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Spacetime as a quantum error-correcting code?

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ABSTRACT

This essay considers an interpretation of the AdS/CFT correspondence under which the bulk and the boundary emerge from a more fundamental discrete system that realizes the structure of an erasure-protection quantum error-correcting code (QECC). I consider the extent to which this view underwrites the claim that spacetime is a QECC (as some authors have suggested), and how it fits into recent schemes of realist interpretations of dualities.

1. Introduction

The AdS/CFT correspondence is a dictionary that relates a $(d+1)$-dimensional bulk theory of gravity in anti-de Sitter spacetime (AdS) to a d-dimensional boundary conformal field theory (CFT). This correspondence is an example of a duality between a theory that is only partially constructed (AdS gravity), and a theory for which the full mathematical structure is currently available (CFT).[1] It is thus significant as a means toward a quantum theory of gravity, or minimally, toward a reconciliation between general relativity and quantum field theory. Philosophers of physics have not been lax in analyzing the significance of this duality; in particular, how it might admit a realist interpretation (Le Bihan & Read, 2018), and how the relation between the bulk theory and the boundary theory might be understood in terms of emergence (De Haro, 2017; De Haro, Mayerson, & Butterfield, 2016; Dieks, van Dongen, & de Haro, 2015; Rickles, 2013; Teh, 2013; Vistarini, 2017).

Recently a proposal to interpret the AdS/CFT correspondence as an erasure-protection quantum error-correcting code (QECC) has generated interest (Almheiri, Dong, & Harlow, 2015; Pastawski, Yoshida, Harlow, & Preskill, 2015; Harlow, 2018; Wolchover, 2019). An erasure-protection QECC is a procedure for encoding information in the elements of a subspace of a multi-qudit Hilbert space in such a way that errors due to erasure can be detected and corrected.[2] The proposal has elicited the informal claim that "spacetime is a QECC" (Preskill, 2017; Wolchover, 2019). The goal of the current essay is to assess this claim: What could it mean to say spacetime is a QECC, and to what extent is this justified by the QECC interpretation of the AdS/CFT correspondence?

The QECC interpretation was presented in Almheiri et al. (2015) as a solution to an apparent problem with the AdS/CFT dictionary referred to as the "bulk locality paradox": under reasonable assumptions about locality, a standard way of representing a local bulk field on the boundary, called the AdS-Rindler representation, is trivial.[3] This triviality, together with the redundant nature of the relation between d-dim boundary operators and $(d+1)$-dim bulk fields, is suggestive of characteristics of a QECC. Under Almheiri et al.'s proposal, the collection of bulk states with support in a bulk region called a causal wedge consists of a subset of boundary states on a corresponding boundary region that forms an erasure-protection QECC, and this means that these bulk states encode the information associated with the corresponding boundary states in a redundant way that protects it against erasure.

A QECC is not the sort of thing associated with a spacetime. Thus informal claims to the effect that "spacetime is a QECC" appear perplexing. A QECC can be realized by physical systems which can possess spatiotemporal properties. In the QECC interpretation of AdS/CFT, these physical systems are described by the boundary CFT, and there is a common claim in the physics literature that the bulk emerges from the boundary. So perhaps the claim that spacetime is a QECC is better understood as the claim that spacetime in the bulk emerges from boundary systems that realize the structure of a QECC. On the other hand, many authors have observed that the

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[1] As Le Bihan and Read (2018, pg. 6) point out "… our understanding of the AdS side of the AdS/CFT duality is inherently perturbative …", as opposed to a mathematically exact understanding of the CFT side.

[2] A qudit is a state of a d-state quantum system. The $d = 2$ case is called a qubit.

[3] Note that this motivation for the QECC interpretation of AdS/CFT is independent of concerns that stem from the black hole information loss paradox. In that context, AdS/CFT has been cited as evidence that black hole evaporation is a unitary process (see, e.g., Wallace, 2018, p. 17).

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duality between the boundary and the bulk is exact and thus symmetrical; hence to the extent that a notion of emergence requires an asymmetrical relation between a fundamental physical system and an emergent physical system, it cannot be underwritten solely by duality (Teh, 2013; Dieks et al. 2015, De Haro et al., 2016a; De Haro, 2017).

In what follows, I will suggest that this concern can be addressed by adopting an emergentist view of Pastawski et al.'s (2015) realization of the QECC interpretation in the form of a discrete system of qudits on a negatively curved lattice, what they refer to as the "HaPPY code" (after the authors' initials). One can imagine taking a continuum limit of this discrete system to recover the AdS/CFT correspondence, and the claim then is that such a continuum limit suffices to underwrite a notion of emergence. Thus under what I will call the HaPPY code interpretation of AdS/CFT, both the boundary spacetime and the bulk spacetime emerge in a continuum limit from a fundamental discrete lattice system that realizes the structure of an erasure-protection QECC. Under this view, a more accurate version of the claim "spacetime is a QECC" might be "spacetime emerges from a QECC".

