Effective Field Theories

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Abstract: An effective field theory (EFT) of a physical system is a theory of the dynamics of the system at energies small compared to a given cutoff. For some systems, low-energy states with respect to this cutoff are effectively independent of ("decoupled from") states at high energies. Hence one may study the low-energy sector of the theory without the need for a detailed description of the high-energy sector. Systems that admit EFTs appear in both relativistic quantum field theory (RQFT) and condensed matter physics. In some cases, the high-energy theory is known and the effective theory may be obtained by a process in which high-energy effects are systematically eliminated. In other cases, the high-energy theory may not be known, and the effective theory may then be obtained by imposing symmetry and "naturalness" constraints on candidate Langrangians. Many physicists currently believe that the Standard Model of particle physics is an example of such a bottom-up EFT. In both cases, the nature of the intertheoretic relation between an EFT and its (possibly hypothetical) high-energy theory is complex and arguably cannot be described in terms of standard accounts of reduction. This essay provides a review of this relation and what it suggests about the ontological status of EFTs and the extent to which the notion of emergence can be associated with them.
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1. Introduction

An effective field theory (EFT) of a physical system is a theory of the dynamics of the system at energies small compared to a given cutoff. For some systems, low-energy states with respect to this cutoff are effectively independent of ("decoupled from") states at high energies. Hence one may study the low-energy sector of the theory without the need for a detailed description of the high-energy sector. Systems that admit EFTs appear in both relativistic quantum field theories (RQFTs) and condensed matter physics. When an underlying high-energy theory is known, an effective theory may be obtained in a "top-down" approach by a process in which high-energy effects are systematically eliminated. When an underlying high-energy theory is not known, it may still be possible to obtain an EFT by a "bottom-up" approach in which symmetry and "naturalness" constraints are imposed on candidate Lagrangians. In both cases, the intertheoretic relation between the EFT and its (possibly hypothetical) high-energy theory is complicated, and, arguably, cannot be described in terms of traditional accounts of reduction. This has suggested to some authors that the EFT intertheoretic relation (and/or the phenomena associated with it) should be described in terms of a notion of emergence. Other authors have described the process of constructing an EFT as one in which idealizations are made in order to produce a computationally tractable, yet inherently approximate, theory that is empirically equivalent (to a given range of accuracy) to a typically computationally more complex, but complete, high-energy theory. One such claim is that, in the context of RQFTS, the set of possible worlds associated with an EFT are ones in which space is discrete and finite.

This essay reviews effective field theory techniques, focusing on the intertheoretic relation that links an EFT with its (possibly hypothetical) high-energy theory. The goal is to contribute to discussions on how EFTs can be interpreted, and, in particular, to investigate the extent to which a notion of emergence is viable in such interpretations. Section 2 sets the stage by reviewing the general steps in the construction of an EFT in the top-down and bottom-up approaches. Section 3 then reviews the extent to which an EFT can be said to be empirically equivalent to its high-energy theory: Typical EFTs are nonrenormalizable and thus break down at high-energies; however, this has not stopped physicists from using them to derive predictions for low-energy phenomena. Section 4 indicates the extent to which the explicit form of an EFT depends on the type of renormalization scheme one employs to handle divergent integrals that can arise when one uses the EFT to calculate the values of observable quantities. It is argued that the choice of renormalization scheme is irrelevant for calculating such values. However, to the extent that this choice determines the explicit form of the EFT, arguably, it has non-trivial consequences when it comes to the question of how the EFT can be interpreted. These consequences are investigated in Section 5. Finally, Section 6 takes up the task of assessing the extent to which the intertheoretic relation between an EFT and its high-energy theory can be described in terms of emergence.

2. The Nature of EFTs

The construction of an EFT follows one of two general procedures, top-down and bottom-up, depending on whether a high-energy theory is known. Both procedures are based on an expansion of the effective action (which formally represents the EFT) in terms of a (possibly infinite) sum of local operators, constrained by symmetry and "naturalness" considerations. They differ on how the effective action is obtained: the top-down approach obtains it by eliminating degrees of freedom from the action of the high-energy theory, whereas the bottom-up approach constructs it from scratch.

2.1. Top-Down

The top-down approach starts with a known theory and then systematically eliminates degrees of freedom associated with energies above some characteristic energy scale E_0. The practical goal is to obtain a low-energy theory that allows one to more easily calculate the values of observable quantities associated with energies below E_0 than in the original theory. Intuitively, calculations in such a low-energy "effective" theory have fewer parameters to deal with (namely, all those parameters associated with high energy degrees of freedom), and thus are simpler than calculations in the original theory. However, the construction of a low-energy effective theory that accomplishes this is not just a matter of simply ignoring the high-energy degrees of freedom, for they may be intimately tangled up with the low-energy degrees of freedom in non-trivial ways. (As we'll see below, one way to distinguish a "renormalizable" theory from a "non-renormalizable" theory is that in the former, the high-energy degrees of freedom are independent of the low-energy degrees of freedom, whereas in the latter, they are not.) One method of disentangling the high-energy and low-energy degrees of freedom was pioneered by Wilson and others in the 1970s. In the following, I will refer to it as the Wilsonian approach to EFTs. It typically involves two steps: (I) The high-energy degrees of freedom are identified and integrated out of the action. These high-energy degrees of freedom are referred to as the high momenta, or "heavy", fields. The result of this integration is an *effective* action that describes non-local interactions between the low-energy degrees of freedom (the low momenta, or "light", fields). (II) To obtain a local effective action (i.e., one that describes local interactions between low-energy degrees of freedom), the effective action from Step I is expanded in terms of local operators. The following describes these steps in slightly more technical detail.[1]

(I) Given a field theory described by an action S and possessing a characteristic energy scale E_0, suppose we are interested in the physics at a lower scale $E \ll E_0$. First choose a cutoff Λ at or slightly below E_0 and divide the fields ϕ into high and low momenta parts with respect to Λ: $\phi = \phi_H + \phi_L$, where ϕ_H have momenta $k > \Lambda$ and ϕ_L have momenta $k < \Lambda$. Now integrate out the high momenta fields. In the path integral formalism, one does the integral over the ϕ_H. Schematically

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[1] This exposition is based on Polchinski (1993), and Campbell-Smith and Mavromatos (1998). See, also Burgess (2004, 2007), Dobado, *et al.* (1997), Manohar (1997), Pich (1998), Rothsein (2004), and Schakel (2008).

$$\int \mathcal{D}\phi_L \int \mathcal{D}\phi_H \; e^{iS[\phi_H,\phi_L]} = \int \mathcal{D}\phi_L e^{iS_\Lambda[\phi_L]} \;, \tag{1}$$

where the Wilsonian effective action is given by $e^{iS_\Lambda[\phi_L]} = \int \mathcal{D}\phi_H \; e^{iS[\phi_L,\phi_H]}$. The effective Lagrangian density $\mathcal{L}_{eff}$ is thus given by $S_\Lambda[\phi_L] = \int d^Dx \, \mathcal{L}_{eff}[\phi_L]$, where D is the dimension of the spacetime.

(II) Typically, the integration over the heavy (i.e., high momenta) fields will result in a non-local effective action (*i.e.*, one in which terms occur that consist of operators and derivatives that are not all evaluated at the same spacetime point). In practice, this is addressed by expanding the effective action in a set of local operators:

$$S_\Lambda = S_0(\Lambda,\, g^*) + \textstyle\sum_i \int d^Dx \; g^i \mathcal{O}_i \;, \tag{2}$$

where the sum runs over all local operators $\mathcal{O}_i$ allowed by the symmetries of the initial theory, and the g_i are coupling constants. Assuming weak coupling, the expansion point S_0 may be taken to be the free action of the initial theory, so that $g^* = 0$.

To see how the effective action relates to the initial high-energy action, one can perform a dimensional analysis on the operators that appear in (2). This allows one to obtain information about their behavior as the cutoff Λ is lowered. This analysis involves three steps:

(i) Choose units in which the action is dimensionless ($\hbar = 1$, $c = 1$). In such units, energy has dimension $+1$ while length has dimension -1. The free action can now be used to determine units for the field operators.[2] This then determines units for the coupling constants, and subsequently, for terms in the expansion (2). For instance, if an operator $\mathcal{O}_i$ has been determined to have units E^{δ_i} (thus dimension δ_i), then its coupling constant g_i has units $E^{D-\delta_i}$, and the magnitude of the ith term is $\int d^Dx \, \mathcal{O}_i \sim E^{\delta_i - D}$.

[2] Consider, for instance, a scalar field theory with free action $S = (1/2)\int d^Dx \, (\partial_\mu\phi)^2$. This action contains D powers of the spacetime coordinate from d^Dx (with total energy units E^{-D}), and -2 powers from the two occurances of the spacetime derivative $\partial_\mu \equiv \partial/\partial x^\mu$ (with total units E^2). Thus, in order for the action to be dimensionless (with units E^0), the field ϕ must have units E^y satisfying $E^{-D}E^2E^yE^y = E^0$, and thus dimension $y = D/2 - 1$.

(ii) To make the cutoff dependence of the terms explicit, one can define dimensionless coupling constants by $\lambda_i = \Lambda^{\delta_i - D} g_i$. The order of the ith term in (2) is then

$$\lambda_i (E/\Lambda)^{\delta_i - D} \,. \tag{3}$$

(iii) The terms in the expansion (2) can now be classified into three types:

- *Irrelevant*: $\delta_i > D$. This type of term falls at low energies as $E \to 0$. Such terms are suppressed by powers of E/Λ.
- *Relevant*: $\delta_i < D$. This type of term grows at low energies as $E \to 0$.
- *Marginal*: $\delta_i = D$. This type of term is constant and equally important at low and high energies (in so far as quantum effects can modify its scaling behavior towards either relevancy or irrelevancy).

