CHAPTER **16**

Condensed Matter Physics and the Nature of Spacetime

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Abstract

This essay considers the prospects of modeling spacetime as a phenomenon that emerges in the low-energy limit of a quantum liquid. It evaluates three examples of spacetime analogues in condensed matter systems that have appeared in the recent physics literature, indicating the extent to which they are viable, and considers what they suggest about the nature of spacetime.

1. INTRODUCTION

In the philosophy of spacetime literature not much attention has been given to concepts of spacetime arising from condensed matter physics. This essay attempts to address this. It looks at analogies between spacetime and a quantum liquid that have arisen from effective field theoretical approaches to highly correlated many-body quantum systems. Such approaches have suggested to some authors that spacetime can be modeled as a phenomenon that emerges in the low-energy limit of a quantum liquid with its contents (matter and force fields) described by effective field theories (EFTs) of the low-energy excitations of this liquid. In the following, these claims will be evaluated in the context of three examples. Section 2 sets the stage by describing the nature of EFTs in condensed matter systems and how Lorentz-invariance typically arises in low-energy approximations. Section 3 looks at two examples of spacetime analogues in superfluid Helium: analogues of general relativistic spacetimes in superfluid Helium 4 associated with the "acoustic" spacetime programme (e.g., Barceló et al., 2005), and analogues of the Standard Model of particle physics in superfluid Helium 3 (Volovik, 2003). Section 4 looks at a twistor analogue of spacetime in a 4-dimensional quantum Hall liquid (Sparling, 2002). It will be seen that these examples possess limited viability

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as analogues of spacetime insofar as they fail to reproduce all aspects of the appropriate physics. On the other hand, all three examples may be considered part of a condensed matter approach to quantum gravity; thus to the extent to which philosophers should be interested in concepts of spacetime associated with approaches to quantum gravity, spacetime analogues in condensed matter should be given due consideration.

2. EFFECTIVE FIELD THEORIES IN CONDENSED MATTER SYSTEMS

In general, an effective field theory (EFT, hereafter) of a physical system is a theory of the dynamics of the system at energies close to zero. For some systems, such low-energy states are effectively independent of ("decoupled from") states at high energies. Hence one may study the low-energy sector of the theory without the need for a detailed description of the high-energy sector. Systems that admit EFTs appear in both quantum field theory and condensed matter physics. It is systems of the latter type that will be the focus of this essay.

In particular, the condensed matter systems to be discussed below are highly-correlated quantum many-body systems; that is, many-body systems that display macroscopic quantum effects. Typical examples include superfluids, superconductors, Bose–Einstein condensates, and quantum Hall liquids. The low-energy states described by an EFT of such a system take the form of collective modes of the ground state, generically referred to as "quasiparticles". Such quasiparticles may be either bosonic or fermionic. In the examples below, under the intended interpretation, the latter correspond to the fermionic matter content of spacetime (electrons, neutrinos, etc.), whereas the former correspond to gauge fields (gravitational, electromagnetic, Yang–Mills, etc.) and their quanta (gravitons, photons, etc.).[1] Intuitively, one considers the system in its ground state and tickles it with a small amount of energy. The low-energy ripples that result then take the above forms. To construct an EFT that describes such ripples, the system must first possess an analytically well-defined ground state.[2] An EFT can then be constructed as a *low-energy approximation* of the original theory. One method for doing so is to expand the initial Lagrangian in small fluctuations in the field variables about their ground state values, and then integrate out the high-energy fluctuations. An example of this will be the construction of the EFT for superfluid Helium 4 below.

In this example and the others reviewed in Sections 3 and 4 below, a Lorentz invariant relativistic theory is obtained as the low-energy approximation of a non-relativistic (i.e., Galilei-invariant) theory. Before considering some of the details of

[1] A third type of low-energy state that may arise in condensed matter EFTs takes the form of topological defects of the ground state, the simplest being vortices. This type will not play a role in the following discussion.

[2] In general this is typically not the case. A necessary condition for the existence of an EFT, so characterized, is that the associated system exhibit *gapless* excitations; i.e., low-energy excitations arbitrarily close to the ground state. This notion of an EFT is that described by Polchinski (1993) and Weinberg (1996, p. 145). For Polchinski, an EFT must be "natural" in the sense that all mass terms should be forbidden by symmetries. Mass terms correspond to *gaps* in the energy spectrum insofar as such terms describe excitations with finite rest energies that cannot be made arbitrarily small. For Weinberg, RG theory should only be applied to EFTs that are massless or nearly massless. (Note that this does not entail that massive theories have no EFTs insofar as mass terms that may appear in the high-energy theory may be encoded as interactions between massless effective fields.)

these examples, it may be helpful to get a feel for just how this can come about. It turns out that this is not that uncommon in many non-relativistic condensed matter systems.

Relativistic phenomena are governed by a Lorentz invariant energy dispersion relation of the standard form $E^2 = m^2c^4 + c^2p^2$. This reduces in the massless case to a linear relation between the energy and the momentum: $E^2 = c^2p^2$. It turns out that such a linear relation is a generic feature of the low-energy sector of Bose–Einstein condensates and (bosonic) superfluids. The general form of the dispersion relation for the quasiparticles of these systems is given by $E^2 = c_s^2p^2 + c_s^2p^4/K^2$, where c_s is the quasiparticle speed, and K is proportional to the mass of the constituent bosons (see, e.g., Liberati et al., 2006, p. 3132). In a low-energy approximation, one may assume the quasiparticle momentum is much smaller than the mass of the constituent bosons; i.e., $p \ll K$, and thus obtain a massless relativistic quasiparticle energy spectrum, $E^2 \approx c_s^2p^2$. In Section 3.1, we'll see how this is encoded in the EFT for superfluid Helium 4.

For fermionic quantum liquids, the Fermi surface plays an essential role in the low-energy approximation. For a non-interacting Fermi gas, the Fermi surface is the boundary in momentum space that separates occupied states from unoccupied states and is characterized by the Fermi momentum p_F. In the corresponding EFT, the Fermi surface becomes the surface on which quasiparticle energies vanish. The energy spectrum of low-energy fermionic quasiparticles then goes as $E(p) \approx v_F(p - p_F)$, where $v_F \equiv (\partial E/\partial p)|_{p=pF}$ is the Fermi velocity.[3] This linear dispersion relation suggests the relativistic massless case and figures into the recovery of the relativistic Dirac equation in one- and two-dimensional systems.[4] The massless Weyl equation that describes chiral fermions can also be recovered in the 1-dim case and this will be relevant in the example of a spacetime analogue in a quantum Hall liquid in Section 4. In this example, a $(1 + 1)$-dim relativistic EFT can be constructed for the edge of a 2-dim quantum Hall liquid, and this can then be extended to a $(3 + 1)$-dim EFT for the edge of a 4-dim QH liquid, with an associated notion of spacetime. In three-dimensional systems, the analysis is a bit more complex. An example of a 3-dim system with a relativistic EFT is the A-phase of superfluid Helium 3, which is a fermionic system in which a finite gap exists between the Fermi surface and the lowest energy level, except at two points. When the energy is linearized about these "Fermi points", it takes the form of a dispersion relation formally identical to that for $(3 + 1)$-dim massless relativistic fermions coupled to a 4-potential field that can be interpreted as an electromagnetic potential field. A sketch of the details will be provided in Section 3.2 below.

3. SPACETIME ANALOGUES IN SUPERFLUID HELIUM

This section reviews two examples of spacetime analogues in superfluid Helium: acoustic spacetimes in superfluid Helium 4, and the Standard Model and gravity

[3] Near the Fermi surface, the energy can be linearly expanded as $E(p) = E(p_F) + (\partial E/\partial p)|_{p=p_F}(p - p_F) + \cdots$. Quasiparticle energies vanish on the Fermi surface, hence to second order, $E(p) = v_F(p - p_F)$.

[4] See Zee (2003, p. 274) for the recovery in a system of electrons hopping on a 1-dim lattice, and Zhang (2004, pp. 672–675) for the recovery in current models of 2-dim high temperature superconductors.

in superfluid Helium 3-A. In both of these examples, the low-energy EFT of the system is formally identical to a relativistic theory. The EFT for superfluid Helium 4 is formally identical to a theory describing a massless scalar field in Minkowski spacetime (to first order) or in a curved spacetime (to second order); and the EFT for superfluid Helium 3-A is formally identical to (relevant aspects of) the Standard Model. Associated with these EFTs are concepts of spacetime, and the extent to which the EFTs are adequate analogues of spacetime will depend, in part, on one's prior convictions on how best to model spacetime. For the superfluid Helium examples, these convictions are:

(a) that spacetime is best modeled by (a given aspect of) the solutions to the Einstein equations in general relativity;
(b) that spacetime is best modeled by the ground state for quantum field theories of matter, gauge, and metric fields.

The examples can be judged on the degree to which they reproduce the appropriate physics (general relativity, the Standard Model), as well as the feasibility of the convictions that motivate them. We'll see that spacetime analogues in superfluid Helium 4 are wanting insofar as they do not completely reproduce general relativity, while spacetime analogues in superfluid Helium 3-A are wanting for the same reason, as well as for some qualified reasons concerning the extent to which they reproduce the Standard Model. In the following I will first explain relevant features of each example and then discuss its viability in providing an analogue of spacetime. Section 3.3 will then take up the question of what these examples suggest about the nature of spacetime.

3.1 "Acoustic" spacetimes and superfluid Helium 4

The ground state of superfluid Helium 4 is a Bose–Einstein condensate consisting of ^{4}He atoms (Helium isotopes with four nucleons). It can be characterized by an order parameter that takes the form of a "macroscopic" wavefunction $\varphi_0 = (\rho_0)^{1/2}e^{i\theta}$ with condensate particle density ρ_0 and coherent phase θ. An appropriate Lagrangian describes *non-relativistic* neutral bosons interacting *via* a spontaneous symmetry breaking potential with coupling constant κ (see, e.g., Zee, 2003, pp. 175, 257),

$$\mathcal{L}_{^4\mathrm{He}} = i\varphi^\dagger \partial_t \varphi - \frac{1}{2m}\partial_i\varphi^\dagger \partial_i\varphi + \mu\varphi^\dagger\varphi - \kappa(\varphi^\dagger\varphi)^2, \quad i = 1,2,3. \tag{1}$$

Here m is the mass of a ^{4}He atom, and the term involving the chemical potential μ enforces particle number conservation. This is a thoroughly non-relativistic Lagrangian invariant under Galilean transformations.