This emergentist understanding of the relation between the HaPPY code lattice system and the physical systems described by the AdS/CFT correspondence is not explicitly considered by Pastawski et al. (2015), and it arguably represents a novel way of interpreting AdS/CFT, at least with respect to a number of interpretive options that have recently been discussed in the philosophy of physics literature (Le Bihan & Read, 2018).

The plan of the essay is as follows. In section 2, I describe the bulk locality paradox, and in section 3, I describe the QECC interpretation as an attempt to address it. In section 4, I discuss Pastawski et al.'s (2015) realization of the QECC interpretation by the HaPPY code. In section 5, I consider two options for understanding the claim "spacetime is a QECC". The first is a proposal by Van Raamsdonk (2010) under which the degree of entanglement of boundary states tracks the degree of connectedness of the bulk spacetime. The second is the HaPPY code interpretation, under which the spacetime associated ^gwith the AdS/CFT correspondence emerges from a discrete system that realizes a QECC. Ultimately, I will argue that the latter is the best way of understanding the claim that "spacetime is a QECC". Finally, I indicate the sense in which the HaPPY code interpretation is distinct from two realist interpretive options discussed by Le Bihan and Read (2018) referred to as "common core" and "overarching theory".

2. The bulk locality paradox

The QECC interpretation of the AdS/CFT correspondence is a response to the bulk locality paradox. This stems from the general problem of representing local bulk degrees of freedom by local boundary degrees of freedom. The context is as follows: The AdS/CFT dictionary, as typically presented, provides a means of using bulk quantities to calculate vacuum expectation values of products of boundary operators, referred to as correlation functions. These are typically the observables of interest in a local quantum field theory. However, some authors have expressed dissatisfaction with this version of the dictionary, insofar as some bulk observables of interest cannot be represented by correlation functions on the boundary. This has motivated a "reconstruction" program that attempts to represent a local bulk field in its entirety on the boundary (as opposed to only those bulk elements that go into calculating boundary correlation functions). The bulk locality paradox is a problem associated with the standard approach to this reconstruction program, as the remainder of this section attempts to explain.

The AdS/CFT dictionary comes in two versions: a "differentiate" version and an "extrapolate" version (see, e.g., Harlow, 2016, p. 33; Harlow & Stanford, 2011). The purpose of both versions is to express boundary correlation functions in terms of bulk quantities. In the differentiate version (Gubser, Klebanov, & Polyakov, 1998; Witten, 1998), one identifies the partition functions of the bulk and boundary theories in the following way:

$$Z_{bulk}[\phi_0] = Z_{CFT}[\phi_0] \tag{1}$$

where, on the left, ϕ_0 is the boundary value of a bulk field $\phi(r,x)$, while, on the right, it is a source term that couples to a local boundary operator $\mathscr{O}(x)$.[4] In the path integral formulation of quantum field theory, correlation functions are calculated by taking functional derivatives with respect to source terms of the partition function. In particular, given (1), boundary correlation functions can be expressed by

$$big\langle \mathscr{O}(x_1)\ldots\mathscr{O}(x_n)\rangle_{CFT} = \left[\frac{\delta}{\delta\phi_0(x_1)}\cdots\frac{\delta}{\delta\phi_0(x_n)}Z_{bulk}[\phi_0]\right]_{\phi_0=0} \tag{2}$$

Thus (1) provides a "differentiate" method of calculating boundary correlation functions in terms of bulk quantities, and this is important insofar as correlation functions are typically the observables of interest in a local quantum field theory. Indeed, this differentiate version of the AdS/CFT dictionary is stressed by much of the recent philosophy of physics literature (see, e.g., De Haro, 2017; De Haro et al., 2016a; De Haro et al., 2016b; De Haro et al., 2017; Dieks et al. 2015; Teh, 2013).