This dimensional analysis indicates that, in cases of physical relevance, there will only be a finite number of relevant and marginal terms in (2).[3] In such cases, the low-energy EFT will only depend on the underlying high-energy theory through a finite number of parameters. It is typical in the literature on EFTs to elevate these considerations to a principle. Polchinski (1993, pg. 6) articulates such a principle in the following way:

> *The low energy physics depends on the short distance theory only through the relevant and marginal couplings, and possibly through some leading irrelevant couplings if one measures small enough effects.* (Polchinski 1993, pg. 6.)

Note that arbitrarily many irrelevant terms can occur in (2), but they are suppressed at low energies by powers of E/Λ. Moreover, the cut-off can be used as a regulator for any divergences associated with these terms in calculations of the values of observable quantities. Thus, "... even though the [effective] Lagrangian may contain arbitrarily many terms, and so potentially arbitrarily many coupling constants, it is nonetheless predictive so long as its predictions are only made for low-energy processes, for which $E/\Lambda \ll 1$" (Burgess 2004, pg. 17). A little more will be said on this matter in Section 3 below.

Finally, in addition to symmetry considerations, a further constraint is typically applied to the expansion (2) of the effecctive action: One assumes that the dimensionless coefficients λ_i are of order 1. This is associated with a hypothesis of "naturalness", in so

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[3] Consider, again, scalar field theory. From footnote 2, the dimension of a scalar field ϕ is given by $D/2 - 1$; hence, in general, an operator $\mathcal{O}_i$ constructed from M ϕ's and N derivatives will have dimension $\delta_i = M(D/2 - 1) + N$. For $D \geq 3$, there are only a finite number of ways in which $\delta_i < D$ and $\delta_i = D$.

far as it rules out the presence of very large or very small numbers, relative to the cutoff, in the expansion.[4] Intuitively, a "natural" EFT should only involve quantities that are small, but not too small, relative to the cutoff. An immediate consequence of this is that mass terms, which have coefficients proportional to powers of the cutoff, cannot appear in (2). Thus naturalness is typically formulated in terms of the following condition:[5]

> *EFTs must be natural, meaning that all possible masses must be forbidden by symmetries.*

Note that this does not preclude the existence of massive objects (fields, particles, etc.) in an EFT description of low-energy phenomena. Rather, it constrains such descriptions to those in which mass terms are generated by broken high-energy symmetries. Thus, according to the Standard Model, massive vector bosons (the W and Z bosons) exist at low energies (with respect to the appropriate cutoff) due to electroweak symmetry breaking, even though gauge invariance prohibits massive terms in the electroweak action, and massive fermions exist similarly due to chiral symmetry breaking.[6]

At this point, the following qualifications should be made concerning the above Wilsonian approach to top-down EFTs:

(a) First, in Step (I), the identification of the appropriate heavy and light field variables is not always self-evident. For example, the weakly coupled EFT for quantum chromodynamics (QCD) known as chiral perturbation theory is written in terms of pion fields as opposed to quark and gluon fields; and Landau's Fermi liquid theory of conductors can be written as an EFT of weakly interacting quasiparticles, as opposed to strongly interacting electrons (Manohar 1997, pg. 321).[7]

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[4] See, e.g., Neubert (2006, pg. 155).

[5] Polchinski (1993, pg. 9). Another way to motivate this restriction is by noting that mass terms correspond to gaps in the energy spectrum in so far as such terms describe excitations with finite rest energies that cannot be made arbitrarily small. These gaps create problems when taking a smooth low-energy limit (in the sense of a smooth renormalization group evolution of parameters). Thus for Weinberg (1996, pg. 145), renormalization group theory can only be applied to EFTs that are massless or nearly massless.

[6] While these aspects of the Standard Model suggest it can be viewed as a natural EFT, other aspects famously preclude this view. In particular, terms representing massive scalar particles like the Higgs boson are not protected by any symmetry and thus should not appear in an EFT. That they do, and that the order of the Higgs term is proportional to the electroweak cutoff, generates the "hierarchy problem" for the Standard Model.

[7] Another example is Non-Relativistic QCD (NRQCD), which is an EFT of quark/gluon bound systems for which the relative velocity is small. The low-energy fields are obtained by splitting the gluon field

(b) Second, in Step (I), the path integral over the heavy fields is typically performed in practice using a saddle-point approximation (Dobado *et al.* 1997, pg. 4). This involves expanding the action of the high-energy theory about a given configuration of the heavy fields chosen to be a "saddle point" (or "stationary point"; *i.e.*, a global extremum). To second order in this expansion, the integral over the heavy fields takes the form of a Gaussian integral, which has a well-defined solution.

(c) Third, in Step (II), the dimensional method of justifying the finite dependence of an EFT on its high-energy theory is based on using the free theory to determine units, and this assumes the high-energy theory is weakly coupled. Strong interactions may have the effective of changing the scaling behavior of terms.

(d) Finally, the dimensional assignments in Step (II) work when using the EFT to make simple "tree-level" calculations of the values of observed quantities. However, for higher-order "loop" corrections to such calculations, scaling based on dimensional analysis may break down, and one may have to appeal to a particular renormalization scheme in order to justify the explicit reliance of an EFT on a finite number of parameters (see Section 4 below).[8]

2.2. Bottom-Up

The procedure outlined above for constructing an EFT requires having in one's possession the action (or Lagrangian density) of a high-energy theory. In some cases of interest, the fundamental high-energy theory is not known, but an EFT is, nonetheless, still constructible. One simply begins with the operator expansion (2) and includes all terms consistent with the naturalness constraint and with the symmetries and interactions assumed to be relevant at the given energy scale. One can then determine how these terms scale when a given cutoff is raised (as opposed to lowered). Examples of such "bottom-up" EFTs include the Fermi theory of low-energy weak interactions (as it was originally constructed); and, in the view of many physicists, the Standard Model itself (see, *e.g.*, Hartmann 2001 for discussion). Another example is effective field theoretic formulations of general relativity in which the Hilbert action is identified as the first term of the expansion (2) (Burgess 2004).

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into four modes and identify three of these as light variables. Rothsein (2003, pg. 61) describes this process of identification as an "art form" as opposed to a systematic procedure.

[8] Tree-level calculations are contributions to the perturbative expansion of a physical quantity (like a scattering cross-section) that do not involve integrating over the internal momenta of virtual processes. Loop calculations, on the other hand, involve possibly divergent integrals over internal momenta, and are typically associated with higher-order corrections to tree-level calculations. (The terminology is based on the graphical representation of perturbative calculations by Feynman diagrams.)

2.3. Example: Low-Energy Superfluid Helium-4 Film

An example of a top-down EFT constructed *via* the method of Section 2.1 is the low-energy theory of a superfluid Helium-4 (^4He) film. The effective Lagrangian density that describes this system is formally identical to the Langrangian density for (2+1)-dimensional quantum electrodynamics (QED$_3$). This is somewhat surprising, given that the underlying "high-energy" theory is non-relativistic. This example, in which a relativistic EFT might be said to *emerge* from a non-relativistic high-energy theory, will be instructive in the discussion of the concept of emergence in EFTs in Section 6 below.

At low temperatures, the liquid state of ^4He becomes a superfluid characterized by dissipationless flow and quantized vortices. The phase transition between the normal liquid and superfluid states is encoded in an order parameter that takes the form of a macroscopic wave function $\varphi_0 = (\rho_0)^{½} e^{i\theta}$ describing the coherent ground state of a Bose condensate with density ρ_0 and coherent phase θ. An appropriate Lagrangian density describes *non-relativistic* neutral bosons (*viz.*, ^4He atoms) interacting *via* a spontaneous symmetry breaking potential with coupling constant κ (Zee 2003, pp. 175, 257),

$$\mathcal{L}_{4He} = i\varphi^\dagger \partial_t \varphi - (1/2m)\partial_i \varphi^\dagger \partial_i \varphi + \mu \varphi^\dagger \varphi - \kappa(\varphi^\dagger \varphi)^2 \ , \qquad i = 1, 2, 3. \tag{4}$$

Here m is the mass of a ^4He atom, and the term involving the chemical potential μ enforces particle number conservation. This is a thoroughly non-relativistic Lagrangian density invariant under a global $U(1)$ symmetry and Galilean transformations. We now consider (4) as representing an underlying "high-energy" theory and seek to construct a low-energy EFT *via* the top-down approach outlined above.

To investigate the low-energy behavior of (4), one first needs to identify appropriate dynamical variables by means of which a distinction can be made between high- and low-energy degrees of freedom.[9] Since the ground state φ_0 is a function only of the phase, low-energy excitations take the form of phase fluctuations. This suggests rewriting the field variable φ in terms of density and phase variables $\varphi = (\rho)^{½} e^{i\theta}$, and identifying the high energy degrees of freedom with the density ρ. The next task is to integrate the density field out of (4).[10] This can be done by expanding the variables as

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[9] The following exposition draws on Wen (2004, pp. 82-83; 259-264) and Zee (2003, pp. 257-258; 314-316).

