropy of the matter in its exterior, and where the entropy of a black hole is given by Hawking's formula: a quarter of its surface area.[19]

This suggests that the holographic principle really requires *three* steps:

(i) Positing a relation between the number of fundamental degrees of freedom N of a system, and the number of its states $\mathcal{N}$.
(ii) Using this relation to identify N with the Boltzmann entropy of the system.
(iii) Assuming the entropy of matter in the Generalized Second Law is the Boltzmann entropy.

Here are some concerns with these steps. First, Step (i) is motivated by theories that only have Boolean degrees of freedom. A Boolean degree of freedom only takes one of two values (think of a degree of freedom as an essential property). For theories with only Boolean degrees of freedom, the number of possible states $\mathcal{N}$ and the total number of Boolean degrees of freedom n are related by $\mathcal{N}=2^n$, or $n=\log_2\mathcal{N}$. This inspired t Hooft's (1993), p. 4 original formulation of the holographic principle:

> *The total number of Boolean degrees of freedom in any region of spacetime surrounding a black hole cannot exceed A/(4ln 2), where A is the horizon area.*[20]

For theories with non-Boolean degrees of freedom, this suggests (7) is inappropriate, and should be replaced with the naive generalization,

$$N=\log_m\mathcal{N} \tag{8}$$

where m is the number of values a non-Boolean degree of freedom can take. This then generates a concern with Step (ii). The naive generalization (8) is now disanalogous with the definition of Boltzmann entropy $S_B=\ln\mathcal{N}$. Recall that one motivation for the latter is that S_B is supposed to be an additive version of $\mathcal{N}$.[21] But N (the number of degrees of freedom, or essential properties) is conceptually distinct from $\mathcal{N}$ (the number of possible states) and not just an additive version of it.

Finally, Step (iii) requires a Boltzmann version of black hole entropy, and this requires identifying the microstates of a black hole and relating them to surface area. Charitably, there are some results in string theory and loop quantum gravity to this effect (Strominger & Vafa, 1996, Ashetekar, Baez, Corichi, & Krasnov, 1998). But I'd suggest that the general upshot of these concerns is to question the conceptual significance of the holographic principle, beyond a correspondence in some formulations of physical systems between bulk properties and edge states.

[19] The GSL is supposed to protect against apparent violations of the 2nd Law in which a physical system with a given amount of entropy falls into a black hole. To demonstrate the spherical entropy bound, suppose a spacetime region $\mathcal{O}$ with radius R can have more entropy than a black hole with the same radius. Now consider a process in which a region $\mathcal{O}$ with radius R and entropy $S>S_{bh}(R)$ collapses to form a black hole with radius $R'<R$ and entropy $S_{bh}(R')$. Hawking's area formula entails $S_{bh}(R)>S_{bh}(R')$; thus $S_{bh}(R')-S<0$. The process is thus characterized by $\Delta S_{bh}=S_{bh}(R')$, and $\Delta S=-S$; which entails $\Delta S_{bh}+\Delta S<0$, and this violates the GSL.

[20] For theories with only Boolean degrees of freedom, $S_B=\ln\mathcal{N}=n\ln 2$. Thus $n=S_B/\ln 2$, where $S_B\leq A/4$ due to the GSL.

[21] For instance, the number of possible states $\mathcal{N}_{12}$ of a system comprised of two subsystems with possible states $\mathcal{N}_1$, $\mathcal{N}_2$ is given by $\mathcal{N}_{12}=\mathcal{N}_1\times\mathcal{N}_2$. The corresponding Boltzmann entropies are then related by $S_{12}=S_1+S_2$.