An alternative version of the AdS/CFT dictionary is referred to as the "extrapolate" version, and it is based on identifying a local boundary operator with the boundary value of a bulk field, given by the following limit,

$$\lim_{r\to\infty} r^{\Delta}\phi(r,x) = \mathscr{O}(x) \tag{3}$$

where Δ is the scaling dimension of $\mathscr{O}$ (Banks, Douglas, Horowitz, & Martinec, 1998). This identification can then be used to calculate boundary correlation functions by "extrapolating" bulk correlation functions to the boundary:

$$\langle \mathscr{O}(x_1)\ldots\mathscr{O}(x_n)\rangle_{CFT} = \lim_{r\to\infty} r^{n\Delta}\langle\phi(r,x_1)\ldots\phi(r,x_n)\rangle_{bulk} \tag{4}$$

Harlow and Stanford (2011) showed that (2) and (4) are equivalent methods for calculating boundary correlation functions. Moreover, Harlow (2018) suggests that (4) can be thought of as analogous to the LSZ formula that is used in quantum field theory to describe the results of scattering experiments:

> The dictionary we have developed so far is sufficient for discussing 'scattering' experiments where we act with boundary local operators at some early time, wait for a while, and then measure some boundary operators to see what comes out (Harlow, 2018, p. 8).

On the other hand Harlow goes on to suggest that this analogy has limitations:

> Unfortunately it does not seem to be the case that all interesting experiments in the bulk can be so easily related to local correlation functions in the CFT. The problem is that the output of an experiment which happens behind a horizon is by definition

[4] The bulk field is a function of bulk global coordinates (x,r). In these coordinates, the boundary is at $r\to\infty$.

unable to reach the boundary, at least not if the semiclassical picture of the spacetime is correct (Harlow, 2018, p. 12).

Insofar as local bulk fields deep within the bulk, and bulk observables behind, say, the event horizon of a bulk black hole are interesting, the differentiate and extrapolate dictionaries are of limited use. This is an important observation. A number of authors have recently advocated a "common core" interpretation of the AdS/CFT correspondence, and I will suggest in Section 5 below that this has been informed, in part, by the differentiate version of the dictionary.[5] The QECC interpretation, on the other hand, is motivated by taking these limitations seriously, and, as we'll see in Sections 3.2 and 5, it speaks against a "common core" understanding of AdS/CFT duality.

Taking the limitations of (2) and (4) seriously suggests that "… we'd like to back off of the extrapolate dictionary" (Harlow, 2018, p. 12). Backing off leads to a research program referred to as "reconstruction", the goal of which is to construct a boundary representation of a bulk field located *anywhere* in the bulk. This involves solving the bulk equations of motion with the boundary condition given by the extrapolate dictionary (3), and results in what is called the AdS-Rindler representation of a bulk field, defined in the following way. Let Σ be the intersection on the boundary with a bulk time slice, and let R be a subregion of Σ. Now define the "causal wedge" $C[R]$ of R by,

$$C[R] = J^{+}_{bulk}[D_{bnd}[R]] \cap J^{-}_{bulk}[D_{bnd}[R]] \tag{5}$$

where $D_{bnd}[R]$ is the domain of dependence, in the boundary, of R (i.e., the set of boundary points X such that any inextendible causal curve through X intersects R), and J^{+}_{bulk} and J^{-}_{bulk} are the causal future and causal past in the bulk.[6] If one takes the domain of dependence of R to be the set of points that are causally determined by R, then $C[R]$ is the set of bulk points that are causally accessible from the boundary region that is causally determined by R. The AdS-Rindler representation $O_{(\phi;R)}$ of a bulk field $\phi(x)$ localized in $C[R]$ is then given by smearing a local boundary operator $O(X)$ at boundary point X over $D_{bnd}[R]$. Formally,

$$\phi(x)|_{x\in C[R]} = \int_{D_{bnd}[R]} K(x;X)O(X)dX \equiv O_{(\phi;R)} \tag{6}$$

where $K(x;X)$ is a smearing function that depends on the type of bulk field. The AdS-Rindler representation (6) plays a central role in the following.

Causal Wedge Reconstruction Conjecture: For any boundary spatial region R, any bulk field in $C[R]$ can be represented by a CFT operator with support on R.[7]

The Causal Wedge Reconstruction Conjecture adds a powerful entry to the AdS/CFT dictionary. Again, in principle, it is meant to address concerns with the differentiate and extrapolate dictionaries: it is supposed to provide the tools necessary to extend the AdS/CFT correspondence to "interesting" bulk observables deep within the bulk. However, it faces the following complication (Almheiri et al. 2015, p. 6; Pastawski et al. 2015, p. 18; Harlow, 2018, p. 22):

Claim: Under reasonable assumptions about locality in the bulk and on the boundary, the AdS-Rindler representation of a bulk field must be a multiple of the identity.