A low-energy approximation of (1) can be obtained in a two-step process:[5]

(a) One first writes the field variable φ in terms of density and phase variables, $\varphi = (\rho)^{1/2}e^{i\theta}$, and expands the latter linearly about their ground state values, $\rho = \rho_0 + \delta\rho$, $\theta = \theta_0 + \delta\theta$ (where $\delta\rho$ and $\delta\theta$ represent fluctuations in density and phase above the ground state).

[5] The following draws on Wen (2004, pp. 82–83) and Zee (2003, pp. 257–258).

(b) After substituting back into (1), one identifies and integrates out the high-energy fluctuations.

Since the ground state φ_0 is a function only of the phase, low-energy excitations take the form of phase fluctuations $\delta\theta$. To remove the high-energy density fluctuations $\delta\rho$, one "integrates" them out: One way to do this is by deriving the Euler–Lagrange equations of motion for the density variable, solving for $\delta\rho$, and then substituting back into the Lagrangian. The result schematically is a sum of two terms: $\mathcal{L}_{^4\mathrm{He}} = \mathcal{L}_0[\rho_0, \theta_0] + \mathcal{L}'_{^4\mathrm{He}}[\delta\theta]$, where the first term describes the ground state of the system and is formally identical to (1), and the second term, dependent only on the phase fluctuations, describes low-energy fluctuations above the ground state. This second term represents the effective field theory of the system and is generally referred to as the effective Lagrangian. To second order in $\delta\theta$, it takes the form,

$$\mathcal{L}'_{^4\mathrm{He}} = \frac{1}{4\kappa}(\partial_t\theta + v_i\partial_i\theta)^2 - \frac{\rho_0}{2m}(\partial_i\theta)^2, \tag{2}$$

with $\delta\theta$ replaced by θ for the sake of notation. Here the second order term depends explicitly on the superfluid velocity $v_i \equiv (1/m)\partial_i\theta$. One now notes that (2) is formally identical to the Lagrangian that describes a massless scalar field in a $(3+1)$-dim curved spacetime:

$$\mathcal{L}'_{^4\mathrm{He}} = \frac{1}{2}\sqrt{-g}\, g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta, \quad \mu, \nu = 0, 1, 2, 3, \tag{3}$$

where the curved effective metric depends explicitly on the superfluid velocity v_i:

$$g_{\mu\nu}\, dx^\mu\, dx^\nu = (\rho/cm)\left[-c^2\, dt^2 + \delta_{ij}\left(dx^i - v^i\, dt\right)\left(dx^j - v^j\, dt\right)\right], \tag{4}$$

where $(-g)^{1/2} \equiv \rho^2/m^2c$, and $c^2 \equiv 2\kappa\rho/m$ (see, e.g., Barceló et al., 2001, pp. 1146–1147). One initial point to note is that, if the original Lagrangian had been expanded to 1st order in $\delta\theta$, the second order term dependent on v_i would vanish in both the effective Lagrangian and the effective metric, and the latter would be formally identical to a flat Minkowski metric (up to conformal constant).[6] This suggests an interpretation of the effective metric (4) as representing low-energy curvature fluctuations (due to the superfluid velocity) above a flat Minkowski background. This is formally identical to the linear approximation of solutions to the Einstein Equations in general relativity, which can likewise be approximated by low-energy fluctuations in curvature above a flat Minkowski background metric. This formal equivalence has been exploited to probe the physics of black holes and the nature of the cosmological constant.

(i) *Acoustic Black Holes*. The general idea is to identify the speed of light in the relativistic case with the speed of low-energy fluctuations, generically referred to as sound modes, in the condensed matter case; hence the terms "acoustic" spacetime and "acoustic" black hole. In general, acoustic black holes are regions in the

[6] When the v_i term is suppressed in (2), the Lagrangian describes a massless field with energy spectrum $E^2 = c^2p^2$. This is the linearly dispersing relation associated with low-energy quasiparticles in Bose–Einstein condensates and bosonic superfluids mentioned in Section 2.

background condensate from which low-energy fluctuations traveling at or less than the speed of sound cannot escape. This can be made more precise with the definitions of acoustic versions of ergosphere, trapped region, and event horizon, among others. A growing body of literature seeks to exploit such formal similarities between relativistic black hole physics and acoustic "dumb" hole physics (see, e.g., Barceló et al., 2005). The primary goal is to provide experimental settings in condensed matter systems for relativistic phenomena such as Hawking radiation associated with black holes.

(ii) *The Cosmological Constant*. Volovik (2003) has argued that the analogy between superfluid Helium and general relativity provides a solution to the cosmological constant problem. The latter he takes as the conflict between the theoretically predicted value of the vacuum energy density in quantum field theory (QFT), and the observational estimate as constrained by general relativity: The QFT theoretical estimate is 120 orders of magnitude greater than what is observed. Volovik sees this as a dilemma for the marriage of QFT with general relativity. If the vacuum energy density contributes to the gravitational field, then the discrepancy between theory and observation must be addressed. If the vacuum energy density is not gravitating, then the discrepancy can be explained away, but at the cost of the equivalence principle. Volovik's preferred solution is to grab both horns by claiming that both QFT and general relativity are EFTs that emerge in the low-energy sector of a quantum liquid.

(a) The first horn is grasped by claiming that QFTs are EFTs of a quantum liquid. As such, the vacuum energy density of the QFT does not represent the true "trans-Planckian" vacuum energy density, which must be calculated from the microscopic theory of the underlying quantum liquid. At $T = 0$, the pressure of such a liquid is equal to the negative of its energy density (Volovik, 2003, pp. 14, 26). This relation between pressure and vacuum energy density also arises in general relativity if the vacuum energy density is identified with the cosmological constant term. However, in the case of liquid ^{4}He in equilibrium, the pressure is zero (Volovik, 2003, p. 29); hence, so is the vacuum energy density.
(b) The second horn is grasped simply by claiming that general relativity is an EFT. Thus, we should not expect the equivalence principle to hold at the "trans-Planckian" level, and hence we should not expect the true vacuum energy density to be gravitating.

Limitations

The implicit claim associated with both the acoustic black hole program and Volovik's solution to the cosmological constant problem is that acoustic spacetimes can be considered analogues of general relativistic spacetimes. One way to assess this claim is by considering the notion of *background structure* in acoustic spacetimes and in general relativity. Note that the acoustic metric arises in a *background-dependent* manner. The acoustic metric (4) is obtained ultimately by imposing particular constraints on *prior* spacetime structure; it is not obtained *ab* *initio*.[7] A natural question then is *What should be identified as the background structure of acoustic spacetimes*? The answer to this question will affect the extent to which acoustic spacetimes effectively model general relativity.

One option is to identify Minkowski spacetime as the background structure of acoustic spacetimes. This might be motivated by the explicit form of the acoustic metric (4). As indicated above, it can be interpreted as describing low-energy curvature fluctuations, due to the superfluid velocity, above a flat Minkowski background metric. In particular, (4) can be written in the suggestive form $g_{\mu\nu}\, dx^\mu\, dx^\nu = \eta_{\mu\nu}\, dx^\mu\, dx^\nu + g'_{\mu\nu}\, dx^\mu\, dx^\nu$, where the first term on the right is independent of the superfluid velocity and is identical to a Minkowski metric, and the second term depends explicitly on the superfluid velocity. (The issue of general covariance will be addressed in the subsequent discussion below.)

A second option, however, is to identify the background structure of acoustic spacetimes with (Galilei-invariant) Neo-Newtonian spacetime. This is motivated by considering the procedure by which the acoustic metric was derived. This starts with the Galilei-invariant Lagrangian (1). Low energy fluctuations of the ground state to first order obey the Lorentz symmetries associated with Minkowski spacetime, and low energy fluctuations to second order obey the symmetries of the curved acoustic metric (4).[8] From this point of view, the relation between acoustic spacetimes and Minkowski spacetime is one in which both supervene over a background Neo-Newtonian spacetime. This second option seems the more appropriate: If acoustic metrics are to be interpreted as low-energy fluctuations above the ground state of a condensate, then the background structure of such spacetimes should be identified with the spatiotemporal structure of the condensate ground state, which obeys Galilean symmetries.[9]

This response has implications for the question of the viability of acoustic spacetimes as models of general relativity. Note first that acoustic metrics are not obtained as solutions to the Einstein equations; they are derived *via* a low-energy approximation from the Lagrangian (1) (and similar Lagrangians for other types of condensed matter systems). As noted above, this approximation results schematically in the expansion $\mathcal{L}_{^4\mathrm{He}} = \mathcal{L}_0[\rho_0, \theta_0] + \mathcal{L}'_{^4\mathrm{He}}[\delta\theta]$. To make contact with the Lagrangian formulation of general relativity, Volovik (2003, p. 38) interprets $\mathcal{L}_{^4\mathrm{He}}$ as comprised of a "gravitational" part $\mathcal{L}_0$ describing a background spacetime expressed in terms of the variables θ_0, ρ_0, with gravity being simulated by the superfluid velocity, and a "matter" part $\mathcal{L}'_{^4\mathrm{He}}$, expressed in terms of the variable $\delta\theta$. To obtain the "gravitational" equations of motion, one can proceed in analogy with general relativity by extremizing $\mathcal{L}_{^4\mathrm{He}}$ with respect to θ_0, ρ_0. This results in a set of equations that are quite different in form from the Einstein equations (Volovik, 2003, p. 41), and this indicates explicitly that the *dynamics* of acoustic spacetime EFTs does not reproduce general relativity. Hence acoustic spacetimes cannot be considered *dynamical* analogues of general relativistic spacetimes.

[7] For the moment I will leave aside the question of how this structure can be interpreted. In particular, as will be made explicit in Section 3.3 below, background-dependence of a spacetime theory does not necessarily imply a substantivalist interpretation, any more than background-independence necessarily implies a relationalist interpretation.

[8] Whether or not (4) exhibits non-trivial symmetries will depend on the explicit form of the superfluid velocity.

[9] Again, I will postpone discussion of how this structure can be interpreted until Section 3.3.