Two caveats should be made at this point. First, as indicated above, to gain traction, the holographic principle requires implicit assumptions about the nature of information and the nature of physical systems; in particular, it assumes that physical systems can be described wholly in terms of Boolean degrees of freedom. If it turns out that any theory with non-Boolean degrees of freedom can ultimately be reduced to a theory with only Boolean degrees of freedom, then the concerns voiced above lose some of their bite. Second, even if the argument for holography from black hole thermodynamics fails, this is not to say that holographic duality as exhibited in other ways, is bankrupt. Teh (2013), pp. 3–4 for instance observes that the holographic duality exhibited by the AdS/CFT correspondence "…goes far beyond this [*i.e.*, the statement about degrees of freedom of the bulk being equivalent to the degrees of freedom of the boundary] by giving a precise account of the theories that live on the bulk and its boundary respectively, and how one can construct a 'dictionary' that relates the two."

6. Conclusion

This essay has briefly looked at three principles of quantum gravity in the context of the condensed matter approach. I've suggested that both versions of the condensed matter approach should aspire to be asymptotically safe, but I've questioned whether an asymptotically safe theory can also be considered an EFT. A comprehensive answer will have to involve fleshing out interpretative options surrounding the relation between an EFT and a high-energy theory, a central aspect of which may depend on the development of a notion of emergence appropriate for EFTs. I've also observed that while condensed matter approaches to quantum gravity based on fractional quantum Hall liquids may informally be said to satisfy the holographic principle, a deeper information-theoretic significance of the latter is questionable.

Thus asymptotic safety and holography seem to have limited applicability to the condensed matter approach. On the other hand, the manner in which relative locality expresses itself in the condensed matter approach perhaps offers more fertile ground for philosophical analysis, including, among other things, the extent to which momentum space state descriptions are more fundamental than configuration space state descriptions, the relation between phase space realism and wavefunction realism, and the extent to which spacetime can be said to emerge in the relative locality limit. I've suggested that both versions of the condensed matter approach satisfy the principle of relative locality, to the extent that they encode relevant quantities in momentum space topological invariants, and these invariants generate nontrivial momentum space curvature. But I've questioned whether relative locality underwrites realism with respect to phase space, as its advocates suggest, as opposed to fundamentalism with respect to momentum space. In answering these questions, further work, again, needs to done on the relation between an EFT and a high-energy theory, and on the distinctions between fundamental and derived properties, absolute and relative properties, and, arguably, global and local properties.

References

Altland, A., & Simons, B. (2010). *Condensed matter field theory* ((2nd ed.)). Cambridge University Press.
Amelino-Camelia, G. (2008) Quantum gravity phenomenology. arXiv: 0806.0339v1 [gr-qc].
Amelino-Camelia, G., & Smolin, L. (2009). Prospects for constraining quantum gravity dispersion with near term observations. *Physical Review D: Particles and Fields*, *80*, 084017.
Amelino-Camelia, G., Freidel, L., Kowalski-Gllikman, J., & Smolin, L. (2011a). Principle of relative locality. *Physical Review D: Particles and Fields*, *84*, 084010.

Amelino-Camelia, G., Freidel, L., Kowalski-Glikman, J., & Smolin, L. (2011b). Relative locality: A deepening of the relativity principle. *General Relativity and Gravitation*, *43*, 2547–2553.

Ashetekar, A., Baez, J., Corichi, A., & Krasnov, K. (1998). Quantum geometry and black hole entropy. *Physical Review Letters*, *80*, 904–907.

Avron, J. D., Osadchy, & Seller, R. (2003). A topological look at the quantum Hall effect. *Physics Today*, *56*, 38–42.

Bain, J. (2012). The emergence of spacetime in condensed matter approaches to quantum gravity. *Studies in History and Philosophy of Modern Physics* http://dx.doi.org/10.1016/j.shpsb.2012.05.001.

Bain, J. (1999). Weinberg on QFT: Demonstrative induction and underdetermination. *Synthese*, *117*, 1–30.

Barcelo, C., Liberati, S., & Visser, M. (2011). Analogue gravity. *Living Reviews in Relativity*, 14. 〈http://www.livingreviews.org/lrr-2011-3〉.

Bekenstein, J. (1973). Black holes and entropy. *Physical Review D: Particles and Fields*, *7*, 2333–2346.