A sketch of the proof is as follows: First, one can show that for any local bulk field $\phi(x)$ and any local boundary operator $O(Y)$ on the same time slice as $\phi(x)$, there is an AdS-Rindler representation $O_{(\phi;R)}$ of $\phi(x)$, such that $O(Y)$ lies in the complement $\overline{R}$ of R on the boundary time slice.[8] One implication of this is that if we assume local commutativity holds for the CFT, then, since points in R and its complement are spacelike separated, $O_{(\phi;R)}$ and $O(Y)$ must commute.[9] Another implication is that a bulk field admits more than one AdS-Rindler representation $O_{(\phi;R)}, O_{(\phi;R')}, \ldots$. In other words, a given bulk field can lie in more than one causal wedge, each associated with a different boundary subregion $R, R', \ldots$. Moreover, by the previous implication, each of these representations commutes with some arbitrary boundary operator on the same time slice. Now suppose all the AdS-Rindler representations of $\phi(x)$ associated with different boundary subregions are equivalent: $O_{(\phi;R)} = O_{(\phi;R')} = \ldots$; i.e., suppose for any given bulk field, there is a unique AdS-Rindler representation. Then it should commute with all local CFT operators on the same time slice. This is problematic: according to the time slice axiom for a local quantum field theory, the algebra of observables generated by operators within an arbitrarily small time slice coincides with the algebra of all observables. The latter acts irreducibly on the theory's Hilbert space of states, and Schur's Lemma entails that any operator that commutes with all irreducible representations of an algebra is a multiple of the identity. The upshot then is that the following assumptions entail that the AdS-Rindler representation of a bulk field must be a multiple of the identity:

(a) The boundary CFT obeys local commutativity (alternatively, the bulk theory obeys "radial" local commutativity).
(b) The boundary CFT obeys the time slice axiom.
(c) The AdS-Rindler representation of a bulk field is unique.

Thus, if we've adopted the AdS-Rindler representation as a way of adding interesting bulk observables (that cannot be computed using correlation functions) to the AdS/CFT dictionary, then we're in trouble. Unless, that is, we can come up with a way of understanding how uniqueness (c) might fail.[10] Various authors suggest uniqueness is a reasonable assumption to make. Almheiri et al. (2015, pg. 7) indicate that "… in the bulk theory it seems that the [representations] are equivalent"; the intuition being that they all represent the same bulk field. Harlow (2018, pg. 22) concurs: "From the bulk side we expect these different representations to be equivalent, technically since they are just different Bogoliubov representations of the same field …". Almheiri et al.'s QECC

[5] These authors include Vistarini (2017), De Haro (2019), and De Haro and Butterfield (2018, 2019). On the other hand, some authors who take the differentiate dictionary to be the statement of AdS/CFT duality do acknowledge its limitations (e.g., De Haro, 2019; Teh, 2013).

[6] See, e.g., Harlow (2018, pg. 20). A causal wedge is a slight generalization of an AdS-Rindler wedge, which is a region in AdS spacetime that corresponds to a Rindler wedge in Minkowski spacetime. In particular, the bulk region of a causal wedge is isomorphic to an AdS-Rindler wedge (Morrison, 2014, p. 12).

[7] See, e.g., Harlow (2018, pg. 21). The intuition underlying the conjecture is that as long as a bulk field lies in $C[R]$, it has an AdS-Rindler representation using Heisenberg boundary operators with support on $D_{bnd}[R]$, and these can be evolved to Schrödinger boundary operators with support on R (Harlow, 2018, p. 19).

[8] Form an open neighborhood S_Y surrounding the boundary point Y, and then take R to be the complement of S_Y on the boundary time slice containing Y. For S_Y sufficiently small, it will be the case that the bulk point x lies in $C[R]$; hence an AdS-Rindler representation of $\phi(x)$ can be constructed.

[9] This boundary locality assumption is made by Almheiri et al. (2015, pg. 6). Alternatively Harlow (2018, pg. 21) assumes "radial locality" for the bulk field; i.e., commutativity follows from x and Y being spacelike separated.

[10] Alternatively, we might drop the locality assumptions (a) and (b), but locality of the CFT is one of the essential characteristics of the boundary theory that is supposed to inform us, under the AdS/CFT Correspondence, about the mysterious bulk.

interpretation is intended to address these concerns: it is an attempt to understand the non-uniqueness of the AdS-Rindler representation of a bulk field in terms of the redundancy of an erasure-protection QECC.

3. Resolution: the boundary as a QECC

Suppose we want to encode k "logical" qudits in $n > k$ "physical" qudits in such a way that we can recover the former if we only have access to some set R of $m < n$ of the latter; suppose, in particular, that the $(n-m)$ qudits in the complement $\overline{R}$ of R are erased. The goal of an erasure-protection QECC is to devise an encoding that is sufficiently redundant to protect against erasure. To accomplish this, one first defines the physical Hilbert space as an n-qudit product space $\mathscr{H}^{(n)} = \mathscr{H}_R^{(m)} \otimes \mathscr{H}_{\overline{R}}^{(n-m)}$, where $\mathscr{H}_R^{(m)}$ and $\mathscr{H}_{\overline{R}}^{(n-m)}$ are m-qudit and $(n-m)$-qudit Hilbert spaces of qudits with support in R and $\overline{R}$, respectively. The logical qudits are then encoded in the physical qudit elements of a subspace $\mathscr{H}_C \subset \mathscr{H}^{(n)}$, called the "codespace". One can show that an erasure-protection encoding is possible if and only if the set R can be decomposed into sets R_1 and R_2, with k and $(m-k)$ qudits, respectively, in such a way that