[10] Formally this involves calculating the functional integral $e^{iS_{eff}[\theta]} = \int \mathcal{D}\rho e^{iS_{4He}[\theta,\rho]}$, where $S_{eff}[\theta]$ is the effective low-energy action, and $S_{4He}[\theta, \rho] = \int d^4x\, \mathcal{L}_{4He}$ is the action of the high-energy theory. As mentioned at the end of Section 2.1, such integrals can be calculated using a saddle-point approximation, which, in this context, is equivalent to the semiclassical expansion method outlined above (Schakel 2008, pg. 75).

$\rho = \rho_0 + \delta\rho$, $\theta = \theta_0 + \delta\theta$, where $\delta\rho$, $\delta\theta$ represent small fluctuations in the density and phase about their stationary ground state values ρ_0 , θ_0. Substituting these into (4), one obtains the equivalent of (2) for the effective Lagrangian density: $\mathcal{L}_{4He} = \mathcal{L}_0[\rho_0, \theta_0] + \mathcal{L}'_{4He}[\delta\rho, \delta\theta]$, where the first term reproduces (4) without the interaction term, and the second term describes fluctuation contributions. The high energy fluctuations $\delta\rho$ can be eliminated by deriving the Euler-Lagrange equations of motion for the density variable and solving for $\delta\rho$. After substitution back into $\mathcal{L}'_{4He}$, and in two dimensions, the result is,

$$\mathcal{L}'_{4He} = (1/4\kappa)(\partial_t\theta)^2 - (\rho_0/2m)(\partial_i\theta)^2 + \ldots \ , \qquad i = 1, 2, \tag{5}$$

with $\delta\theta$ replaced by θ for the sake of notation. Ignoring the higher order terms, (5) formally describes a scalar field propagating at speed $c^2 = 2\kappa\rho_0/m$. For units in which $c = 1$, it can be re-written as

$$\mathcal{L}'_{4He} = (1/4\kappa)\eta^{\mu\nu}\partial_\mu\theta\partial_\nu\theta + \ldots \ , \qquad \mu, \nu = 0, 1, 2, \tag{6}$$

where $\eta^{\mu\nu}$ is the (2+1)-dim Minkowski metric. (6) is manifestly Lorentz invariant; in fact, it is formally identical to the Lagrangian density for a massless scalar field propagating in (2+1)-dim Minkowski spacetime. To obtain QED_3, consider the first term in (5). In so far as this represents the kinetic energy density, one can identify a superfluid velocity variable by $v_i \equiv (1/m)\partial_i\theta$. The fact that the macroscopic wave function is unique up to phase then entails that the superflow in a multiply-connected domain is quantized,

$$\oint \vec{v} \cdot d\vec{x} = (1/m)\oint \vec{\partial}\theta \cdot d\vec{x} = (1/m)2\pi q \ , \tag{7}$$

around a closed path encircling a "hole", where q is an integer. Such holes may be interpreted as *vortices* -- points where the real and imaginary parts of φ_0 vanish.[11] Then (7) entails that the superflow about a vortex is quantized. More importantly in this context, (7) suggests an analogy with Gauss's Law in which a vortex plays the role of a charge carrier and the superfluid velocity plays the role of the electric field. To further cash out this analogy, note that in two dimensions, the magnetic field is a scalar, whereas the electric field is a 2-vector. This motivates the following identifications:

$$B \equiv \partial_0\theta = -2\kappa(\rho - \rho_0) \tag{8a}$$

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[11] More precisely, vortices are soliton solutions to the equations of motion of (4) characterized by $\varphi = f(r)e^{i\theta}$, with boundary conditions $f(0) = 0$ and $f(r) \to \psi_0$, as $r \to \infty$. Intuitively, these conditions describe a localized wave with finite energy that does not dissipate over time.

$$E_i \equiv (1/m)\epsilon_{ij}\partial_j\theta = \epsilon_{ij}v_j \tag{8b}$$

in which the magnetic field is identified with the density, and the electric field with the superfluid velocity (here ϵ_{ij} is the skew volume 2-form). Substituting into (6), one obtains the Lagrangian density for sourceless QED_3

$$\mathcal{L}'_{\mathrm{QED}_3} = (1/4\kappa)\eta^{\mu\sigma}\eta^{\nu\lambda}F_{\sigma\lambda}F_{\mu\nu} + \ldots\ , \qquad \mu, \nu = 0, 1, 2, \tag{9}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, with the potential A_μ defined by $E_i = \partial_0 A_i - \partial_i A_0$, $B = \partial_1 A_2 - \partial_2 A_1$. One may further note that (7) entails that the density for "elementary" vortices ($q = \pm 1$) is given by $(1/2\pi)\vec{\partial}\times\vec{\partial}\theta$. This can be identified as the 0th component of a vortex current $j_v{}^\mu = (1/2\pi)\epsilon^{\mu\nu\lambda}\partial_\nu\partial_\lambda\theta$, where $\epsilon^{\mu\nu\lambda}$ is the skew volume 3-form.[12] This vortex current is the dual of the electromagnetic current, in so far as adding a source term $A_\mu j_v{}^\mu$ to (9) and extremizing with respect to A_μ produces the Maxwell equations with a source.

In summary, we started with the non-relativistic Lagrangian density (4) for a superfluid ^4He film and found that, to lowest order, its EFT takes the form of the relativistic Lagrangian density for (2+1)-dim quantum electrodynamics. This was motivated by the formal similarity between vortex quantization (7) and Gauss' Law. This similarity was exploited in terms of a *duality* transformation under which vortices become the sources of a gauge field formally identical to the Maxwell field. Under a literal interpretation of this dual representation (8), low energy excitations of a superfluid ^4He film take the form of electric and magnetic fields, the former being given by the superfluid velocity, and the latter being given by the superfluid density. Moreover, topological defects (*i.e.*, elementary vortices) take the form of charge-carrying electrons.

3. Renormalizability and Predictability

Historically the Wilsonian approach to EFTs outlined in Section 2.1 had its origin in the development of renormalization group (RG) techniques by Wilson and others in the 1970s (see, *e.g.*, Huggett and Weingard 1995; Cao and Schweber 1993). These techniques were originally developed to study the low-energy behavior of condensed matter systems, and were subsequently applied to the problem of renormalization in relativistic quantum field theories; *i.e.*, the appearance of integrals that diverge at high energies when one uses a quantum field theory to calculate the values of observable quantities. This is related to the issue of predictability, in so far as a theory that "blows up" at high energies cannot be used to make high energy predictions. This section

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[12] Note that the form of $j_v{}^\mu$ contracts over skew and symmetric indices; however it is not identically zero, since for vortices, θ is not a globally defined function.

considers the issues of renormalizability and predictability in the context of EFTs. In particular, given that typical EFTs are not renormalizable, how does this affect their ability to make predictions?

In the RG approach to renormalization, the intent is to analyze the behavior of a theory at different energy scales s. One thus uses a scale-dependent momentum cutoff $\Lambda(s)$ as the basis for an initial distinction between high and low energy modes, and the heavy modes with respect to an initial energy Λ are then integrated out of the theory. The cutoff is now lowered to $\Lambda(s) = s\Lambda$ and the parameters of the theory are then rescaled to formally restore the cutoff back to Λ. Successive iterations of this procedure generate a flow in the space of parameters of the theory. Scale-dependent parameters can then be classified as relevant (shrinking in the high energy limit as $s \to \infty$), irrelevant (growing as $s \to \infty$), or marginal (constant under scale transformation).[13] A theory is now said to be renormalizable if it contains no irrelevant parameters. Intuitively, such a theory is cutoff independent, in so far as its parameters become independent of $\Lambda(s)$ in the high-energy limit $s \to \infty$. A non-renormalizable theory, on the other hand, is one in which there are (scale-dependent) irrelevant parameters. Such parameters cannot be ignored at high energies and thus contribute to ultraviolet divergent integrals.

EFTs can be either renormalizable or non-renormalizable in the above sense, depending on whether they contain irrelevant terms, although typically the construction outlined in Section 2.1 above produces an infinite number of the latter. However, as eluded to in Section 2.1, the appearance of an infinite number of irrelevant terms in an EFT need not signal a break-down in predictability. After Manohar (1997, pg. 322), the effective Lagrangian density associated with (2) can be represented schematically by the sum:

$$\mathcal{L}_{eff} = \mathcal{L}_{\leq D} + \mathcal{L}_{D+1} + \mathcal{L}_{D+2} + \ldots, \tag{10}$$

where $\mathcal{L}_{\leq D}$ contains terms with dimension $\leq D$, $\mathcal{L}_{D+1}$ contains terms with dimension $D+1$, and so on, and, as in Section 2.1 above, D is the dimension of spacetime (recall that an operator with dimension δ is deemed irrelevant, relevant, or marginal, depending on whether δ is greater than, less than, or equal to D, respectively). In this sum, each summand contains a *finite* number of terms with coefficients that are powers of the ratio (s/Λ). The first summand consists of a finite number of relevant and/or marginal terms to order zero in (s/Λ) (thus such terms are scale-independent). Each summand thereafter contains a finite number of irrelevant terms to a higher order in (s/Λ) (thus such terms are scale-dependent). A renormalizable Lagrangian density consists of only the first summand, thus when it is used to derive predictions, they will be scale-independent. Non-renormalizable Lagrangians include irrelevant terms, and

[13] In this context, relevant terms are called "super renormalizable", irrelevant terms are called "non-renormalizable", and marginal terms are called "renormalizable".

predictions derived from them will be scale-dependent. In general, to compute the value of an observable quantity to a given order r in (s/Λ), one should retain terms up to $\mathcal{L}_{D+r}$.