While acknowledging that acoustic spacetimes do not model the dynamics of general relativity, some authors have insisted, nonetheless, that acoustic spacetimes account for the *kinematics* of general relativity:

> ... the features of general relativity that one typically captures in an "analogue model" are the *kinematic* features that have to do with how fields (classical or quantum) are defined on curved spacetime, and the *sine qua non* of any analogue model is the existence of some "effective metric" that captures the notion of the curved spacetimes that arise in general relativity. (Barceló et al., 2005, p. 7.)

> The acoustic analogue for black-hole physics accurately reflects half of general relativity—the kinematics due to the fact that general relativity takes place in a Lorentzian spacetime. The aspect of general relativity that does not carry over to the acoustic model is the dynamics—the Einstein equations. Thus the acoustic model provides a very concrete and specific model for separating the kinematic aspects of general relativity from the dynamic aspects. (Visser, 1998, p. 1790.)

Caution should be urged in evaluating claims like these. First, if the kinematics of general relativity is identified with Minkowski spacetime, as linear approximations to solutions to the Einstein equations might suggest, then acoustic spacetimes cannot be considered kinematical analogues of general relativity. And this is because, as argued above, the background structure of acoustic spacetimes should be identified with Neo-Newtonian spacetime and not Minkowski spacetime. More importantly, just what the kinematics of general relativity consists of is open to debate, particularly if we look beyond the linear approximation and consider solutions to the Einstein equations in their full generality. Rather than engage in this debate, I will restrict my comments to two points. First, to the extent that general solutions to the Einstein equations are background independent, they will obviously not be modeled effectively by background dependent acoustic spacetimes. Second, to the extent that the Einstein equations are diffeomorphism invariant, they will not be modeled effectively by acoustic spacetimes, insofar as the low-energy EFT (2) is not diffeomorphism invariant.[10] Thus, insofar as the kinematics of general relativity involves either (or both) of the properties of background independence and diffeomorphism invariance, acoustic spacetimes cannot be said to be kinematical analogues of general relativity.

I would thus submit that acoustic spacetimes provide neither dynamical nor kinematical analogues of general relativity. In fact this sentiment has been expressed in the literature. Barceló et al. (2004) suggest that acoustic spacetimes simply demonstrate that some phenomena typically associated with general relativity really have nothing to *do* with general relativity:

> Some features that one normally thinks of as intrinsically aspects of gravity, both at the classical and semiclassical levels (such as horizons and Hawking radiation), can in the context of acoustic manifolds be instead seen to be

[10] More precisely, the low-energy EFT (2) does not obey "substantive" (as opposed to "formal") general covariance in Earman's (2006, p. 4) sense; i.e., diffeomorphisms are not a local gauge symmetry of (2).

rather generic features of curved spacetimes and quantum field theory in curved spacetimes, that have nothing to do with gravity *per se*. (Barceló et al., 2004, p. 3.)

This takes some of the initial bite out of Volovik's solution to the cosmological constant problem. If acoustic spacetimes really have nothing to do with general relativity, their relevance to reconciling the latter with QFT is somewhat diminished. While they might provide useful analogues for investigating features of quantum field theory in curved spacetime, extending their use to descriptions of gravitational effects and problems associated with such effects is perhaps not warranted. On the other hand, Volovik's solution to the cosmological constant problem is meant to carry over to other analogues of general relativity besides superfluid ^{4}He. In particular, it can be run for the case of the superfluid ^{3}He-A, which differs significantly from ^{4}He in that fields other than massless scalar fields arise in the low-energy approximation. The fact that these fields model aspects of the dynamics of the Standard Model perhaps adds further plausibility to Volovik's solution. To investigate further, I now turn to ^{3}He.

3.2 The Standard Model and gravity in superfluid Helium 3-A

The second example of a spacetime analogue in a condensed matter system concerns the Standard Model of particle physics and the A-phase of superfluid Helium 3. Since ^{3}He atoms are fermions, they can only condense as a Bose–Einstein condensate if they group themselves into bosonic pairs. Thus the particle content of the superfluid consists of pairs of ^{3}He atoms. These pairs are similar to the electron Cooper pairs described by the standard Bardeen–Cooper–Schrieffer (BCS) theory of conventional superconductors. ^{3}He Cooper pairs, however, have additional spin and orbital angular momentum degrees of freedom, and this allows for a number of distinct superfluid phases. In particular, the A-phase is characterized by pairs of ^{3}He atoms spinning about anti-parallel axes that are perpendicular to the plane of their orbit.[11]

The (second-quantized) Hamiltonian that describes such ^{3}He-A Cooper pairs takes the following schematic form:

$$H_{^3\text{He-A}} = \chi^{\dagger}\mathcal{H}\chi, \quad \mathcal{H} = \sigma^b g_b(\vec{p}), \quad b = 1,2,3, \tag{5}$$

where the χ's are (non-relativistic) 2-spinors that encode creation and annihilation operators for ^{3}He atoms, σ^a are Pauli matrices, and g_b are three functions of momentum that encode the kinetic energy and interaction potential for ^{3}He-A Cooper pairs.[12] This Hamiltonian can be diagonalized to obtain the quasiparticle

[11] ^{3}He Cooper pairs are characterized by spin triplet ($S = 1$) states with p-wave ($l = 1$) orbital symmetry. There are thus nine distinct types of ^{3}He Cooper pairs, characterized by 3 spin ($S_z = 0, \pm 1$) and 3 orbital ($l_z = 0, \pm 1$) momentum eigenvalues. In ^{3}He-A Cooper pairs, there are no $S_z = 0$ substates, and the orbital momentum axis is aligned with the axis of zero spin.

[12] For details consult Volovik (2003, pp. 82, 96). For inquiring minds, $g_1 = \vec{p}\cdot(\Delta_0/p_F)(\vec{\sigma}\cdot\hat{d})\hat{m}$, $g_2 = \vec{p}\cdot(\Delta_0/p_F)(\vec{\sigma}\cdot\hat{d})\hat{n}$, and $g_3 = (p^2/2m) - \mu$. In these expressions, the unit vector d encodes the direction of zero spin, the cross product of the unit vectors m, n encodes the orbital momentum vector, and the constant Δ_0 plays the role of a gap in the BCS energy spectrum for quasiparticle excitations above the Cooper pair condensate. Eq. (5) essentially is a modification of the standard BCS Hamiltonian to account for the extra degrees of freedom of ^{3}He-A Cooper pairs.

energy spectrum. One finds that it vanishes in 3-momentum space at two "Fermi points", call them $p_i^{(a)}$, $i = 1,2,3$, $a = 1,2$. This is due in particular to the directional dependence of the Hamiltonian on the orbital momentum degrees of freedom.

The presence of Fermi points in the energy spectrum is significant for two primarily reasons.[13] First, they are topologically stable insofar as they define singularities in the one-particle Feynman propagator $\mathcal{G} = (ip_0 - \mathcal{H})^{-1}$ that are insensitive to small perturbations. This means pragmatically that the general form of the energy spectrum remains unchanged even when the system undergoes (small) interactions. Second, near the Fermi points $p_\mu^{(a)} = (0, p_i^{(a)})$ in 4-momentum space, the form of the inverse propagator can be expanded as

$$\mathcal{G}^{-1} = \sigma^b e_b^\mu (p_\mu - p_\mu^{(a)}), \quad b = 0,1,2,3 \tag{6}$$

(where the tetrad field e_b^μ encodes the linear approximations of the g_b functions). The quasiparticle energy spectrum is given by the poles in the propagator, and hence takes the general form,

$$g^{\mu\nu}(p_\mu - p_\mu^{(a)})(p_\nu - p_\mu^{(a)}) = 0, \tag{7}$$

where $g^{\mu\nu} = \eta^{ab} e_a^\mu e_b^\nu$. Here the parameters $g^{\mu\nu}$ and $p_\mu^{(a)}$ are dynamical variables insofar as small perturbations of the system are concerned. Again, such perturbations cannot change the fact that Fermi points exist in the energy spectrum; what they *can* change, however, are the positions of the zeros in the energy spectrum, as given by the values of $p_\mu^{(a)}$, or the slope of the curve of the energy spectrum in momentum space, as dictated by the values of $g^{\mu\nu}$.[14]

The Lagrangian corresponding to the energy spectrum (7) can be written as,

$$\mathcal{L}'_{^3\text{He-A}} = \bar{\Psi}\gamma^\mu(\partial_\mu - q^{(a)}A_\mu)\Psi, \tag{8}$$

where $\gamma^\mu = g^{\mu\nu}(\sigma_\nu \otimes \sigma_3)$ are Dirac γ-matrices, the Ψ's are relativistic Dirac 4-spinors (constructed from pairs of the 2-spinors in (5)), and $q^{(a)}A_\mu = p_\mu^{(a)}$. This describes massless Dirac fermions interacting with a 4-vector potential A_μ in a curved spacetime with metric $g_{\mu\nu}$. (8) would be formally identical to the Lagrangian for massless quantum electrodynamics (QED), except for the fact that it does not have a term describing the Maxwell field (i.e., the gauge field associated with the potential A_μ).

It turns out that a Maxwell term arises naturally as a vacuum correction to the coupling between the quasiparticle matter field Ψ and the potential field A_μ. This

[13] The following exposition relies on Volovik (2003, pp. 99–101), and the review in Dreyer (2006, pp. 3–4). Fermi points also occur in the energy spectrum of the sector of the Standard Model above electroweak symmetry breaking (the sector that contains massless chiral fermions). This leads to a theory of universality classes of fermionic vacua based on momentum space topology (Volovik, 2003, Ch. 8). The significance of this theory for the present essay is that superfluid ^{3}He-A and the sector of the Standard Model above electroweak symmetry breaking belong to the same universality class, hence can be expected to exhibit the same low-energy behavior.

[14] This suggests interesting interpretations of the electromagnetic potential and the spacetime metric. To the extent that they can be identified with the objects $p_\mu^{(a)}$ and $g^{\mu\nu}$ in (7), respectively, the electromagnetic potential "...is just the dynamical change in the position of zero in the energy spectrum [of fermionic matter coupled to an electromagnetic field]", and the spacetime metric's role is to change the slope of the energy spectrum (Volovik, 2003, p. 101). The extent to which these identifications are viable is discussed in the following.

is demonstrated by applying the low-energy approximation method outlined in Section 3.1 to the potential field variable: One expands (8) in small fluctuations in A_μ about its ground state value, and then integrates out the high-energy fluctuations. The result is a term that takes the form of the Maxwell Lagrangian in a curved spacetime $\mathcal{L}_{max} = (4\beta)^{-1}\sqrt{-g}\,g^{\mu\nu}g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}$, where $F_{\mu\nu}$ is the gauge field associated with the potential A_μ, and β is a constant that depends logarithmically on the cut-off energy.[15] Combining this with (8), the effective Lagrangian for ^{3}He-A then is formally identical to the Lagrangian for massless (3 + 1)-dim QED in a curved spacetime.