$$|\tilde{i}\rangle = U_R\left(|i\rangle_{R_1} \otimes |\chi\rangle_{R_2\overline{R}}\right) \tag{7}$$

where $|\tilde{i}\rangle$ is an n-qudit basis state of $\mathscr{H}_C$, $|i\rangle_{R_1}$ is a k-qudit state on R_1, $|\chi\rangle_{R_2\overline{R}}$ is an $(n-k)$-qudit state on $R_2 \cup \overline{R}$, and U_R is a unitary transformation that acts nontrivially only on R (Harlow, 2018, p. 29).[11] One can further show that the existence of an encoding (7) is equivalent to all of the following conditions (Almheiri et al. 2015, pp. 1011, 13):

(a) QECC Condition. Within $\mathscr{H}_C$, any $(n-m)$-qudit operator $O_{\overline{R}}$ with support on $\overline{R}$ is a multiple of the identity:

$$\langle\tilde{i}|O_{\overline{R}}|\tilde{j}\rangle = c\delta_{ij}, \qquad \text{for constant } c.$$

(b) Erasure-Protection Condition. Any n-qudit operator O that acts on $\mathscr{H}_C$ (called an "encoded logical operator") can be represented by an m-qudit operator $O_R = U_R O_{R_1} U_R^\dagger$ with support on R, in the sense that:

$$O_R|\tilde{\psi}\rangle = O|\tilde{\psi}\rangle, \qquad O_R^\dagger|\tilde{\psi}\rangle = O^\dagger|\tilde{\psi}\rangle, \qquad \text{for all } |\tilde{\psi}\rangle \in \mathscr{H}_C$$

(c) Within $\mathscr{H}_C$, any n-qudit encoded logical operator O commutes with all $(n-m)$-qudit operators $O_{\overline{R}}$ with support on $\overline{R}$:

$$\langle\tilde{i}|[O, O_{\overline{R}}]|\tilde{j}\rangle = 0.$$

Condition (a) is typically referred to as the QECC condition in the more general context in which errors are not necessarily due to erasure.[12] In the general case, a QECC protects against errors if and only if any local operator that detects and corrects an error acts as a multiple of the identity on the codespace. Condition (b) captures the sense in which an erasure-protection QECC protects information in a redundant way: the information associated with m-qudit observables (with support on R) is encoded redundantly in $\mathscr{H}_C$ in n-qudit observables, where $n > m$. This protects against the erasure of up to $(n-m)$ qudits. Finally, condition (c) is a further characterization of an erasure-protection QECC that, as we'll now see, helps to clarify the bulk locality paradox.

First note that Condition (a) is suggestive of the claim underwriting the bulk locality paradox that the AdS-Rindler representation of a bulk field is a multiple of the identity. Condition (b) is suggestive of the Causal Wedge Reconstruction Conjecture; i.e., the claim that any bulk field in the causal wedge of a boundary spatial subregion R can be expressed by a boundary operator on R. These suggestions can be made more explicit in the following way: Suppose the boundary CFT is characterized by a product Hilbert space $\mathscr{H} = \mathscr{H}_R \otimes \mathscr{H}_{\overline{R}}$, where R is a subregion of a boundary time slice (as in the AdS-Rindler representation), and $\mathscr{H}_R$ and $\mathscr{H}_{\overline{R}}$ consist of states of boundary subsystems localized in R and its complement $\overline{R}$. Suppose further that the states of bulk subsystems localized in the causal wedge $C[R]$ of R form a subspace $\mathscr{H}_C \subset \mathscr{H}$ of boundary states. An operator O that acts on $\mathscr{H}_C$ then corresponds to a bulk field operator with support in $C[R]$, and the Causal Wedge Reconstruction Conjecture entails the bulk field operator can be represented by a boundary operator with support on R, which is a statement of the Erasure-Protection Condition (b). This is equivalent to Condition (c), which can now be interpreted as stating that the bulk field operator O that admits an AdS-Rindler representation O_R with support on R commutes with all local boundary operators $O_{\overline{R}}$ with support on $\overline{R}$, when restricted to the causal wedge $C[R]$. This addresses the bulk locality paradox in the sense that the AdS-Rindler boundary representation of a local bulk field only commutes with all local boundary operators with support on the codespace, and this does not entail that it must be trivial when acting on other states outside the codespace. Finally, we can now appeal to the QECC Condition (a) to understand the limited sense in which locality in the bulk and on the boundary entails triviality of certain boundary operators, and only when acting on the code subspace. In particular, boundary operators $O_{\overline{R}}$ with support on $\overline{R}$ act on code states like multiples of the identity.