Bouatta, N. &J. Butterfield (2012) On emergence in gauge theories at the 't Hooft limit'. [Preprint] URL: 〈http://philsci-archive.pitt.edu/id/eprint/9288〉.

Bousso, R. (2002). The holographic principle. *Review of Modern Physics*, *74*, 825–874.

Brown, H., & Pooley, O. (2006). Minkowski space-time: A glorious non-entity. In: D. Dieks (Ed.), *The ontology of spacetime*, vol. I (pp. 67–89). Elsevier.

Donoghue, J. (1995) Introduction to the effective field theory description of gravity. arXiv: gr-qc/9512024v1.

Ney, A., & Albert, D. (2013). *The wave function: Essays on the metaphysics of quantum mechanics*. Oxford University Press.

North, J. (2009). The 'structure' of physics: A case study. *Journal of Philosophy*, *106*, 57–88.

Percacci, R. (2009). Asymptotic safety. In: D. Oriti (Ed.), *Approaches to quantum gravity* (pp. 111–128). Cambridge University Press.

Polchinski, J. (1993). Effective field theory and the Fermi surface. arXiv: Hep-th/92110046.. In: J. Harvey, & J. Polchinski (Eds.), *Proceedings of 1992 theoretical advanced studies institute in elementary particle physics*. World Scientific.

Strominger, A., & Vafa, C. (1996). Microscopic origin of the Bekenstein–Hawking entropy. *Physics Letters B*, *379*, 99–104.

Susskind (1995). The world as a hologram. *Journal of Mathematical Physics*, *36*, 6377–6396.

Teh, N. (2013). Holography and emergence. *Studies in History and Philosophy of Modern Physics* http://dx.doi.org/10.1016/j.shpsb.2013.02.006.

t Hooft, G. (1993) Dimensional reduction in quantum gravity, arXiv: gr-qc/9310026v2.

Thouless, D., Kohmoto, M., Nightingale, M., & den Nijs, M. (1982). Quantized Hall conductance in a two-dimensional periodic potential. *Physical Review Letters*, *49*, 405–408.

Volovik, G. (2003). *The universe in a helium droplet*. Oxford University Press.

Wallace, D., & Timpson, C. (2010). Quantum mechanics on spacetime I: Spacetime state realism. *British Journal for Philosophy of Science*, *61*, 697–727.

Weinberg, S. (2009) Effective field theory, past and future, arXiv:0908.1964v3 [hep-th].

Weinberg, S. (1997). What is quantum field theory and what did we think it was?. In: T. Cao (Ed.), *Conceptual foundations of quantum field theory* (pp. 241–251). Cambridge University Press.

Weinberg, S. (1979). Ultraviolet divergences in quantum theories of gravitation. In: S. Hawking, & W. Israel (Eds.), *General relativity: An Einstein centenary survey* (pp. 790–901). Cambridge University Press.

Wen, X.-G. (2004). *Quantum field theory of many-body systems*. Oxford University Press.

Wen, X.-G. (1990). Topological orders in rigid states. *International Journal of Modern Physics B: Condensed Matter Physics, Statistical Physics, Applied Physics*, *4*, 239–271.

Wen, X.-G., & Niu, Q. (1990). Ground-state degeneracy of the fractional quantum Hall states in the presence of a random potential and on high-genus Riemann surfaces. *Physical Review B: Condensed Matter*, *41*, 9377–9396.

Witten, E. (2004). Perturbative gauge theory as a string theory in twistor space. *Communications in Mathematical Physics*, *252*, 189–258.

Zhang, S. -C. (2002). To see a world in a grain of sand. In: J. D. Barrow, P. C.W. Davies, & C. L. Harper (Eds.), *Science and ultimate reality: Quantum theory, cosmology and complexity* (pp. 667–690). Cambridge University Press.

Zhang, S. -C., & Hu, J. (2001). A four-dimensional generalization of the quantum Hall effect. *Science*, *294*, 823–828.

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