Volovik (2003, pp. 114–115) now indicates how this can be extended to include $SU(2)$ gauge fields, and, in principle, the relevant gauge fields of the Standard Model. The trick is to exploit an additional degree of freedom associated with the quasiparticles described by (8). In addition to their charge, such quasiparticles are also characterized by the two values ± 1 of their spin projection onto the axis of zero spin of the underlying ^{3}He-A Cooper pairs. This two-valuedness can be interpreted as a quasiparticle $SU(2)$ isospin symmetry and incorporated explicitly into (8) by coupling Ψ to a new effective field W^i_μ identified as an $SU(2)$ potential field (analogous to the potential for the weak force). Expanding this modified Lagrangian density in small fluctuations in the W-field about the ground state then produces a Yang–Mills term. The general moral is that discrete degeneracies in the Fermi point structure of the energy spectrum induce local symmetries in the low-energy sector of the background liquid (Volovik, 2003, p. 116). For the discrete two-fold ($\mathbb{Z}_2$) symmetry associated with the zero spin axis projection, we obtain a low-energy $SU(2)$ local symmetry; and in principle for larger discrete symmetries $\mathbb{Z}_N$, we should obtain larger local $SU(N)$ symmetry groups. In this way the complete local symmetry structure of the Standard Model could be obtained in the low-energy limit of an appropriate condensed matter system.

Limitations

There are complications to the above procedure, however. The Standard Model has gauge symmetry $SU(3) \otimes SU(2) \otimes U(1)$ with the electroweak sector given by $SU(2) \otimes U(1)$. The electroweak gauge fields belong to non-factorizable representations of $SU(2) \otimes U(1)$, and hence cannot be simply reconstructed from representations of the two separate groups.[16] This suggests that the low-energy EFT of ^{3}He-A does not completely reproduce all aspects of the Standard Model. In fact, it can be demonstrated explicitly that the ^{3}He-A EFT is formally identical only to the sector of the Standard Model above electroweak symmetry breaking, given that both have in common the same Fermi point momentum space topology (see footnote 13).

Moreover, it turns out that general relativity is not fully recovered either. A low-energy treatment of the ^{3}He-A effective metric does not produce the Einstein–Hilbert Lagrangian of general relativity. Under this treatment, one expands the Lagrangian density in small fluctuations in the effective metric $g_{\mu\nu}$

[15] See, e.g., Volovik (2003, p. 112). A detailed derivation is given in Dziarmaga (2002). This method of obtaining the Maxwell term as the second order vacuum correction to the coupling between fermions and a potential field was proposed by Zeldovich (1967).

[16] Thanks to an anonymous referee for making this point explicit.

about the ground state and then integrates out the high-energy terms. This follows the procedure of what is known as "induced gravity", after Sakharov's (1967) derivation of the Einstein–Hilbert Lagrangian density as a vacuum correction to the coupling between quantum matter fields and the spacetime metric. In Sakharov's original derivation, the metric was taken to be Lorentzian, and the result included terms proportional to the cosmological constant and the Einstein–Hilbert Lagrangian density (as well as higher-order terms). In the case of the ^{3}He-A effective metric, the result contains higher-order terms dependent on the superfluid velocity v_i, and these terms dominate the Einstein–Hilbert term.[17] To suppress these terms, Volovik (2003, pp. 130–132) considers the limit in which the mass of the constituent ^{3}He-A atoms goes to infinity (since the superfluid velocity is inversely proportional to this mass, this entails that $v_i \to 0$). In such an "inert vacuum", the Einstein–Hilbert term can be recovered. Since this limit involves no superflow, Volovik's (2003, p. 113) conclusion is that our physical vacuum cannot be completely modeled by a superfluid. This is suggestive of the formal properties a condensed matter system must possess in order to better model the Standard Model and gravity. In particular, it must possess Fermi points that do not arise *via* symmetry breaking (as the Fermi points of superfluid ^{3}He-A do). From a physical point of view, however, it remains unclear what kind of condensate could possess the property of having infinitely massive constituent particles.

3.3 Interpretation

As has been seen, both of the examples of spacetime analogues in superfluid Helium have their limitations, primarily when it comes to reproducing the relevant physics. To the extent to which they fail to do this, one might question the relevance such examples have to debates over the ontological status of spacetime. On the other hand, both of the above examples, to varying degrees, can be seen to fall within the auspices of a condensed matter approach to quantum gravity. This is explicitly acknowledged by Volovik's (2003) analysis of superfluid ^{3}He-A, and to a lesser extent by the researchers engaged in the acoustic spacetime program (see, e.g., Liberati et al., 2006). This quantum gravity research programme seeks to determine the appropriate condensed matter system that reproduces the matter, gauge and metric fields of current physics in its low-energy approximation, thereby providing a common origin for both quantum field theory and general relativity.[18] Given that all current approaches to quantum gravity are incomplete in one sense or another, the incompleteness of the above examples may thus perhaps be excused. Furthermore, given that philosophers of spacetime should be

[17] See, e.g., Volovik (2003, p. 113). Sakharov's original procedure results in a version of semiclassical quantum gravity, insofar as it describes quantum fields interacting with a classical, unquantized spacetime metric. In the condensed matter context, the background metric is not a classical background spacetime, but rather arises as low-energy degrees of freedom of a quantized non-relativistic system (the superfluid). Hence one could argue this condensed matter version of induced gravity is not semiclassical.

[18] See, e.g., Smolin (2003, pp. 57–58). Thus, to be more precise, the condensed matter programme is an approach to reconciling general relativity and quantum theory, as opposed to an approach to a quantum theory of gravity. Ultimately it suggests gravity need not be quantized, since it claims that gravity emerges in the low energy limit of an already quantized system.

$$\underbrace{\textit{liquid He} \xrightarrow[\textit{(phase transition)}]{\textit{lower energy}} \textit{superfluid He} \xrightarrow[\textit{to zero}]{\textit{lower energy}} \textit{ground state}}_{\textit{Galilei-invariant initial Lagrangian}}$$

$$\xrightarrow[\textit{slightly!}]{\textit{raise energy}} \underbrace{\textit{relativistic system}}_{\substack{\textit{Lorentz-invariant} \\ \textit{effective Lagrangian}}}$$

FIGURE 16.1 The relation between the initial Lagrangian and the effective Lagrangian for superfluid Helium.

interested in concepts of spacetime associated with approaches to quantum gravity, they should be interested in concepts of spacetime associated with the above examples, incomplete though they might be. With this attitude in mind, I will now consider what such concepts might look like.

The condensed matter approach to quantum gravity is a background dependent approach to general relativity and the standard model. Under a literal interpretation, it is characterized by the following. First, it suggests that the vacuum of current physics is the Galilei-invariant ground state of a condensate. The Galilei-invariant spatiotemporal structure of the condensate is thus literally interpreted as background spacetime structure. Low-energy collective excitations above the ground state, in the form of fermionic and bosonic quasiparticles, are interpreted as matter, potential, and metric field quanta, respectively; and induced vacuum corrections to the interactions between matter and potential fields are interpreted as gauge fields (the electromagnetic field, the gravitational field, etc.). Note in particular how this picture views violations of Lorentz invariance. It suggests such violations occur at *low* energies, relative to the vacuum; i.e., they occur as one decreases the energy from the realm of the Lorentz-invariant EFT to the Galilei-invariant ground state. Violations of Lorentz-invariance also occur at high energies, relative to the vacuum: they occur as one increases the energy from the realm of the relativistic low-energy EFT to the realm of typical excited states of the condensate. In the example of superfluid Helium, for instance, typical excited superfluid states for temperatures below the critical temperature T_c, will be described by the Galilei-invariant Lagrangian (1). When the energy is increased even more, we eventually pass through the phase transition at T_c and back to the normal liquid state, which, again, is described by the Galilei-invariant Lagrangian (1) (see Figure 16.1).

How this literal interpretation is further qualified; in particular, how one interprets the spatiotemporal structure of the condensate and the nature of, for instance, the low-energy excitation corresponding to the metric field, will depend on one's proclivities, be they relationalist or substantivalist. Let's consider how these further qualifications could play themselves out.

First, any relationalist interpretation should award ontological status *just* to the condensate: relationalists will not countenance interpretations in which the condensate exists in a background spacetime, for instance. A relationalist interpretation might then be based on the following claims:

(1) The background structure consists of the (Galilei-invariant) spatiotemporal relations between the parts of the ground state of the condensate.
(2) Physical fields (matter, gauge, and metric) are low-energy collective excitations of the condensate.
(3) Relativistic spacetime structure consists of the spatiotemporal properties of low-energy excitations.

How Claim (3) gets further qualified may depend on the convictions one possesses on how best to model spacetime in the relativistic context. For instance, if one is convinced that relativistic spacetime is best modeled by the spatiotemporal properties of the ground state for quantum field theories, then one might identify the relativistic spacetime structure of Claim (3) with the spatiotemporal properties of *all* low-energy excitations identified with physical fields. Such convictions underlie a view, common among string theorists, of the relation between general relativity and quantum field theory that prioritizes the latter and Lorentz symmetries. On the other hand, if one is convinced that relativistic spacetime is best modeled by (some aspect of) the solutions to the Einstein equations in general relativity, then the relativistic spacetime structure of Claim (3) might be identified solely with the spatiotemporal properties of that particular low-energy excitation of the condensate identified as the metric field. Convictions of this sort underlie the canonical loop approach to quantum gravity, for which Rovelli (2006) offers a typical relationalist interpretation. Note that having a condensate at the base of everything would make the life of relationalists of the latter stripe a bit more easy. Such relationalists must provide stories that allow them to treat the metric field on par, ontologically, with the other physical fields in nature, and such stories tend to be difficult in the telling (issues such as the non-local nature of the energy associated with the metric field prevent a complete analogy between it and other physical fields, for instance). If there is a condensate substrate common to all physical fields, including the metric field, presumably the latter obtains just as much ontological underpinning from it as the other fields.