To recap so far, Almheiri et al.'s QECC interpretation of the AdS/CFT correspondence can be thought of as a dictionary between structural aspects of an erasure-protection QECC and structural aspects of the AdS/CFT correspondence.[13] According to this QECC dictionary, for every decomposition $\mathscr{H} = \mathscr{H}_R \otimes \mathscr{H}_{\overline{R}}$ of the boundary CFT Hilbert space, where R is a boundary spatial subregion with complement $\overline{R}$, there is a collection of bulk states of subsystems localized in the causal wedge $C[R]$ of R that forms a subspace $\mathscr{H}_C \subset \mathscr{H}$, and a collection of local bulk fields with support on $C[R]$ that act on $\mathscr{H}_C$. The subspace $\mathscr{H}_C$ forms an erasure-protection QECC codespace that encodes boundary states in a redundant way that protects them against erasure. More precisely, the CFT Hilbert space represents a "physical" Hilbert space of qudits, and the CFT states in the "codespace" $\mathscr{H}_C$ represent "encoded logical qudits", these corresponding to bulk states in a causal wedge. The "logical" qudits that are encoded in the bulk states in $C[R]$ evidently are boundary degrees of freedom associated with a subset (indicated by R_1 above) of the boundary region R. Thus, according to the QECC interpretation, the bulk AdS theory of gravity is a way of encoding boundary CFT degrees of freedom localized in a boundary

[11] In expression (7), $i = 0, \ldots, d-1$, where d is the dimension of the qudits. An example of (7) is the three qutrit code for which $d = 3$, $k = 1$, $n = 3$, and $m = 2$ (Harlow, 2018, p. 24; Almheiri et al. 2015, p. 7). In this code, a single logical qutrit is encoded in three physical qutrits in such a way that it can be recovered if access is limited to only two of the physical qutrits.

[12] See, e.g., Terhal (2015, pg. 317).

[13] One initial concern with this dictionary is that in order to identify the full Hilbert space of the boundary CFT with a physical qudit space of a QECC, the former has to be finite-dimensional. Harlow (2018, pg. 45) suggests this can be addressed by regulating the CFT at a high energy scale.

subregion R (or more precisely, R_1) against the erasure of boundary degrees of freedom localized in its complement $\overline{R}$.

Note that there are two senses of redundancy associated with the AdS-Rindler representation:

1. *Bulk redundancy.* A bulk field in a given causal wedge $C[R]$ redundantly represents a boundary operator with less degrees of freedom on R.
2. *Boundary redundancy.* A bulk field can be localized in different causal wedges $C[R]$, $C[R']$, *etc.*, and hence can be redundantly represented by different boundary operators on R, R', *etc.*

Under the QECC interpretation, bulk redundancy corresponds to the redundancy associated with a QECC in which encoded logical qudits redundantly represent logical qudits. Boundary redundancy is due to the non-uniqueness of the AdS-Rindler representation of a bulk field, and it is a type of redundancy over and above that of a QECC. Thus, it is important to keep these senses of redundancy distinct. One way some authors have sought to understand the second sense is by viewing the codespace as a space of low-energy CFT states (Almheiri et al. 2015, p. 19). The action on this codespace by a bulk field can be realized by distinct boundary CFT operators if the latter are interpreted as acting on high-energy CFT states differently.

3.1. The QECC interpretation and the Ryu-Takayanagi formula

There is another entry in the QECC dictionary that should be mentioned at this point, since, as we'll see in Section 5, it underwrites one attempt to understand the claim that "spacetime is a QECC". This entry is due to Harlow (2018, 2017) and takes the form of an equivalence between the Erasure-Protection Condition (b) and a finite-dimensional version of the Ryu-Takayanagi (RT) formula, the latter being a generalization of the Bekenstein-Hawking black hole entropy. To state this equivalence first requires an extension of the notion of a causal wedge to what is called an "entanglement wedge". Let R be a boundary spatial subregion as in section 2, let γ_R be the minimal bulk spatial hypersurface with the same boundary as R, and let H_R be the bulk hypersurface bounded by $\gamma_R \cup R$.[14] The entanglement wedge $W[R]$ of R is defined as the bulk domain of dependence of H_R [15]

$$W[R] = D_{bulk}[H_R] \tag{8}$$

For simple boundary regions R, one can show that $W[R]$ coincides with the causal wedge $C[R]$, but there can be regions for which $W[R]$ is larger than $C[R]$ (one example is when R is the union of two disjoint regions). Thus in general, $C[R] \subset W[R]$ and this suggests an extension of the Causal Wedge Reconstruction Conjecture to:

Entanglement Wedge Reconstruction Conjecture: For any boundary spatial region R, any bulk field in $W[R]$ can be represented by a CFT operator with support on R.