To consider how this analysis of renormalizability relates to predictability, note first that renormalizability, as defined above, is predicated on the property of being scale-independent. A renormalizable theory is independent of energy scale, whereas a non-renormalizable theory is not. So in order to articulate the relation between renormalizability and predictability, one needs to articulate the relation between scale-independence and predictability. An extreme view might require scale-independence (and hence renormalizability) to be a necessary condition for predictability. The argument might run something like this: If a theory is scale-dependent, then using a cutoff to regulate divergent integrals will be of no help, in so far as (a) the cutoff must be taken to infinity at the end of the day; and (b) doing so will cause scale-dependent terms (which are well-behaved at low-energies with respect to the cutoff) to blow up. One intuition underlying this argument is that the cutoff must, in fact, be taken to infinity at the end of the day; otherwise, we would not end up with the continuum theory we began with. This appears to be the argument underlying Huggett and Weingard's response to their "Problem Number two of understanding renormalization", namely, "...why do actual physical theories depend on only a finite number of parameters? A slogan: why is the world renormalisable?" (Huggett and Weingard 1995, pg. 179). Their answer to this problem is the following:

> ... purely relevant trajectories terminate in the continuum limit -- call this 'asymptotic safety'. Any irrelevant dependent theories... do not terminate in this way and are not asymptotically safe. They generate indefinitely long trajectories with ever varying [parameters], either periodically or ever growing. Either way, unphysical singularities are likely. Thus, while asymptotically safe relevant theories are potentially physical, irrelevant theories are not -- just the result we hoped for to answer the second question. (Huggett and Weingard 1995, pg. 183.)

If one takes "potentially physical" to mean "scale-independent", then Huggett and Weingard's claim that irrelevant (*i.e.*, non-renormalizable) theories are not potentially physical is correct. However, if one takes "potentially physical" to mean "capable of producing finite predictions", then Huggett and Weingard's claim does not go through: non-renormalizable theories are capable of producing finite predictions, with the qualification that such predictions are scale-dependent.

Manohar suggests there is nothing wrong with such a notion of predictability, in so far as there is no reason to expect potentially physical theories to be scale-independent:

> A non-renormalizable theory is just as good as a renormalizable theory for computations, provided one is satisfied with a finite accuracy... While exact computations are nice, they are irrelevant. Nobody knows the exact theory up to infinitely high energies. Thus any realistic calculation is done using an effective field theory. (Manohar 1997, pg. 322.)

Burgess likewise suggests that the distinction between a renormalizable (*viz.*, scale-independent) theory and a non-renormalizable (*viz.*, scale-dependent) theory is a matter of degree rather than kind:

> Because... only a finite number of terms in $\mathcal{L}_{eff}$ contributes to any fixed order in $[s/\Lambda]$, and these terms need appear in only a finite number of loops, it follows that only a finite amount of labor is required to obtain a fixed accuracy in observables. Renormalizable theories represent the special case for which it suffices to work only to zeroth order in the ratio $[s/\Lambda]$. This can be thought of as the reason why renormalizable theories play such an important role throughout physics... Thus, although an effective Lagrangian is not renormalizable in the traditional sense, it nevertheless is predictive in the same way a renormalizable theory is. (Burgess 2007, pg. 349.)

The suggestion here is that, to the extent that scale-dependent predictions and scale-independent predictions are both calculated in the same manner (*i.e.*, by applying an appropriate renormalization scheme to divergent integrals), they are of the same kind. Thus,

> ... nonrenormalizable theories are not fundamentally different from renormalizable ones. They simply differ in their sensitivity to more microscopic scales which have been integrated out. (Burgess 1998, pg. 13.)

This tolerant view of non-renormalizable theories, and in particular, the predictability of EFTs, has arguably become the norm among physicists. What it implies about the ontology of EFTs, and in particular, the nature of the Wilsonian cutoff Λ, will have to wait until Section 5. Section 4 provides a brief review of two explicit ways of deriving predictions using EFTs. My ultimate claim in Section 5 will be that the method one chooses to derive predictions from an EFT (*i.e.*, the renormalization scheme one adopts), will influence the possible ways of interpreting it.

4. On Renormalization Schemes and Types of EFTs

As Manohar (1997, pg. 326) observes, knowing the Lagrangian density of a quantum field theory is not enough to calculate the values of observable quantities. To accomplish the latter (using perturbative techniques) requires expanding the Green's function that represents a particular observable quantity in an infinite series in which, typically, divergent integrals appear.[14] This is the problem of renormalization in quantum field theory. Thus in addition to knowing the Lagrangian density of a quantum field theory, one needs to specify a renormalization scheme. This is a method that specifies (i) a means of regulating divergent integrals, and (ii) a means of subtracting the associated infinities in a systematic way. There are a number of different methods that accomplish this, two of which are important in the context of interpreting EFTs. The first adopts momentum cutoff regularization and a mass-dependent method of subtraction, and is used (at least implicitly) in the Wilsonian approach to constructing EFTs (outlined above in Section 2). The second adopts dimensional regularization and a mass-dependent method of subtraction, and is associated with what Georgi (1992, pg. 1; 1993, pg. 215) has called "continuum EFTs".

4.1. Mass-Dependent Schemes and Wilsonian EFTs

In a Wilsonian EFT, the explicit appearance of the cutoff Λ that defines the border between the low-energy physics and the high-energy physics suggests employing it as a means to regulate the particular type of divergent integrals that appear in calculations of the values of observable quantities. Given such a divergent integral of the schematic form $\int_0^\infty d^D p \kappa(p)$, where D is the dimension of spacetime and $\kappa(p)$ is a particular function of momentum p, one can insert the cutoff Λ and rewrite the integral as the sum of a finite piece and an infinite piece:

$$\int_0^\Lambda d^D p \kappa(p) + \int_\Lambda^\infty d^D p \kappa(p). \tag{11}$$

For the types of divergent integrals under consideration, the infinite piece can be absorbed into a redefinition of the parameters of the theory through the introduction of renormalization constants. It turns out that, in this method of regularization, these constants are dependent on the heavy masses that appear in the high-energy theory; hence, the manner in which they are defined is referred to as a *mass-dependent* subtraction scheme.[15]

There are two main advantages of employing this type of renormalization scheme in the context of EFTs. First, it is conceptually consistent with the image of an EFT as a

[14] A Green's function is a vacuum expectation value of field operators.

[15] More precisely, a mass-dependent subtraction scheme is one in which anomalous dimensions and renormalization group β functions explicitly depend on μ/M, where μ is the renormalization scale and M is the heavy mass (Georgi 1993, pg. 221).

low-energy approximation to a high-energy theory based on a restriction of the latter to a particular energy scale. This scale is explicitly represented by the cutoff Λ which thus plays a *double role* in designating the appropriate energy scale, and in cutting-off divergent integrals. The second advantage of using this renormalization scheme is that it guarantees that the Decoupling Theorem holds, given a few other assumptions.

The Decoupling Theorem is due to Appelquist and Carazzone (1975). Hartmann (2001) describes it thusly:

> For two coupled systems with different energy scales m_1 and m_2 ($m_2 > m_1$) and described by a renormalizable theory, there is always a renormalization condition according to which the effects of the physics at scale m_2 can be effectively included in the theory with the smaller scale m_1 by changing the parameters of the corresponding theory. (Hartman 2001, pg. 283.)

This theorem is a formal guarantee of the informal EFT "ideology" of Polchinski (1993, pg. 6), stated above in Section 2.1. Hartmann (2001, pg. 284) is careful to note that it requires that there is an underlying high-energy theory that is renormalizable, and that different mass scales exist in this theory. Moreover, as Georgi (1992, pg. 3) indicates, the renormalization condition that the theorem refers to is, in fact, a mass-dependent subtraction scheme.

The above advantages of cutoff regulated, mass-dependent renormalization schemes are balanced by the following disadvantages:

(1) A momentum cutoff regularization method violates Poincaré invariance of the underlying high-energy theory, as well as any gauge invariance it may possess.

(2) Mass-dependent subtraction schemes typically prevent the justification, based on dimensional analysis, that allows one to ignore the potentially infinite number of irrelevant terms in the effective action (2) from being extended from tree-level calculations to higher-order loop corrections. The reason is that in mass-dependent schemes, the simple tree-level dependence of irrelevant terms on orders of $1/\Lambda$ can break down when doing higher-order loop corrections. In particular, in these higher-order corrections, the dependence of irrelevant terms on the cutoff may be of order 1 (in general, such terms have a power law dependence on the cutoff), and thus such terms cannot be ignored (Manohar 1997, pp. 327-228; Pich 1998, pg. 14). Note that this does not prevent loop calculations from proceeding in mass-dependent schemes; rather it makes them more difficult (Manohar 1997, pg. 329).

4.2. Mass-independent Schemes and Continuum EFTs

To address Problem (2), many authors suggest adopting a mass-independent renormalization scheme. In this type of scheme, the dimensional parameter μ (analogous to the momentum cutoff Λ in the cutoff approach) only appears in loop corrections in logarithms, and not powers, thus the relevant integrals are small at scales much smaller than the heavy fields (Manohar 1997, pg. 238; Pich 1998, pg. 15). This allows one to effectively ignore the contributions of irrelevant terms, not only at tree-level as naive dimensional analysis allows, but also for higher-order loop corrections as well.