Substantivalists of any stripe should award ontological status to both the condensate and spacetime. One can imagine various ways of doing so. A conservative substantivalist, for instance, might adopt the relationalist's Claims (2) and (3) while replacing Claim (1) with

(1′) The background structure consists of the properties of a substantival Neo-Newtonian spacetime.

How Claim (3) gets cashed out by a conservative substantivalist might follow the same maneuvers as the relationalist above. A more intrepid substantivalist might insist on maintaining an ontological distinction between matter and spacetime at all energy scales. One way to do this is to adopt Claims (1′) and (2), but replace (3) with

(3′) Relativistic spacetime structure consists of the properties of a low-energy emergent substantival spacetime.

The full explication of (3′) would require fleshing out a notion of "low-energy emergence". In fact, the examples of low-energy EFTs in superfluid Helium above (as well as the example in quantum Hall liquids below) have suggested to some authors that novel phenomena including fields, particles, symmetries, twistors, *and* spacetime, *emerge* in the low-energy sector of certain condensed matter systems.[19] Doing justice to this notion of low-energy emergence is perhaps best left to another essay; however, one thing that should be said is that it is distinct from typical notions of emergence associated with phase transitions in condensed matter systems. As Figure 16.1 suggests, typical superfluids can be described by a single Lagrangian that encodes both the normal liquid phase and the superfluid phase, as well as the phase transition between the two. This Lagrangian is formally distinct from the effective Lagrangian of the low-energy sector of the superfluid (when it exists analytically). Thus to the extent that these distinct Lagrangians encode different theories, low-energy emergence can be thought of as a relation between theories, as opposed to a particular interpretation of a single theory (as typical notions of emergence associated with symmetry-breaking phase transitions appear to be).

At this point, it might be appropriate to consider possible motivations for the above substantivalist interpretations. It might not be clear how the roles that typical substantivalists require spacetime to play are accomplished in the condensed matter context. One such role is to provide the ontological substrate for physical fields. Typical substantivalist interpretations of general relativity, for instance, are motivated by a literal interpretation of the representations of physical fields as tensor fields that quantify over the points (or regions) of a differentiable manifold. In the condensed matter context, this intuition might be applied to the field representations of the constituent particles of the condensate as quantifying over the points or regions of Neo-Newtonian spacetime. A conservative substantivalist might claim that, in order to support the condensate, we must postulate the existence of a substantival Neo-Newtonian spacetime. In the case of an intrepid substantivalist, this intuition might be extended to the effective fields of the EFT and the low-energy emergent substantival spacetime; however, it will only do work if the notion of low-energy emergence is cashed out in such a way that the effective fields (and the emergent relativistic spacetime) are sufficiently ontologically distinct from the condensate. Otherwise, relationalists might claim the condensate itself provides the necessary ontological support for the effective fields.

A different type of substantivalist motivation comes from a desire to explain inertial motion in terms of background spacetime structure. A substantivalist might suggest that the coordinated behavior of test particles undergoing inertial motion is mysterious, since such particles have no inertial "antennae" to detect each other, and is explained if we posit a substantival spacetime endowed with an affine con-

[19] In their review of models of analogue gravity, Barceló et al. (2005) speak of "emergent gravitational features in condensed matter systems" (p. 84), and "emergent spacetime symmetries" (p. 89); Dziarmaga (2002, p. 274) describes how "...an effective electrodynamics emerges from an underlying fermionic condensed matter system"; Volovik (2003) in the preface to his text on low-energy properties of superfluid helium, lists "emergent relativistic quantum field theory and gravity" and "emergent non-trivial spacetimes" as topics to be discussed within; Zhang (2004) provides "examples of emergence in condensed matter physics", including the 4-dim quantum Hall effect; and Zhang and Hu (2001, p. 825) speak of the "emergence of relativity" at the edge of 2-dim and 4-dim quantum Hall liquids.

nection that singles out the privileged inertial trajectories.[20] I now want to argue that this motivation for substantivalism fails in the condensed matter context. Note first that it will not do work for a conservative substantivalist. For such a substantivalist, the condensate exists in Neo-Newtonian spacetime, and fields and test particles are low-energy ripples in the condensate. However, the relativistic inertial structure experienced by the ripples is not that possessed by Neo-Newtonian spacetime: According to Claim (3), it consists of the properties of the ripples themselves.[21] An intrepid substantivalist may on first glance fare a bit better: Claim (3′) guarantees that there are substantival privileged inertial trajectories in the relativistic context. In fact, an intrepid substantivalist might even claim to be able to address a key criticism of this motivation; namely, that to explain the origin of inertial motion by referring to privileged inertial trajectories in a substantival spacetime is simply to replace mysterious inertial antennae with mysterious spacetime "feelers" (Brown and Pooley, 2006, p. 72). An intrepid substantivalist might claim to have the basis for an explanation of these feelers: Low-energy ripples of the condensate, viewed as low-energy emergent phenomena, might be expected to coordinate themselves with a low-energy emergent substantival spacetime, given the common origin of the two. Again, whether this basis can be fleshed out into a legitimate explanation will depend on how the notion of low-energy emergence is cashed out. But even if a legitimate explanation in terms of low-energy emergence is forthcoming, it will do no work in distinguishing an intrepid substantivalist from a conservative substantivalist, and hence, in distinguishing substantivalism from relationalism in this context. Note first, that for any notion of low-energy emergence that the intrepid substantivalist adopts, a conservative substantivalist may appropriate it to flesh out Claim (2) and the origin of physical fields. She will then be able to explain the mysterious inertial antennae of such fields in terms of their common substrate origin, to the same degree that the intrepid substantivalist can explain the mysterious spacetime feelers of physical fields in terms of their common origin with (relativistic) spacetime itself. In other words, any legitimate intrepid substantivalist explanation of spacetime feelers will map onto an equally legitimate conservative substantivalist explanation of inertial antennae. And, obviously, a relationalist may engage in the same practice as the conservative substantivalist in this context.

Thus, of the two standard motivations for substantivalism, only the motivation from fields is relevant in the condensed matter context, and intrepid substantivalists will be fairly hard-pressed to make it work for them. Of course this is not to say there may be other motivations for intrepid substantivalism (again, an insistence on a separation between matter and spacetime at all energy scales may be one).[22]

[20] In other words, spacetime has privileged "ruts" along which test particles are constrained to move in the absence of external forces. See, e.g., Brown and Pooley (2006), where this motivation is identified and critiqued.

[21] Note that the "Newtonian limit", $v/c \to 0$, for these relativistic low-energy ripples will consist of non-relativistic low-energy ripples that do experience Neo-Newtonian inertial structure, but again, given the nature of Claim (3), according to a conservative substantivalist, this structure is not to be attributed to the container Neo-Newtonian spacetime, but to properties of the ripples.

[22] For the sake of completeness, two further substantivalist positions can be identified. A super substantivalist might interpret spacetime simply as the condensate itself, with matter fields and gauge fields identified as low-energy aspects of spacetime. Arguably, such a super substantivalist would be hard-pressed to distinguish herself from the relationalist. Both

One might now compare the above notions of spacetime with notions of spacetime associated with other approaches to quantum gravity. Rather than explicitly doing so, the remainder of this section will simply indicate how the condensed matter approach compares conceptually with the two most popular approaches; namely, the background independent canonical loop approach, and background dependent approaches like string theory. The intent is to distinguish these approaches in terms of how they deal with the issues of prior spacetime structure and the nature and status of spacetime symmetries.

(a) The condensed matter approach is distinct from the canonical loop approach, insofar as it is background-*dependent*, the background being the spatiotemporal structure of the condensate. Moreover, while both the condensed matter approach and the loop approach predict violations of Lorentz invariance, these predictions differ in their details. First, as indicated above (see, e.g., Figure 16.1), the condensed matter approach predicts such violations both at low energies (as we approach the ground state), and at high energies (as we approach typical excited states of the condensate and beyond). The loop approach predicts violations only at high energies (scales smaller than the Planck scale) at which it predicts spacetime becomes discrete. Second, the condensed matter approach explains the violation of Lorentz invariance in terms of the existence of a preferred frame; namely the frame defined by the spatiotemporal properties of the condensate, whereas the loop approach explains the violation in terms of background-independence: at the Planck scale, there are no frames, whether Lorentzian or otherwise.[23]

(b) The condensed matter approach differs from background-dependent approaches like string theory in three general respects. First, as is evident in the previous sections, the condensed matter approach differs from string theory in that the structure it attributes to the background is not Minkowskian: Given that the fundamental condensate is a *non-relativistic* quantum liquid, the background will be Neo-Newtonian. Second, while background-dependent approaches that are ultimately motivated by quantum field theory (as string theory is) typically view QFTs as low-energy EFTs of a more fundamental theory, such approaches view the latter as a theory of *high-energy* phenomena (strings, for example). The phenomena of experience, as described by current QFTs, are then interpreted as emerging *via* a process of symmetry breaking. The condensed matter approach, on the other hand, views QFTs and general relativity as EFTs of a more fundamental *low-energy* theory (relative to the vacuum), and the process by which the former arise is a low-energy emergent process that is not to be associated with symmetry breaking. Finally, in general, the condensed matter approach can be characterized by placing less ontological significance on the notion of symmetry than background-dependent approaches in at least two major respects.

First, background-dependent approaches that view QFTs as EFTs describe the phenomena of experience as obeying "imperfect" (gauge) symmetries that result

make the same ontological Claims (1)–(3) and differ only on terminology. A hybrid substantivalist might adopt Claims (1), (2) and (3′); but such a beast would also be hard to motivate: Hybrids cannot consistently appeal to the motivation from fields, given that Claim (1) entails they reject it at the level of the condensate.