$W[R]$ is intimately related to the Ryu-Takayanagi (RT) formula. The latter states that, for a boundary spatial subregion R and the density operator ρ for a boundary CFT state, the von Neumann entropy of the reduced density operator ρ_R of a subsystem localized in R is given by

$$S(\rho_R) = \mathrm{tr}(\rho \mathscr{L}_R) + S(\rho_{H_R}) \tag{9}$$

where $\mathscr{L}_R \equiv \mathrm{Area}(\gamma_R)/4G$ is a bulk operator, G is the Newtonian gravitational constant, and ρ_{H_R} is the reduced density operator for a subsystem localized on H_R.[16] This formula has been proven to hold for the von Neumann algebra of observables on $W[R]$ (Harlow, 2018, p. 45). It can be understood in the following way: Let the boundary CFT system be decomposable into two subsystems with product Hilbert space $\mathscr{H} = \mathscr{H}_R \otimes \mathscr{H}_{\overline{R}}$. Then $S(\rho_R)$ is the entanglement entropy of the subsystem associated with $\mathscr{H}_R$.[17] The term $\mathrm{tr}(\rho \mathscr{L}_R)$ is the expectation value of the "area" operator $\mathscr{L}_R$ when the system is in the state ρ. The term $S(\rho_{H_R})$ is the entanglement entropy of ρ on the algebra of observables on $W[R]$. Thus the RT formula equates the entanglement entropy of a CFT subsystem on a boundary subregion R, to the entanglement entropy of the subsystem on the bulk subregion $W[R]$, plus a term that depends on the area of the boundary γ_R between the bulk region $W[R]$ and its bulk complement.

Harlow (2018, pg. 44; 2017, pg. 884) has shown that the Erasure-Protection Condition (b) of an erasure-protection QECC is equivalent to the following finite-dimensional version of the RT formula (9):

$$S(\tilde{\rho}_R) = \mathrm{tr}(\tilde{\rho} \mathscr{L}_R) + S(\tilde{\rho}) \tag{10}$$

where $\tilde{\rho}$ is a state on $\mathscr{H}_C \subset \mathscr{H}^{(n)} = \mathscr{H}_R^{(m)} \otimes \mathscr{H}_{\overline{R}}^{(n-m)}$, and $\mathscr{L}_R$ is an operator in the center of the (finite-dim) von Neumann algebra on $\mathscr{H}_C$.[18] This equivalence between a feature (9) of the AdS/CFT correspondence and a feature (10) of an erasure-protection QECC is an additional entry in the QECC dictionary.

According to Harlow (2018), this entry in the QECC dictionary establishes the Entanglement Wedge Reconstruction Conjecture in the following sense: The Entanglement Wedge Reconstruction Conjecture is underwritten by a correspondence between a subregion R of the boundary, and a subregion $W[R]$ of the bulk. Harlow (2018, pg. 38) refers to this correspondence as "subregion duality", and views the Erasure-Protection Condition (b) as a statement of it: "it says that all entanglement wedge operators can be represented in R". Thus:

> Since the Ryu-Takayanai formula has been independently established …, theorem (5.2) [i.e., the equivalence between the Erasure-Protection Condition (b) and (10)] establishes subregion duality in the entanglement wedge once and for all: we now know precisely which bulk subregion is dual to any boundary subregion. So far this is probably the biggest achievement of the quantum error correction perspective on holography (Harlow, 2018, p. 45.).

[14] γ_R is the smallest bulk hypersurface that separates R from its complement $\overline{R}$ (see, e.g., Jaksland, 2018, p. 10).

[15] The entanglement wedge was introduced in Headrick, Hubeny, Lawrence, and Rangamani (2014) where it got its name from its connection to the entanglement entropy associated with the boundary region R (as discussed below).

[16] See, e.g., Harlow (2018, pg. 40). This is a modification, due to Faulkner, Lewkowycz, and Maldacena (2013), of the original formula given by Ryu and Takayanagi (2006) that adds a "quantum correction" to the latter.