Mass-independent renormalization schemes are typically associated with the method of regulating divergent integrals known as dimensional regularization. This method takes advantage of the mathematical fact that the particular types of divergent integrals that arise in quantum field theoretic calculations, again represented schematically by $\int_0^\infty d^D p\kappa(p)$, will converge for sufficiently small values of D. Formally, one lets $D = 4 - \epsilon$ in the integral (where ϵ is a very small constant), and then analytically continues D to 4. This process picks up poles in D-dimensional momentum space, and these can be absorbed into a redefinition of the parameters of the theory. In this case, such redefinitions are independent of the masses, hence the term *mass-independent subtraction scheme.*

In the context of EFTs, there are two main advantages of employing a mass-independent renormalization scheme based on dimensional regularization. First, dimensional regularization respects Poincaré and gauge invariance. Second, as indicated above, mass-independent renormalization schemes allow one to truncate the effective action (2) to a finite list of terms, not only for tree-level calculations, but also for higher-order loop calculations. However, it turns out that this simplification comes at the cost of having terms that explicitly include the heavy fields appear in this finite list (see, *e.g.*, Burgess 2004, pg. 19). This has the following consequences:

(1') Many authors consider mass-independent schemes to violate the "spirit" of an EFT, to the extent that the latter is based on the notion of a cutoff, below which the physics is explicitly described by only the light fields (Burgess 2004, pg. 19; Burgess 2007, pg. 343; Polchinski 1993, pg. 5).

(2') Perhaps more importantly, the presence of heavy field terms in an effective action employing a mass-independent renormalization scheme prevents the application of the Decoupling Theorem (Georgi 1993, pg. 225; Manohar 1997, pg. 329). As mentioned above in Section 3.1, the latter holds only for mass-dependent schemes.

It turns out that Problem (2') can be addressed by inserting decoupling by hand into an EFT that employs a mass-independent scheme, but this requires a slight reconceptualization of the nature of an EFT. This results in what Georgi (1992, pg. 1; 1993, pg. 215) refers to as a "continuum EFT".

How can an EFT be constructed without initial appeal to a cutoff? Briefly, for top-down constructions, the initial momentum splitting of the fields in a Wilsonian EFT and the integration over the heavy modes, is replaced in a continuum EFT by the following steps (after Georgi 1993, pg. 228; see, also, Burgess 2004, pp. 19-20; Burgess 2007, pg. 344):

(I') Start with a dimensionally-regularized theory with Lagrangian density $\mathcal{L}_H(\chi,\phi) + \mathcal{L}(\phi)$ at a large scale s, where $\mathcal{L}(\phi)$ describes the light fields and $\mathcal{L}_H(\chi,\phi)$ describes everything else (where χ are the heavy fields of mass M). Now evolve the theory to lower scales using the renormalization group: This allows you to go from scale s to scale $s - ds$ without changing the content of the theory.

(II') When s gets below M, the effective theory is changed to a new one without the heavy fields: $\mathcal{L}(\phi) + \delta\mathcal{L}(\phi)$, where $\delta\mathcal{L}(\phi)$ encodes a "matching correction" that includes any new nonrenormalizable interactions that may be required. The matching correction is made so that the physics of the light fields is the same in the two theories at the boundary $s = M$. To explicitly calculate $\delta\mathcal{L}(\phi)$, one expands it in a complete set of local operators in the same manner that the expansion (2) for Wilsonian EFTs is performed:

$$\mathcal{L}_{eff} = \mathcal{L}(\phi) + \sum_n \delta\mathcal{L}^n(\phi) \ . \tag{12}$$

Dimensional analysis can now be applied to determine the scaling behavior of the terms in (12), in the same way it is applied in Wilsonian EFTs. Again, in the case of a continuum EFT, this analysis is valid not just for tree-level calculations, but also for higher-order loop calculations as well.

To summarize and compare, in the construction of a Wilsonian EFT, the heavy fields are first integrated out of the underlying high-energy theory and the resulting Wilsonian effective action is then expanded in a series of local operator terms. The cutoff Λ in a Wilsonian EFT plays a double role: first, through the definition of the heavy and light fields, in explicitly demarcating the low-energy physics from the high-energy physics; and second, in regulating divergent integrals in the calculation of observable quantities. In the construction of a continuum EFT, the heavy fields are initially left alone in the underlying high-energy theory, which is first evolved down to the appropriate energy scale. The continuum EFT is then constructed by completely removing the heavy fields from the high-energy theory, as opposed to integrating them out; and this removal is compensated for by an appropriate matching calculation (this latter is ultimately responsible for the appearance of heavy modes into the operator expansion (12)). In a continuum EFT, the first role of the Wilsonian cutoff Λ is played by the renormalization scale s that demarcates the low-energy physics from the high-energy physics. The second role of the Wilsonian cutoff is dropped, the procedure of dimensional regularization taking its place.

This last observation suggests the motivation for Georgi's phrase "continuum EFT". In a Wilsonian EFT, the regularization of divergent integrals is performed by restricting the range of momentum variables in integrals over momentum space. The Fourier transform equivalent of this procedure is a restriction of the range of coordinate variables in integrals over coordinate space. Hence regularization in a Wilsonian EFT is analogous to placing a high-energy continuum theory on a discrete lattice, and this is the reason why momentum cutoff regularization violates Poincaré invariance. In contrast, in a "continuum EFT", the regularization of divergent integrals is performed by calculating them in a continuous spacetime of dimension D (the Fourier transform equivalent of D-dimensional momentum space). This is the reason why dimensional regularization does not violate Poincaré invariance.

5. Ontological Implications

Different renormalization schemes ultimately all agree on the values of physical quantities. In particular, both mass-dependent and mass-independent schemes will, at the end of the day, agree on all empirically measured quantities.[16] Thus a Wilsonian EFT for a given physical system is empirically equivalent to that system's continuum EFT. On the other hand, the fact that these types of EFT place different emphasis on the nature of the cutoff suggests they can be interpreted as telling us different things about the world. I now consider some of the implications this has for debates over the ontological status of EFTs.

5.1. Decoupling and Quasi-Autonomous Domains

Cao and Schweber emphasize the role of the Decoupling Theorem in understanding the ontological significance of EFTs:

> Thus, with the decoupling theorem and the concept of EFT emerges a hierarchical picture of nature offered by QFT [quantum field theory], one that explains why the

[16] Burgess (2004, pg. 20) explains this in the following way: "... the difference between the cutoff- and dimensionally regularized low-energy theory can itself be parameterized by appropriate local effective couplings within the low-energy theory. Consequently, any regularization-dependent properties will necessarily drop out of final physical results, once the (renormalized) effective couplings are traded for physical observables."

> description at any one level is so stable and is not disturbed by whatever happens at higher energies, and thus justifies the use of such descriptions. (Cao and Schweber 1993, pg. 64.)

> In this picture, the [physical world] can be considered as layered into quasi-autonomous domains, each layer having its own ontology and associated 'fundamental law'. (Cao and Schweber 1993, pg. 72.)

They further suggest that EFTs entail "... an antifoundationalism in epistemology and an antireductionism in methodology" (Cao and Schweber 1993, pg. 69). Huggett and Weingard (1995, pg. 187) interpret this as a view that "... holds that nature is described by a genuinely never-ending tower of theories, and that the competing possibilities of unification and new physics should be abandoned".

Hartmann (2001, pg. 298) claims that "Cao and Schweber's talk of quasi-autonomous domains rests on the validity of the decoupling theorem", and then rightly points out that the latter is not necessarily valid in all cases in which EFTs exist. In particular, it requires the existence of an underlying high-energy renormalizable theory with different mass scales. This vitiates Cao and Schweber's antifoundationalism and/or antireductionism, if they are taken to entail that there is no underlying theory.

On the other hand, Castellani (2002) appears to endorse an aspect of the "quasi-autonomous domain" interpretation:

> The EFT approach in its extreme version provides a level structure ("tower") of EFTs, each theory connected with the preceding one (going "up" in the tower) by means of the [renormalization group] equations and the matching conditions at the boundary... (Castellani 2002, pg. 263.)

It appears that the participants in this debate are talking past each other, having implicitly adopted different notions of an EFT. Cao and Schweber, in particular, are sometimes ambiguous on what concept of EFT they are employing. On the one hand, in their emphasis on the Decoupling Theorem and momentum cutoff regularization, they appear to adopt a Wilsonian notion of an EFT. For instance, in the context of discussing the ontological significance of momentum cutoff regularization, they observe that

> The following can be stated for other regularization schemes... but not for dimensional regularization, which is more formalistic and irrelevant to the point discussed here. (Cao and Schweber 1993, pg. 92, Footnote 17.)

On the other hand, immediately before introducing the EFT-inspired hierarchical picture of nature, they describe an EFT in the following terms:

> The EFT can be obtained by deleting all heavy fields from the complete renormalizable theory and suitably redefining the coupling constants, masses, and the scale of the Green's functions, using the renormalization group equations. (Cao and Schweber 1993, pg. 64.)

This appears to be a description of the construction of a continuum EFT as outlined in Section 3.2. Hartmann (2001) makes it evident in his critique of Cao and Schweber that he implicitly has adopted the Wilsonian notion of an EFT. Finally, Castellani implicitly adopts a continuum notion of an EFT, describing its construction as one based on matching conditions (see, *e.g.*, Castellani 2002, pg. 262).