23 Smolin (2003, p. 20) indicates that current experimental data on the violation of Lorentz invariance place very restrictive bounds on preferred frame approaches. Nevertheless he suggests the condensed matter approach may provide key information on the way spacetime might emerge in other scenarios; spin foams, for instance.

from a process of symmetry breaking of a "more perfect" fundamental symmetry. Mathematically, the more perfect fundamental symmetry is hypothesized as having the structure of a single compact Lie group with a minimum of parameters. This is then broken into imperfect symmetries that are characterized by product group structure and relatively many parameters. In particular, the gauge field group structure of the Standard Model, below electroweak symmetry breaking, is given by $U(1) \otimes SU(2) \otimes SU(3)$. In the condensed matter approach, the fundamental condensate is *not* expected to have symmetries described by a single compact Lie group. In the case of superfluid Helium 3, for instance, the "fundamental" symmetries *already* have a "messy" product group structure $U(1) \otimes SO(3) \otimes SO(3)$, reflecting the spin and orbital angular momentum degrees of freedom of ^{3}He Cooper pairs. Moreover, in terms of spacetime symmetries in the condensed matter approach, there are also senses in which the low-energy relativistic (viz., Lorentz) symmetries are *more perfect* than the fundamental Galilean symmetries of the condensate. Note first that the Lorentz group can be characterized as leaving invariant a *single* Lorentzian spacetime metric, whereas the Galilei group cannot; the latter leaves *separate* spatial and temporal metrics invariant. Moreover, the Galilei group does not admit unitary representations, whereas the Lorentz group does.[24]

The second way in which the condensed matter approach de-emphasizes the ontological status of symmetries involves viewing it as an alternative logic of nature to the logic of the Gauge Argument, which typically finds adherents in quantum field theory. According to the Gauge Argument, matter fields are fundamental and imposing local gauge invariance on a matter Lagrangian requires the introduction of interactions with potential gauge fields. The emphasis here is on the fundamental role of local symmetries in explaining the origins of gauge fields (see Martin (2002) for a critique of this argument). According to the condensed matter approach, symmetries, both local and global, as well as matter and potential fields, are low-energy emergent phenomena of the fundamental condensate. In particular, local symmetries do not play a fundamental role in the origin of gauge fields.

4. SPACETIME ANALOGUE IN QUANTUM HALL LIQUIDS

A final example of a spacetime analogue in a condensed matter system concerns the twistor formalism and 4-dimensional quantum Hall liquids. In this example, the low-energy EFT of the edge of the system is formally identical to a theory describing massless relativistic $(3+1)$-dim fields of all helicities. The question of how such a model provides an analogue of spacetime is answered by twistor theory, the goal of which is to reconstruct general relativity and quantum field theory from the conformal properties of twistors. This example is thus similar in spirit with the Helium examples insofar as it, too, can be associated with an approach

[24] Of course these senses depend on a more nuanced characterization of "perfection" in group-theoretic terms than in the case of gauge symmetries. Technically, the second sense is based on the fact that the Galilei group has non-trivial exponents, whereas the Lorentz group does not. Unitary representations of the Galilei group up to a phase factor *can* be constructed (so-called projective representations). The importance of unitary representations comes with implementing spacetime symmetries in the context of quantum theory.

to quantum gravity. Moreover, just as with the Helium examples, this example faces limitations of two types. The first involves the extent to which the model reproduces the "appropriate physics" (which in this case is twistor theory), and the second involves the convictions associated with twistor theory as to how best to represent spacetime (in this case, as derivative of twistors). We'll see that the latter limitation is the most severe: twistor theory faces its own problems in reproducing the appropriate physics (general relativity and quantum field theory). These problems will be discussed in Section 4.4. Section 4.1 describes the context in which 2-dim quantum Hall liquids arise, Section 4.2 indicates how this can be extended to four dimensions, and Section 4.3 explains what twistors are and how they are intended to fit into the picture.

4.1 2-dim quantum Hall liquids

Quantum Hall liquids initially arose in explanations of the 2-dimensional quantum Hall effect (QHE). The set-up consists of current flowing in a 2-dim conductor in the presence of an external magnetic field perpendicular to its surface. The classical Hall effect occurs as the electrons in the current are deflected towards the edge by the magnetic field, thus inducing a transverse voltage. In the steady state, the force due to the magnetic field is balanced by the force due to the induced electric field and the *Hall conductivity* σ_H is given by the ratio of current density to induced electric field. The *quantum* Hall effect occurs in the presence of a strong magnetic field, in which σ_H becomes quantized in units of the ratio of the square of the electron charge e to the Planck constant h:

$$\sigma_H = \nu \times (e^2/h), \tag{9}$$

where ν is a constant. The Integer Quantum Hall Effect (IQHE) is characterized by integer values of ν, and the Fractional Quantum Hall Effect (FQHE) is characterized by values of ν given by odd-denominator fractions. Two properties experimentally characterize the system at such quantized values: The current flowing in the conductor becomes dissipationless, as in a superconductor; and the system becomes incompressible.

These effects can be modeled by a condensate referred to as a quantum Hall (QH) liquid. In one formulation, its constituent particles are represented by "composite" bosons: bosons with p quanta of magnetic flux attached to them, where p is an odd integer.[25] The effect of this coupling is to mimic the Fermi–Dirac statistics of the original electrons. One can show that the total magnetic field felt by the composite bosons vanishes when the constant ν in (9) is given by $1/p$, corresponding to the FQHE. At such values, the bosons feel no net magnetic field, and hence can form a condensate at zero temperature. This condensate, consisting of charged bosons, forms the QH liquid, and can be considered to have the same properties as a superconductor; namely, dissipationless current flow and the expulsion of magnetic fields from its interior. The latter property entails there is no

[25] Technically this is achieved by coupling bosons in the presence of a magnetic field to an additional Chern–Simons field. For details, consult Zhang (1992, p. 32).

net internal magnetic field in a QH liquid, and this entails that the particle density is constant.[26] Thus a QH liquid is incompressible.

The fact that a QH liquid is incompressible entails that there is a finite energy gap between the ground state of the condensate and the first allowable energy states. This means a low-energy approximation cannot be constructed; thus there is no low-energy EFT for the bulk liquid. A low-energy EFT can, however, be constructed for the 1-dim edge of the liquid. Wen (1990) assumed edge excitations take the form of low-energy surface waves and demonstrated that the effective Lagrangian for the edge states describes massless chiral fermion fields in $(1+1)$-dim Minkowski spacetime:

$$\mathcal{L}'_{edge} = i\psi^{\dagger}(\partial_t - v\partial_x)\psi, \tag{10}$$

where v is the electron drift velocity.

4.2 4-dim quantum Hall liquids

The $(1+1)$-dim edge Lagrangian (10) tells us little about the ontology of $(3+1)$-dim spacetime. However, it suggests that $(3+1)$-dim massless relativistic fields may be obtainable from the edge states of a 4-dim QH liquid, and this is in fact borne out. Zhang and Hu (2001) provided the first extension of the 2-dimensional QHE to 4-dimensions. In rough outline, they replaced the 2-dim quantum Hall liquid with a 4-dim quantum Hall liquid and then demonstrated that the EFT of the 3-dim edge describes massless fields in $(3+1)$-dim Minkowski spacetime.

In slightly more detail, Zhang and Hu made use of a formulation of the 2-dim QHE in terms of spherical geometry first given by Haldane (1983). Haldane considered an electron gas on the surface of a 2-sphere S^2 with a $U(1)$ Dirac magnetic monopole at its center. The radial monopole field serves as the external magnetic field of the original setup. By taking an appropriate thermodynamic limit, the 2-dim QHE on the 2-plane is recovered.[27] Zhang and Hu's extension to 4-dimensions is based on the geometric fact that a Dirac monopole can be formulated as a $U(1)$ connection on a principle fiber bundle $S^3 \to S^2$, consisting of base space S^2 and bundle space S^3 with typical fiber $S^1 \cong U(1)$ (see, e.g., Nabor, 1997). This fiber bundle is known as the 1st Hopf bundle and is essentially a way of mapping the 3-sphere onto the 2-sphere by viewing S^3 as a collection of "fibers", all isomorphic to a "typical fiber" S^1, and parameterized by the points of S^2. There is also a 2nd Hopf bundle $S^7 \to S^4$, consisting of the 4-sphere S^4 as base space, and the 7-sphere S^7 as bundle space with typical fiber $S^3 \cong SU(2)$. The $SU(2)$ connection on this bundle is referred to as a Yang monopole. Zhang and Hu's 4-dim QHE then consists of taking the appropriate thermodynamic limit of an electron gas on the surface of a 4-sphere with an $SU(2)$ Yang monopole at its center.

[26] Technically this is due to the fact that the Chern–Simons field is determined by the particle density.

[27] The thermodynamic limit involves taking $N \to \infty$, $I \to \infty$, $R \to \infty$, while holding I/R^2 constant (Haldane, 1983, p. 606; see also Meng, 2003, p. 9415). Here I labels representations of $U(1)$ (and is associated with the Dirac monopole field strength), N is the number of states, which in the lowest energy level is given by $2I+1$, and R is the radius of the 2-sphere. In the lowest energy level, the ratio I/R^2 is proportional to the density of states $N/4\pi R^2$, which must be held constant to recover an incompressible liquid.

Some authors have imbued the interplay between algebra and geometry in the construction of the 4-dim QHE with ontological significance. These authors note that there are only four normed division algebras: the real numbers $\mathbb{R}$, the complex numbers $\mathbb{C}$, the quaternions $\mathbb{H}$, and the octonions $\mathbb{O}$.[28] It is then observed that these may be associated with the four Hopf bundles, $S^1 \to S^1$, $S^3 \to S^2$, $S^7 \to S^4$, $S^{15} \to S^8$, insofar as the base spaces of these fiber bundles are the compactifications of the respective division algebra spaces $\mathbb{R}^1$, $\mathbb{R}^2$, $\mathbb{R}^4$, $\mathbb{R}^8$. Finally, one notes that the typical fibers of these Hopf bundles are $\mathbb{Z}_2$, $U(1) \cong S^1$, $SU(2) \cong S^3$, and $SO(8) \cong S^7$, respectively. These patterns are then linked with the existence of QH liquids:

> One, two, and four dimensional spaces have the unique mathematical property that boundaries of these spaces are isomorphic to mathematical groups, namely the groups $\mathbb{Z}_2$, $U(1)$ and $SU(2)$. No other spaces have this property. (Zhang and Hu, 2001, p. 827.)

> The four sets of numbers [viz., $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, $\mathbb{O}$] are mathematically known as division algebras. The octonions are the last division algebra, no further generalization being consistent with the laws of mathematics... Strikingly, in physics, some of the division algebras are realized as fundamental structures of the quantum Hall effect. (Bernevig et al., 2003, p. 236803-1.)