Note that the notion of EFT one adopts should be important in this debate. The Decoupling Theorem was proven in the context of Wilsonian EFTs of a particular kind; namely, those for which there exists an underlying renormalizable high-energy theory with different mass scales. Thus, if by "EFT" Cao and Schweber mean "Wilsonian EFT", then Hartmann's critique goes through: Wilsonian EFTs do not, by themselves, support an ontology of quasi-autonomous domains. On the other hand, the Decoupling Theorem fails for continuum EFTs, but, arguably this does not prevent them from supporting a well-defined notion of quasi-autonomous domains. This is because in continuum EFTs, decoupling is inserted "by hand" in the form of matching calculations. Thus, if by "EFT" Cao and Schweber mean "continuum EFT", then, arguably, Hartmann's critique does not go through: continuum EFTs are, by themselves, capable of supporting an ontology of quasi-autonomous domains. Hence, provided by "EFT", Castellani means "continuum EFT", her endorsement of such an ontology is justified.[17]

5.2. Realistic Interpretations of the Cutoff

In typical expositions of EFTs, emphasis is placed on a realistic interpretation of the cutoff. Many authors claim that such a realistic interpretation is what separates the contemporary concept of an EFT from older views of renormalization (Hartmann 2001, pg. 282; Castellani 2002, pg. 261; Grinbaum 2008, pg. 37). Under these older views, a cutoff, when it occurred in accounts of QFTs, only played a role as a regulator of integrals, and was taken to infinity at the end of the day (if this resulted in a finite theory, then the theory was deemed to be renormalizable). The contemporary concept

[17] Grinbaum (2008, pg. 40), notably, makes a distinction between the "strict" form of decoupling associated with the Appelquist-Carazzone theorem, and a "milder" empirical decoupling thesis, which evidently is to be associated with the matching calculations involved in the construction of continuum EFTs.

of an EFT, so the story goes, is based on viewing the cutoff realistically in a different role; namely, as a means of demarcating low-energy physics from high-energy physics. By now it should be obvious from the discussion in Section 4 that this standard account is only half the story; it, perhaps unfairly, privileges Wilsonian EFTs and the double role the cutoff plays in their construction, over continuum EFTs.

This is not to say that a realistic interpretation of the cutoff cannot be made in the context of continuum EFTs. Recall that in continuum EFTs, the role that the Wilsonian cutoff Λ plays in demarcating low-energy physics from high-energy physics is played by a scaling variable s (*i.e.*, the renormalization scale that appears in the renormalization group equations). Certainly this scaling variable can be realistically interpreted, perhaps as a basis for an ontology of quasi-autonomous domains; and it might even be referred to as a cutoff, in this context. The important point is that, in a continuum EFT, this scaling variable does not play the second role that the Wilsonian cutoff Λ plays; namely, as a regulator of divergent integrals.

Thus to realistically interpret the cutoff in an EFT could mean one of two things:

(a) The Wilsonian regulator Λ should be realistically interpreted.

(b) The constant that demarcates low-energy physics from high-energy physics (given by Λ in Wilsonian EFTs and by a particular value of s in continuum EFTs) should be realistically interpreted.

As the discussion at the end of Section 4.2 suggests, adopting (a) might motivate an ontology in which space is discrete: Momentum cutoff regularization is analogous to placing a continuum theory on a discrete lattice. But such an ontology is *not* forced upon us by a realistic interpretation of the cutoff. Simply put, (b) does not entail (a). One can adopt a realistic interpretation of the cutoff in a continuum EFT and, at the same time, an ontology in which spacetime is continuous.

This conclusion affects part of a recent debate over interpretations of quantum field theory (QFT). Wallace (2006) adopts an interpretation of QFT in which a cutoff is inserted and realistically interpreted and justifies it by appealing to features of EFTs (see, *e.g.*, pg. 43). Fraser (2009) criticizes this "cutoff variant" of QFT in the following way:

> If the cutoffs are taken seriously, then they must be interpreted realistically; that is, space is really discrete and of finite extent according to the cutoff variant of QFT. (Fraser 2009, pg. 552.)

Fraser suggests this makes cutoff QFT unsatisfactory: If the cutoff is not taken seriously, then cutoff QFT reduces to "infinitely renormalized QFT", which is the standard textbook account in which cutoffs, when they appear, are taken to infinity in renormalization schemes. An appeal to Haag's theorem then indicates this textbook account is inconsistent. On the other hand, if the cutoff is taken seriously, then cutoff QFTs avoid Haag's theorem (which, among other things, requires Poincaré invariance); but they entail space is discrete and finite, and "... nobody defends the position that QFT provides evidence that space is discrete and the universe is finite" (Fraser 2009, pg. 552).[18]

Once again, being clear on the type of EFT one adopts on which to base the cutoff variant of QFT will make a difference in this debate. If by "cutoff QFT" one means "Wilsonian EFT", then Fraser's critique, arguably, goes through. However, if by "cutoff QFT" one means "continuum EFT", then the above argument does not go through: continuum EFTs support an ontology in which spacetime is continuous.[19] Thus, provided one can demonstrate that continuum EFTs avoid Haag's theorem, a cutoff version of QFT based on continuum EFTs is a viable alternative to the "formal variant" (*i.e.*, axiomatic QFT) that Fraser (2009, pg. 538) advocates.

5.3. EFTs and Approximation

Finally, two further examples of how the distinction between types of EFTs is important for issues of interpretation involve the notions of idealization and approximation. The fact that Wilsonian EFTs support an ontology in which space is discrete suggests to Fraser (2009, pp. 564) that the cutoff variant of QFT is an indispensable idealization: "[It] ...is an idealization in the sense that the possible worlds in which QFT is true are presumed to be worlds in which space is continuous and infinite", and "[t]his idealization is indispensible insofar as it is not possible to remove the cutoffs entirely..." (since this would turn the cutoff variant into the infinitely renormalized variant). But, again, continuum EFTs do not idealize space as discrete, hence a version of cutoff QFT based on continuum EFTs is not an idealization of this type.

Relatedly, Castellani (2002, pp. 260, 263) suggests that EFTs are "intrinsically approximate and context-dependent". An EFT, under this view, is an approximation of an underlying high-energy theory and is valid only within a specified energy range. Now, arguably, Cao and Schweber's ontology of quasi-autonomous domains is intended

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[18] See, also, Huggett and Weingard (1995, pg. 178) for similar intuitions.

[19] This suggests that the categories of empirically equivalent variants of QFT that Fraser (2009, pg. 538) identifies should be expanded. Her "cutoff QFT" category might be split into "cutoff regularized QFT" and "dimensionally regularized QFT".

in part to address claims of this type. Under Cao and Schweber's view, an EFT describes a quasi-autonomous domain by means of a complete description of phenomena within a given energy range, independent for the most part of descriptions at higher or lower energies (Cao and Schweber 1993, pg. 64, are careful to explain how high-energy effects do make themselves present in relevant and marginal terms of an effective Lagrangian density). Thus, the discussion in Section 5.1 entails that EFTs need not be interpreted as intrinsically approximate, provided one adopts continuum EFTs as the object of one's interpretation.

6. EFTs and Emergence

In the example in Section 2.3 above, the EFT of a superfluid ^4He film took the form (to lowest order) of quantum electrodynamics in (2+1) dimensions. The duality transformations (8) suggested that, at low energies, the density of a superfluid ^4He film behaves like a magnetic field, its velocity behaves like an electric field, and vortices behave like charge carrying electrons. This example of a relativistic EFT of a condensed matter system, and others like it, have suggested to some physicists that novel phenomena (fields, particles, symmetries, spacetime, *etc.*) emerge in the low-energy limit of these systems.[20] On the other hand, in the physics literature, references to emergence are typically not associated with EFTs of relativistic QFTs.[21] In the philosophy of physics literature, the converse is true: Philosophers of physics have considered notions of emergence related to EFTs of relativistic QFTs, but have paid little attention to emergence in EFTs of condensed matter systems.[22] In this section I take it as a given that the formal nature of an EFT is identical in both contexts, and consider the extent to which notions of emergence are applicable, regardless of context.

Consider, first, the view from philosophy of physics: Cao and Schweber, for instance, associate the antireductionism of their quasi-autonomous domains interpretation of EFTs with a notion of emergence:

> ...taking the decoupling theorem and EFT seriously would entail considering the reductionist...program an illusion, and would lead to its rejection and to a point of

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[20] For instance, Zhang (2004, pg. 669) reviews "examples of emergence in condensed matter systems" that take the form of relativistic EFTs, including the QED_3 case. These and other examples in the condensed matter literature are discussed in Bain (2008).

[21] See Castellani (2002) for a review of the 1960s-70s debates between solid-state physicists and particle physicists over the concepts and status of reduction and emergence.

[22] Both philosophers and physicists have considered notions of emergence in condensed matter systems exhibiting spontaneously broken symmetries. But, as Section 6.1 below suggests, this context is distinct from the context in which emergence might be associated with EFTs.

> view that accepts emergence, hence to a pluralist view of possible theoretical ontologies. (Cao and Schweber 1993, pg. 71.)

Likewise, Castellani (2002, pg. 263) suggests that the EFT approach "... provides a level structure of theories where the way in which a theory emerges from another...is in principle describable by using RG [Renormalization Group] methods and matching conditions at the boundary". On the other hand, Castellani argues that the EFT approach does *not* imply antireductionism, in so far as antireductionism is to be associated with the denial of some type of intertheoretic relation:

> The EFT schema, by allowing definite connections between theory levels, actually provides an argument against the basic antireductionist claim... (Castellani 2003, pg. 265.)

Note that while Cao and Schweber take emergence to be descriptive of ontologies (properties, objects, *etc.*), Castellani suggests emergence be viewed as a relation between theories. In both cases, however, the emphasis is on autonomy. Cao and Schweber's emergent ontologies are restricted to quasi-autonomous domains, each described by a distinct EFT. Castellani's emergent theories stand in "definite connections" with each other, but assumedly not so definite as to warrant the label of reduction.

This section will only be concerned with the extent to which the intertheoretic relation between a top-down EFT and its high-energy theory supports a notion of emergence; thus the approach taken will be to view emergence as a relation between theories. Batterman (2002, pg. 115) has suggested that, under a received view in the philosophical literature, such a relation holding between an emergent theory T' and an underlying theory T can mean any or all of the following:

(a) The phenomena of T' cannot be reduced to T.

(b) The phenomena of T' cannot be predicted by T.

(c) The phenomena of T' are causally independent of those of T.

(d) The phenomena of T' cannot be explained by T.

The initial task of this section will be to consider the extent to which the intertheoretic relation between an EFT and its high-energy theory supports these notions of autonomy.

6.1. The EFT Intertheoretic Relation

Sections 2.1 and 4.2 above described the general steps in the construction of a Wilsonian and a continuum EFT, respectively. Although differing in their details, these steps have the following general form: (I) One first identifies and then systematically eliminates high-energy degrees of freedom; and then (II) one expands the resulting effective Lagrangian density (or effective action) in terms of local operators. The intertheoretic relation defined by this procedure has one very important characteristic in the context of a discussion of notions of emergence; namely, its *relata* are *distinct* theories.

To see this, consider the following consequences of Steps (I) and (II):

(1) First, the low-energy degrees of freedom of the EFT are typically formally distinct from the high-energy degrees of freedom. This suggests they admit distinct ontological interpretations (recall, for instance, some of the examples from Section 2: pions versus quarks, quasiparticles versus electrons, and electric and magnetic fields versus ^4He atoms).

(2) Second, the EFT Lagrangian density typically is formally distinct from the high-energy Lagrangian density.

Manohar makes (2) clear in the context of the Fermi EFT of the weak force:

> It is important to keep in mind that the effective theory is a different theory from the full theory. The full theory of the weak interactions is a renormalizable field theory. The effective field theory is a non-renormalizable field theory, and has a different divergence structure from the full theory. (Manohar 1997, pg. 327.)

As another example of (2), consider the EFT of a superfluid ^4He film described in Section 2.3. Above a critical temperature, the system consists of a non-relativistic normal liquid. As the temperature is lowered below the critical value, a phase transition occurs, accompanied by a spontaneously broken symmetry, and the system enters the superfluid phase. If the temperature is lowered further, it's constituents can be described in terms of a relativistic EFT. Importantly, both the normal liquid and the superfluid, as well as the phase transition and the spontaneously broken symmetry, are all encoded in a single Lagrangian density (4). All of these states and processes can thus be said to be described by a *single* theory. On the other hand, the low-energy relativistic system is encoded in the effective Lagrangian density (9), which is sufficiently formally distinct from (4) to warrant viewing it as a different theory (see Figure 1, after Bain 2008, pg. 313).

$$\underbrace{\begin{array}{c}\text{non-relativistic}\\ \text{liquid Helium}\end{array} \xrightarrow[\textit{(phase transition)}]{\textit{lower temp}} \begin{array}{c}\text{non-relativistic}\\ \text{superfluid Helium}\end{array}}_{\textit{initial Lagrangian}} \xrightarrow[\textit{(low energy limit)}]{\textit{lower temp}} \underbrace{\begin{array}{c}\text{relativistic}\\ \text{system}\end{array}}_{\textit{effective Lagrangian}}$$

Figure 1. The relation between the initial Lagrangian and the effective Lagrangian for superfluid Helium.

Note that the claim that an EFT and its high-energy theory are distinct theories is not intended to be based simply on the fact that there is a formal distinction between their respective Lagrangian densities. It is not the case that, in general, there is a 1-1 correspondence between Langrangian densities and theories. For instance, simply changing the interaction term in a Lagrangian density does not, arguably, change the theory it is intended to represent (consider the theory of Newtonian particle dynamics applied to different interactions). However, in the case of an EFT and its high-energy theory, the difference between the two Lagrangian densities is substantial enough to warrant the assumption that one is dealing with two distinct theories. In the ^4He case, the contrast is between a non-relativistic Lagrangian density and a relativistic Lagrangian density; whereas in Manohar's example, the contrast is between a renormalizable Lagrangian density and a nonrenormalizable Lagrangian density. Moreover, as (1) indicates, in both cases, the dynamical variables of the EFT are distinct from those of the high-energy theory.

With the above proviso in mind, in the Lagrangian formalism, a difference in the form of the Lagrangian density entails a difference in the Euler-Lagrange equations of motion for the relevant dynamical variables. One might thus argue that an EFT T' is *derivationally independent* from its associated high-energy theory T, in so far as a specification of the equations of motion of T (together with pertinent initial and/or boundary conditions) will fail to specify solutions to the equations of motion of T'. More generally, the steps involved in the construction of an EFT typically involve approximations and heuristic reasoning. In the case of Wilsonian EFTs, recall that the initial integral over the heavy fields typically involves a saddle-point approximation about the free theory, and even before such an approximation can be constructed, in both the Wilsonian and continuum EFT cases, the task of identifying the relevant high-energy variables must be accomplished. This suggests that, in general, it will be difficult, if not impossible, to reformulate the steps involved in the construction of an EFT (of either the Wilsonian or continuum types) in the form of a derivation.

6.2. Senses of Autonomy

Given that the intertheoretic relation between an EFT and its associated high-energy theory is characterized by derivational independence, what does this suggest about the sense in which the former is autonomous from the latter?

(a) Reductive Autonomy. One might first argue that the relation between an EFT and its high-energy theory cannot be characterized by notions of reduction based on derivability. On the standard (Nagelian) account of reduction, for instance, a necessary condition for a theory T' to reduce to another T is that T' be a *definitional extension* of T (see, *e.g.*, Butterfield and Isham 1999, pg. 115). This requires first that T and T' admit formulations as deductively closed sets of sentences in a formal language (*i.e.*, it assumes a *syntactic* conception of theories), and second that an extension T^* of T can be constructed such that the theorems of T' are a subset of the theorems of T^* (*i.e.*, it requires that T' is a *sub-theory* of T^*). Formally, T^* is constructed by adding to T a definition of each of the non-logical symbols of T'. One might now argue that this cannot be done in the case of a high-energy theory and its EFT. As noted above, in the Lagrangian formalism, differences in the Lagrangian densities representing two theories entail differences in the theories' Euler-Lagrange equations of motion. If one adopts the view that such equations represent the theory's dynamical laws, then the dynamical laws of an EFT and its high-energy theory are different, and a difference in dynamical laws entails a difference in theorems derived from these laws. Thus an EFT is not a sub-theory of its high-energy theory; hence, one cannot say that an EFT reduces to its high-energy theory on this view of reduction.[23]

Note that the above argument does not depend essentially on a syntactic conception of theories. For instance, under a semantic conception of theories, a typical claim is that a theory reduces to another just when models of the first can be embedded in models of the second. This will not suffice to reduce an EFT to its high-energy theory so long as the embedding is required to preserve dynamical laws (and if it is not, then it is unclear whether the term "reduction" for such an embedding is appropriate[24]).

(b) Predictive Autonomy. Predictive autonomy between an EFT and its high-energy theory would seem to be another consequence of derivational independence. Given that the relation between an EFT and its high-energy theory T cannot be described in terms of a derivation in which T is implicated, the phenomena that the EFT describes cannot be derived, and hence predicted, on the basis of T.

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[23] Butterfield and Isham (1999, pg. 122) observe that the standard defintion of supervenience can be characterized in terms of an *infinitistic* definitional extension; thus neither can it be said that an EFT *supervenes* (in this sense) on its associated high-energy theory.

[24] Admittedly this assumes that, whatever else reduction amounts to, it is essentially nomic in nature.

(c) Causal Autonomy. Whether derivational independence of an EFT from its high-energy theory entails causal independence will depend on one's concept of causation. To demonstrate the causal independence of an EFT from its high-energy theory, one would have to provide an account of how the phenomena governed by the EFT are not implicated in the causal mechanisms associated with the relevant high-energy phenomena. The example of superfluid ^4He films is instructive here. Under one interpretation, this EFT suggests that low-energy "ripples" of a superfluid ^4He film behave like relativistic electric and magnetic fields. In so far as ripples in a substrate are implicated in the causal mechanisms that govern the substrate, this suggests causal links between the phenomena of the EFT and the high-energy theory. On the other hand, if one's view of causation is such that the existence of a causal relation requires the existence of a nomic connection (embodied in a dynamical law, say), then one might argue that to the extent to which an EFT and its high-energy theory are nomically independent (in the sense, perhaps, of possessing distinct dynamical laws), they are causally independent, too.

(d) Explanatory Autonomy. Whether or not the phenomena described by an EFT can be explained in terms of the high-energy theory will obviously depend on the notion of explanation one adopts. Arguably, explanatory autonomy will obtain on any account of explanation that requires the *explanandum* to be the product of a derivation in which the *explanans* is implicated; and to the extent to which an EFT is causally independent of its high-energy theory T, it's phenomena cannot be causally explained by T.

6.3. Emergence and Limiting Relations

Evidently there is room to maneuver in addressing the question of whether the intertheoretic relation between an EFT and its high-energy theory can be described in terms of a notion of emergence, at least if such a notion is related to standard accounts of reduction, prediction, causation, and/or explanation. On the other hand, Batterman (2002) has offered a non-standard account of emergence based on the failure of a limiting relation between two theories. This section considers its applicability in the context of an EFT and its high-energy theory.