> Our work shows that QH liquids work only in certain magic dimensions exactly related to the division algebras... (Zhang, 2004, p. 688.)

These comments have philosophical import to the extent that QH liquids play a fundamental role in physics. They suggest, for instance, an explanation for the dimensionality of space. In particular, if spacetime arises from the edge of a QH liquid, and if the latter only exist in the "magic" dimensions one, two and four, then the spatial dimensions of spacetime are restricted to zero, one, or three, respectively (insofar as the edge would have one less spatial dimension than the bulk). Admittedly, these are big "ifs". The extent to which spacetime arises from the edge of a QH liquid will be dealt with in Sections 4.3 and 4.4 below. The following briefly addresses the extent to which QH liquids can be seen as existing only in a limited number of "magic" dimensions.

Note first that Zhang and Hu's statement should be restricted to the compactifications of the spaces $\mathbb{R}^1$, $\mathbb{R}^2$, $\mathbb{R}^4$, and should include the compactification of $\mathbb{R}^8$ as well, the boundary of the latter being isomorphic to the group $SO(8)$. Furthermore, the statements of Bernevig et al. and Zhang should refer to *normed* division algebras. Baez (2001, p. 149) carefully distinguishes between $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, $\mathbb{O}$ as the only normed division algebras, and division algebras in general, of which there are other examples. Baez (2001, pp. 153–156) indicates how the sequence $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, $\mathbb{O}$ can in principle be extended indefinitely by means of the Cayley–Dickson construction. Starting from an n-dim $*$-algebra A (i.e., an algebra A equipped with a conjugation map $*$), the construction gives a new $2n$-dim $*$-algebra A'. The next member of the sequence after $\mathbb{O}$ is a 16-dim $*$-algebra referred to as the "sedenions". The point here is that the sedenions and all subsequent higher-dimensional

[28] A normed division algebra A is a normed vector space, equipped with multiplication and unit element, such that, for all $a, b \in A$, if $ab = 0$, then $a = 0$ or $b = 0$. $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{H}$ are associative, whereas $\mathbb{O}$ is non-associative (see, e.g., Baez, 2001, p. 149).

constructions do not form division algebras; in particular, they have zero divisors. The question therefore should be whether the absence of zero divisors in a normed $*$-algebra has physical significance when it comes to constructing QH liquids.

Zhang (2004, p. 687) implicitly suggests it does. He identifies various quantum liquids with each Hopf bundle: 1-dim Luttinger liquids[29] with $S^1 \to S^1$, 2-dim QH liquids with $S^3 \to S^2$, and 4-dim QH liquids with $S^7 \to S^4$. Bernevig et al. (2003) complete the pattern by constructing an 8-dim QH liquid as a fermionic gas on S^{15} with an $SO(8)$ monopole at its center. But whether this pattern is physically significant remains to be seen. It is not entirely clear, for example, how the bundle $S^1 \to S^1$, and the trivial $\mathbb{Z}_2$ monopole associated with it, is essential in the construction of Luttinger liquids in general. Moreover, while Luttinger liquids arise at the edge of 2-dim QH liquids, this pattern does not carry over to higher dimensions: it is not the case that 2-dim QH liquids arise at the edge of 4-dim QH liquids, nor is it the case that 4-dim QH liquids arise at the edge of 8-dim QH liquids. Furthermore, and more importantly, Meng (2003) demonstrates that higher-dimensional QH liquids can in principle be constructed for any even dimension, and concludes that the existence of division algebras is not a crucial aspect of such constructions (see also Karabali and Nair, 2002). Hence, while the relation between Hopf bundles and normed division algebras on the one hand, and quantum liquids on the other, is suggestive, it perhaps should not be interpreted too literally.

4.3 Edge states for 4-dim QH liquids and twistors

The low-energy edge states of a 2-dim QH liquid take the form of $(1+1)$-dim relativistic massless fields described by (10). These edge excitations can also be viewed as particle-hole dipoles formed by the removal of a fermion from the bulk to outside the QH droplet, leaving behind a hole (see, e.g., Stone, 1990). If the particle-hole separation remains small, such dipoles can be considered single localized bosonic particle states. The stability of such localized states is affected by the uncertainty principle: a stable separation distance entails a corresponding uncertainty in relative momentum, which presumably would disrupt the separation distance. In 1-dim it turns out that the kinetic energy of such dipoles is approximately independent of their relative momentum, hence they are stable. In the case of the 3-dim edge of the 4-dim QH liquid, Zhang and Hu (2001) determined that there is a subset of dipole states for which the isospin degrees of freedom associated with the $SU(2)$ monopole counteract the uncertainty principle. Their main result was to establish that these stable edge states satisfy the $(3+1)$-dim zero rest mass field equations for all helicities, and hence can be interpreted as zero rest mass relativistic fields (see also Hu and Zhang, 2002). These include, for instance, spin-1 Maxwell fields and spin-2 graviton fields satisfying the vacuum linearized Einstein equations, as well as massless fields of all higher helicities. Given that there currently is no evidence for the existence of particles with helicities less than

[29] Wen's (1990) EFT (10) identifies the edge of a 2-dim QH liquid as a Luttinger liquid. A Luttinger liquid is comprised of electrons, but differs from a standard Fermi liquid mathematically in the form of the electron propagator. See Wen (2004, pp. 314–315) for details.

2, the latter fact was recognized by Zhang and Hu (2001, p. 827) as an "embarrassment of riches", and a major difficulty of their model.[30]

By itself, this recovery of (3+1)-dim relativistic zero rest mass fields has limited applicability when it comes to questions concerning spacetime ontology. As with the examples in superfluid Helium, we would like to recover general relativity and the Standard Model in their full glory. This is where twistor theory makes its appearance, the goal of which is to recover general relativity and quantum field theory from the structure of zero rest mass fields. Sparling's (2002) insight was to see that Zhang and Hu's stable dipole states correspond to twistor representations of zero rest mass fields. In particular, Sparling demonstrated that the edge of a 4-dim QH liquid can be identified with a particular region of twistor space $\mathbb{T}$. $\mathbb{T}$ is the carrying space for matrix representations of $SU(2,2)$ which is the double covering group of $SO(2,4)$. Elements Z^α of $\mathbb{T}$ are called twistors and are thus spinor representations of $SO(2,4)$. $\mathbb{T}$ contains a Hermitian 2-form $\sum_{\alpha\beta}$ (a "metric") which splits the space into three regions, $\mathbb{T}^+$, $\mathbb{T}^-$, $\mathbb{N}$, consisting of twistors Z^α satisfying $\sum_{\alpha\beta} Z^\alpha Z^\beta > 0$, $\sum_{\alpha\beta} Z^\alpha Z^\beta < 0$, and $\sum_{\alpha\beta} Z^\alpha Z^\beta = 0$, respectively. The connection to spacetime is based on the fact that $SO(2,4)$ is the double covering group of $C(1,3)$, the conformal group of Minkowski spacetime. This allows a correspondence to be constructed under which elements of $\mathbb{N}$, "null" twistors, correspond to null geodesics in Minkowski spacetime, and 1-dim subspaces of $\mathbb{N}$ (i.e., twistor "lines") correspond to Minkowski spacetime points.[31]

To make the identification of the edge of a 4-dim QH liquid with $\mathbb{N}$ plausible, note that the symmetry group of the edge is $SO(4)$ (which is isomorphic to the 3-sphere S^3) and that of the bulk is $SO(5)$ (which is isomorphic to the 4-sphere S^4). The twistor group $SO(2,4)$ contains both $SO(4)$ and $SO(5)$. Intuitively, the restriction of $SO(2,4)$ to $SO(4)$ can be induced by a restriction of twistor space $\mathbb{T}$ to $\mathbb{N}$.[32] With the edge identified as $\mathbb{N}$, edge excitations are identified as deformations of $\mathbb{N}$ (in analogy with Wenn's treatment of the edge in the 2-dim case). In twistor theory, such deformations take the form of elements of the first cohomology group of projective null twistor space $\mathbb{PN}$, and these are in fact solutions to the zero rest mass field equations of all helicities in Minkowski spacetime (Sparling, 2002, p. 25).

Limitations

The complete recovery of twistors from the edge of a 4-dim QH liquid faces a technical hitch concerning the nature of the thermodynamic limit. In the spherical formulations of the QHE, this limit serves to transform the 2-sphere (resp. 4-sphere) into the 2-plane (resp. 4-plane), while reproducing an incompressible QH liquid (footnote 27). In the 4-dim case, this led to Zhang and Hu's "embarrassment of riches" problem: the thermodynamic limit requires taking the isospin

[30] Hu and Zhang (2002, p. 125301-8) consider possible ways to address this problem. A mechanism is needed under which the higher helicity fermionic states acquire masses (i.e., become "gapped") at low energies and thus decouple from observable interactions.

[31] More precisely, the correspondence is between $\mathbb{PN}$, the space of null twistors up to a complex constant (i.e., "projective" null twistors), and *compactified* Minkowski spacetime (i.e., Minkowski spacetime with a null cone at infinity). This is a particular restriction of a general correspondence between projective twistor space $\mathbb{PT}$ and complex compactified Minkowski spacetime. For a brief review, see Bain (2006, pp. 41–42).

[32] Technically, this restriction corresponds to a foliation of the 4-sphere with the level surfaces of the $SO(4)$-invariant function $f(Z^\alpha) = \sum_{\alpha\beta} Z^\alpha Z^\beta$. These surfaces are planes spanned by null twistors (Sparling, 2002, pp. 18–19, 22).

degrees of freedom associated with the Yang monopole to infinity, allowing for $(3+1)$-dim massless fields of all helicities. In the twistor formulation, it is unclear what this limit corresponds to. One way to see this is to note that the twistor formulation does away with the Yang monopole field. In twistor theory, a general result due to Ward allows one to map the dynamics of anti-self-dual Yang–Mills gauge fields (of which the Yang monopole is a particular example) onto purely geometric structures defined on an appropriate twistor space (see, e.g., Bain, 2006, p. 44, for a brief account). Thus in the twistor formulation, there is no explicit isospin space on which to define a limiting procedure. Assumedly, the isospin limit should have a geometrical interpretation in the twistor formulation, but just what it is, is open to speculation (see Sparling, 2002, pp. 27–28 for discussion).