Batterman's notion of emergence is associated with the failure of what he refers to as the "physicists' sense" of reduction. Under this notion, a "coarse" theory T_c reduces to a "more refined" theory T_f provided a limit of the schematic form $\lim_{\varepsilon\to 0} T_f = T_c$ can be shown to exist, where ε is a relevant parameter.[25] The novelty of emergent properties, according to Batterman, is "...a result of the singular nature of the limiting relationship between the finer and coarser theories that are relevant to the phenomenon of interest"

[25] Batterman's (2002, pg. 78) example is the reduction of special relativity to classical mechanics in the limit $v/c \to 0$, where v is the velocity of a given physical system and c is the speed of light.

(2002, pg. 121). This singular nature, when it exists, is indicative of the existence of a real "physical singularity", according to Batterman. Thus:

> The proposed account of emergent properties has it that genuinely emergent properties, as opposed to "merely" resultant properties, depend on the existence of physical singularities. (Batterman 2002, pg. 125.)

For Batterman, then, there are two necessary conditions for the existence of an emergent property in the context of a fundamental (more refined) theory T and a less fundamental (coarse) theory T':

(a) The physicists' notion of reduction must hold between T and T'; *i.e.*, there must be a limiting relation between T and T'.

(b) The limiting relation must fail in the context with which the emergent property is identified; in particular, there must be a physical singularity associated with the emergent property.

As an example of two theories that satisfy these conditions, Batterman considers thermodynamics (TD) and statistical mechanics (SM). With some qualification, it is possible to define an intertheoretic relation between the TD and SM descriptions of a physical system in terms of the thermodynamic limit N, $V \to \infty$ while $N/V =$ constant, where N and V are the number of particles and volume of the system (Batterman 2002, pg. 123). This limit fails for a thermodynamic system at a critical point at which it undergoes a phase transition. At such a point, the correlation length associated with the system (roughly, the measure of the correlation between spatially separated states) becomes infinite. For Batterman, this is an example of a physical singularity, and he is thus motivated to identify properties associated with phase transitions as emergent properties. (In the ^4He example of Section 2.3, such properties would correspond to the highly correlated phenomena associated with superfluidity.)

Importantly, the intertheoretic relation between TD and SM in this context can be modeled by renormalization group (RG) techniques. The thermodynamic limit generates an RG flow in the parameter space of a TD system. This is analogous to how a scale-dependent momentum cutoff $\Lambda(s)$ generates an RG flow in the parameter space of an RQFT, as described above in Section 3. A TD system at a critical point is then represented by a fixed point in its RG flow. This is a point at which the parameters of the theory remain unchanged under further RG rescaling; *i.e.*, they become scale invariant. As Batterman explains, at a critical point,

> ...there is a loss of a characteristic length scale. This leads to the hypothesis of scale invariance and the idea that the large scale features of a system are virtually

independent of what goes on at a microscopic level. In the thermodynamic case we see that the bulk properties of the thermodynamic systems are independent of the detailed microscopic, molecular constitution of the physical system. (Batterman 2005, pg. 243.)

In the discussion in Section 3 above, a fixed point in the RG flow associated with an RQFT is the explicit indication that the theory is scale-independent and hence renormalizable. For such a theory, the low-energy properties of its constituents are independent of its detailed high-energy constitution. As in the TD/SM case, this also represents the loss of a characteristic scale, in this case an energy scale. Thus the features of a renormalizable theory are independent of what goes on at large energies. And just as in the TD/SM case, scale invariance in a renormalizable RQFT is associated with the existence of physical singularities. In this case, a physical singularity is associated with an observable quantity (like a scattering cross section) that is represented by a divergent Green's function.

This analogy between the intertheoretic relation between TD and SM on the one hand, and the relation between the low-energy and high-energy sectors of a renormalizable RQFT on the other, suggests that Batterman's notion of emergence might be applicable in the latter case. To make this analogy more explicit, consider the following summaries of the relevant features of these examples:

Example 1: $T =$ statistical mechanics (SM). $T' =$ thermodynamics (TD). The limiting relation is the thermodynamic limit: $N,\ V \to \infty$ while $N/V =$ constant.

(i) The thermodynamic limit fails at a fixed point in the associated RG flow in the sense that, at a fixed point, there is no link between the bulk TD properties and the microscopic SM properties. This is a manifestation of scale independence.

(ii) A physical singularity associated with the failure of the thermodynamic limit is a diverging correlation length. Emergent properties are properties associated with the system at the fixed point.

Example 2: $T =$ renormalizable continuum RQFT. $T' =$ cutoff-regulated RQFT. The limiting relation is the continuum limit: $\Lambda(s) \to \infty$. More precisely, to further the analogy with Example 1, the continuum limit can be given schematically by $\Lambda(s) \to \infty$, $[\textit{bare parameters}] \to \infty$, while $[\textit{renormalized parameters}] = [\textit{bare parameters}]/\Lambda(s) =$ constant.[26]

[26] See Stone (2000, pg. 204) for the condensed matter context. The bare parameters are the parameters of the theory before rescaling is performed to restore the cutoff back to its intial value after one iteration of the RG tranformations. The renormalized parameters are the rescaled parameters.

(i) The continuum limit fails at a fixed point in the associated RG flow in the sense that, at a fixed point, there is no link between the low-energy cutoff theory and the high-energy continuum theory. This is a manifestation of scale independence.

(ii) A physical singularity associated with the failure of the continuum limit is represented by a diverging Green's function. Emergent properties are properties associated with the system at a fixed point. In principle, these are properties constructed out of relevant operators.

Fraser has pointed out the following disanalogy between Examples 1 and 2:

> ...whereas the description of a system as containing an infinite number of particles furnished by [statistical mechanics] is taken to be false, the description of space as continuous and infinite that is furnished by QFT with an infinite number of degrees of freedom is taken to be true. (Fraser 2009, pg. 565.)

Thus, in Example 1, the limiting relation is taken to be an idealization, whereas in Example 2 it is not. It would appear, however, that this disanalogy is not relevant to Batterman's notion of emergence, to the extent that the latter is associated with the necessary conditions (a) and (b) above. Condition (a) requires simply that a limiting relation exist, but it says nothing about the status of this relation; in particular, whether it is taken to be an idealization or not.

This suggests that the properties associated with the values of observable quantities constructed from the Green's functions of a renormalizable RQFT are emergent in Batterman's sense. The question now is: To what extent does Example 2 offer insight into the nature of emergence in the context of EFTs?

Two observations appear to be relevant in this context. First, not all EFTs are associated with renormalizable high-energy theories. For those that are not, Batterman's notion of emergence cannot be supported without further ado. Second, even in the case of an EFT with an associated renormalizable high-energy theory, the EFT will typically be formally distinct from the latter. This is a result of the second step in the construction of an EFT (for both Wilsonian or continuum versions) in which the effective Lagrangian density is constructed *via* a local operator expansion. In an RG analysis of a renormalizable continuum RQFT, this step is replaced with a parameter-rescaling procedure by means of which the low-energy cutoff Lagrangian density is transformed back into the initial form of the original Lagrangian density. The upshot is that T and T' in Example 2 are formally identical, whereas an EFT and its high-energy theory are not. Simply put, the cutoff-regulated RQFT of Example 2 is *not* the same mathematical object as an EFT associated with a renormalizable high-energy theory.

This suggests that further work needs to be done if Batterman's notion of emergence is to be applied in the context of an EFT and its high-energy theory.

7. Conclusion

Two general conclusions seem appropriate from this review of effective field theory. First, the discussion in Sections 4 and 5 suggests that, in order to understand how EFTs can be interpreted, one needs to understand the methods that physicists use in applying them. By focusing attention on different renormalization schemes that practicing physicists actually employ, one can discern two types of empirically equivalent EFTs -- Wilsonian EFTs and continuum EFTs. These are non-trivial examples of empirically equivalent theories in so far as, in the context of a given high-energy theory, they make the same low-energy predictions, but they suggest different ontologies. Continuum EFTs support an ontology of quasi-autonomous domains, whereas Wilsonian EFTs do not. Continuum EFTs support an ontology that includes a continuous spacetime, whereas Wilsonian EFTs require space to be discrete and finite. These features of Wilsonian EFTs have contributed to the view that EFTs in general engage in idealization and are inherently approximate. The fact that continuum EFTs do not engage in such idealizations suggests that EFTs do admit interpretations in which they are not considered inherently approximate.

The second conclusion one may draw from this review is that, if one desires to associate (some aspect of) the intertheoretic relation between an EFT and its (possibly hypothetical) high-energy theory with a notion of emergence, then more work has to be done. In the context of standard accounts of emergence, the relevant feature of the EFT intertheoretic relation is that it supports a notion of derivational autonomy (*i.e.*, an EFT cannot be said to be a derivation of its associated high-energy theory). But just how derivational autonomy can be linked with a notion of emergence will depend on such things as how one articulates additional concepts such as reduction, explanation, and/or causation. Section 6 also demonstrated that the EFT intertheoretic relation does not support Batterman's (2002) more formal notion of emergence based on the failure of a limiting relation between two theories. Such a failure of a limiting relation *does* occur between a renormalizable high-energy RQFT and a cutoff-regulated theory obtained from it by renormalization group techniques, but this is a different context than the one in which an EFT is obtained from a high-energy theory. Again, the relevant property of the EFT intertheoretic relation here is that it is a relation between formally distinct, derivationally autonomous, theories.

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