4.4 Interpretation

Even granted that the 4-dim QHE admits a thoroughly twistorial formulation down to the thermodynamic limit, there is still the question of whether spacetime as currently described by general relativity and quantum field theory can be recovered. While Minkowski spacetime can be reconstructed from the space of null twistors, as well as a limited number of field theories, it turns out that no consistent twistor descriptions have been given for massive fields, or for field theories in generally curved spacetimes with matter content. In general, only conformally invariant field theory, and those general relativistic spacetimes that are conformally flat, can be completely recovered in the twistor formalism (see, e.g., Bain, 2006, pp. 45–46 for further discussion). As in the examples of superfluid Helium, one might thus question the relevance that the twistor formulation of the 4-dim QHE has to the ontological status of spacetime. On the other hand, just as with the Helium examples, this twistor example can be viewed as an approach to quantum gravity, and for this reason should be given due consideration.

With this in mind, we may ask what the QH liquid example suggests about the ontological status of spacetime. Taken literally, it suggests that we award fundamental ontological status to a 4-spatial-dimensional quantum Hall liquid. Twistors are then identified as low-energy excitations of the 3-spatial-dimensional edge of this liquid. We then apply the standard practice (and envisioned extensions) of twistor theory to these low-energy excitations to reconstruct spacetime and its contents. On first blush, this interpretation is similar to the superfluid Helium examples in Section 3.3, with twistor theory simply seen as the method for reproducing the relevant physics in the case where the condensed matter system is a QH liquid. Seen in this light, the QH liquid example might be thought to fit within the bounds of Section 3.3's condensed matter approach to quantum gravity. However, the fit is not exact, and consequently how a literal interpretation of the QH liquid example might be further qualified in terms of relationalist and substantivalist options is a bit more nuanced than the superfluid Helium examples. In particular, there are three main differences between the QH liquid example and the superfluid Helium examples.

1. Note first that in the QH liquid example, there is a distinction between the bulk liquid and its edge. Again, spacetime and relativistic field theory are in-

$$\underbrace{\textit{normal state}}_{\textit{Galilei-invariant theory}} \xrightarrow[\textit{transition}]{\textit{phase}} \underbrace{\textit{QH liquid} \rightarrow \textit{edge} \xrightarrow[\textit{to zero}]{\textit{lower energy}} \textit{ground state}}_{\textit{4-dim topological theory, restricted to 3-dim at the edge}}$$

$$\xrightarrow[\textit{slightly!}]{\textit{raise energy}} \underbrace{\textit{relativistic system}}_{\textit{Lorentz-invariant theory}}$$

FIGURE 16.2 The relation between theories for a 4-dim quantum Hall liquid.

terpreted as properties, or constructs, of low-energy excitations of the edge (i.e., properties or constructs of twistors), and not of the bulk liquid itself.

2. Second, unlike the superfluid Helium examples, the QH liquid example is not background dependent, at least under one sense of the term. Technically, the theory of a QH liquid is a topological quantum field theory involving a Chern–Simons gauge field.[33] In such a theory, the spacetime metric does not explicitly appear in the term describing the Chern–Simons field (as it does in the Maxwell term in electrodynamics, for instance). Hence the Chern–Simons field does not obey the symmetries of the spacetime metric. Thus, to the extent that background dependence of a theory entails invariance of the theory under the symmetries associated with a particular spatiotemporal structure as encoded in a metric (or set of metrics as in the Galilei case), the theory of a QH liquid is not background dependent. Intuitively, there is no prior *metrical* geometric structure associated with the theory (although there *is* topological/differentiable structure).

3. A third way in which the QH liquid example differs from the superfluid Helium examples concerns the number of theories involved. In the superfluid Helium example, a single theory describes both the normal liquid and the condensate, and this theory is formally distinct from the low-energy EFT (see Figure 16.1). In the QH liquid example, it turns out that the normal state and the condensate are described by different theories, both of which are distinct from the low-energy EFT of the edge (see Figure 16.2). Briefly, the normal liquid is described by a Galilei-invariant theory of electrons moving in a 4-dim conductor, the QH liquid is described by a 4-dim topological theory, and the low-energy EFT of the edge is, in the first instance, a $(3+1)$-dim Lorentz-invariant theory of massless fields of all helicities.[34]

With these qualifications in mind, one can now imagine relationalist and substantivalist interpretations of the QH liquid example. Relationalists should award ontological status just to the QH liquid and may claim:

(1) Physical fields are properties or constructs of low-energy excitations of the edge of the QH liquid.

[33] For the Chern–Simons theory of a 2-dim QH liquid, see Zhang (1992). For the Chern–Simons theory of a 4-dim QH liquid, see Bernevig et al. (2002).

[34] This difference between the two examples is due to the nature of their phase transitions. In the superfluid Helium case, the phase transition is between systems that possess different (internal) symmetries and is characterized by a broken symmetry. In the QH liquid case, the phase transition is between systems that possess different topological orders and is not characterized by a broken symmetry. For a discussion of the notion of topological order, see Wen (2004, Ch. 8).

(2) Relativistic spacetime structure consists of properties or constructs of low-energy excitations of the edge of the QH liquid.

Unlike the superfluid Helium examples, there is no need to further qualify Claim (2), given the convictions of the twistor theorist about the status of relativistic spacetime; i.e., that it's best modeled by twistors, and not by quantum field theory or general relativity.

Substantivalists should award ontological status to both the QH liquid and spacetime. If a substantivalist seeks to ontologically ground the fields that appear in the theory of a QH liquid, she may reify the 4-spatial-dimensional space associated with the liquid. Before the thermodynamic limit is taken, this is a 4-sphere (conceived, not as a metric space, but as a differentiable manifold). Thus a conservative substantivalist might adopt the relationalist's Claims (1) and (2) and add

(3) Spacetime consists of the properties of a substantival differentiable manifold diffeomorphic to the 4-sphere.

An intrepid substantivalist might adopt Claims (1) and (3), qualifying the latter with a restriction to the appropriate energy scale, and replace (2) with

(2′) Relativistic spacetime structure consists of the properties of a low-energy emergent substantival spacetime.

As in the superfluid Helium examples, this would require fleshing out a notion of low-energy emergence. Note that there is still a distinction between the low-energy relativistic EFT of the edge, and the topological theory of the ground state of the edge (see Figure 16.2); hence low-energy emergence might still be considered as a relation between distinct theories. However, the work done by this concept for an intrepid substantivalist in the QH liquid case will be a bit different from the superfluid Helium examples.

Note first that the reasoning in Section 3.3 concerning the typical motivations for substantivalism applies in the QH liquid example as well: The motivation from fields has the potential to do work, whereas that from inertial motion does not. In the case of an intrepid substantivalist in the superfluid Helium examples, the motivation from fields has to be supplemented with an account of low-energy emergence that allows enough of an ontological distinction between emergent fields and spacetime on the one hand, and the underlying condensate on the other to justify the intrepid's claim that (emergent) fields require the existence of (emergent) spacetime for their ontological support. Moreover, low-energy emergence in this context is associated with the low-energy approximation procedure applied directly to the (theory of the) condensate. In the QH liquid example, there is an extra layer of theoretical structure between the condensate and the emergent fields and spacetime; namely, twistors. Thus the emergence associated with spacetime in Claim (2′) above will have to be predicated on the twistor methods that produce spacetime, and at most, only indirectly on the low-energy approximation methods that produce twistors. Thus, again, the intrepid substantivalist has her work cut out for her.

5. CONCLUSION

Interpreting spacetime as a phenomenon that emerges in the low-energy limit of a quantum liquid is problematic for two reasons. First, it depends on the viability of condensed matter analogues of spacetime, and this was seen to be limited in the examples canvassed in this essay. These limitations manifest themselves in a failure to reproduce all aspects of the appropriate physics. For instance, an interpretation of spacetime as emergent in superfluid Helium 4 might be motivated by a desire to model spacetime as (some aspect of) the solutions to the Einstein equations in general relativity. In Section 3.1, we saw that the effective Lagrangian for superfluid Helium 4 lacks both the dynamics associated with general relativity and, arguably, the kinematics. An interpretation of spacetime as emergent in superfluid Helium 3-A might be motivated by a desire to model spacetime as the ground state for quantum field theories of matter, gauge, and metric fields. In Section 3.2, we saw that, while the effective Lagrangian for superfluid ^{3}He-A does reproduce aspects of the Standard Model, it does not reproduce all aspects; nor does it fully recover general relativity. Finally, an interpretation of spacetime as emergent from the edge of a 4-dimensional quantum Hall liquid might be motivated by a desire to derive spacetime using twistor-theoretic techniques. Here the prospects as noted in Section 4.4 are limited primarily by the limitations of twistor theory: Twistor formulations of general solutions to the Einstein equations, and massive interacting quantum fields, have yet to be constructed.

The second way in which interpretations of spacetime as a low-energy emergent phenomenon are problematic has to do with the notion of low-energy emergence itself; in particular, any such interpretation must provide an account of what low-energy emergence is in the condensed matter context. Section 3.3 offered some initial suggestions, however a full account will require significant work. Moreover, we saw in Sections 3.3 and 4.4 that any such notion by itself is compatible with both relationalism and substantivalism. For a relationalist, it would underlie the claim that spatiotemporal structure consists in the spatiotemporal properties of low-energy emergent physical fields; for a substantivalist, it would underlie the claim that spatiotemporal structure consists in the properties of an emergent substantival spacetime. While this latter view might be the most literal way to conceive spacetime as a low-energy emergent phenomenon, arguably it is the hardest to motivate, as Sections 3.3 and 4.4 indicated.

These results suggest that currently an interpretation of spacetime as a low-energy emergent phenomenon cannot be fully justified. However, this essay also argued that such an interpretation should nevertheless still be of interest to philosophers of spacetime. Each of the examples above may be considered part of a general research programme in condensed matter physics; namely, to determine the appropriate condensed matter system that produces the relevant matter, gauge and metric fields of current physics in its low-energy approximation, thus reconciling quantum field theory with general relativity. This research programme may be seen as one path to quantum gravity in competition, for instance, with the background-independent canonical loop approach, and background-dependent approaches like string theory. Thus to the extent that philosophers of spacetime should consider notions of spacetime associated with approaches to quantum gravity, they should be willing to consider low-energy emergentist interpretations of spacetime.

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