[17] Recall that for a bipartite system AB with density operator ρ_{AB}, the *entanglement entropy* S_A of subsystem A is defined to be the von Neumann entropy S_{vN} of ρ_A: $S_A \equiv S_{vN}(\rho_A) = \mathrm{tr}(\rho_A \log \rho_A)$. One can show that if $|\Psi\rangle$ is a pure state of AB, then it is entangled if and only if $S_{vN}(\rho_{AB}) - S_{vN}(\rho_A) < 0$ (Nielsen & Chuang, 2010; pg. 514). Thus, at least when a bipartite system is in a pure state, S_A is a measure of the degree to which its subsystems are entangled.

[18] Strictly speaking, Harlow proves a slightly stronger claim; namely, an equivalence between the Erasure-Protection Condition (b) applied to operators with support on both R and $\overline{R}$, on the one hand, and the finite-dim version of the RT entropy for reduced states $\tilde{\rho}$ and $\tilde{\rho}_{\overline{R}}$ on the other hand.

Table 1
QECC structure, the HaPPY code realization, and the AdS/CFT correspondence.

QECC structure	HaPPY Code	AdS/CFT
Physical Hilbert space $\mathscr{H}^{(n)} = \mathscr{H}_R^{(m)} \otimes \mathscr{H}_{\overline{R}}^{(n-m)}$.	Hilbert space $\mathscr{H} = \mathscr{H}_R \otimes \mathscr{H}_{\overline{R}}$, for boundary subregion R.	CFT Hilbert space $\mathscr{H} = \mathscr{H}_R \otimes \mathscr{H}_{\overline{R}}$, for boundary subregion R.
Codespace $\mathscr{H}_C \subset \mathscr{H}^{(n)}$.	Subspace $\mathscr{H}_C \subset \mathscr{H}$ of states in $\mathscr{E}[R]$.	Subspace $\mathscr{H}_C \subset \mathscr{H}$ of CFT states that represent bulk states in $W[R]$.
Erasure-Protection Condition: Any n-qudit operator on $\mathscr{H}_C$ can be expressed as an m-qudit operator on R.	Any operator on $\mathscr{E}[R]$ can be pushed out to an operator on R.	*Subregion Duality*: Any bulk field in $W[R]$ can be expressed as a CFT operator on R.
QECC Condition: Any $(n-m)$-qudit operator on $\overline{R}$ acts as a multiple of the identity on $\mathscr{H}_C$.	Any operator on $\overline{R}$ acts as a multiple of the identity on bulk states in $\mathscr{E}[R]$.	Any CFT operator on $\overline{R}$ acts as a multiple of the identity on bulk states in $W[R]$
Finite-dim RT Formula: $S(\tilde{\rho}_R) = \mathrm{tr}(\tilde{\rho}\mathscr{L}_R) + S(\tilde{\rho})$, for state $\tilde{\rho}$ on $\mathscr{H}_C$.	*Lattice RT Formula*: $S(\rho_R) = \lvert\gamma_R\rvert$.	*RT Formula*: $S(\rho_R) = \mathrm{tr}(\rho\mathscr{L}_R) + S(\rho_{H_R})$, for CFT state ρ.

3.2. Ontological implications

The first and third columns in Table 1 summarize the QECC interpretation of the AdS/CFT correspondence. Under this interpretation, the Hilbert space of the boundary CFT is interpreted as a physical qudit Hilbert space decomposable into two factor spaces comprised of the states of subsystems localized in a boundary spatial subregion R and its complement $\overline{R}$. The states of bulk systems in the entanglement wedge $W[R]$ of R form an erasure-protection QECC codespace $\mathscr{H}_C$ that satisfies the QECC Condition and the Erasure-Protection Condition: the former entails that any local CFT operator with support on $\overline{R}$ acts as a multiple of the identity on $\mathscr{H}_C$, and the latter entails that any local bulk operator with support on $W[R]$ can be expressed as a boundary CFT operator with support on R, and this is an expression of Harlow's (2018) subregion duality (or, equivalently, the Entanglement Wedge Reconstruction Conjecture). Finally, the additional "finite-dim RT Formula" property (9) of an erasure-protection QECC entails that the RT formula holds for the states of CFT boundary subsystems localized in R.

Under this QECC interpretation, the AdS/CFT correspondence is a correspondence between one aspect of the boundary CFT with another: the bulk theory describes a particular aspect of the boundary theory, insofar as bulk states are interpreted as a subset of boundary states. Thus the QECC interpretation appears to ontologically prioritize the boundary theory, in the sense that, under it, the bulk theory describes one aspect of the ontology of the boundary theory. In the terminology of Le Bihan and Read (2018, pg. 3), it thus appears that the QECC interpretation adopts the realist option of "discrimination".[19] Under this option, when faced with dual theories (i.e., the boundary CFT and the bulk gravity theory), the realist claims that only one describes a candidate for the actual world.

How, then, should we make sense of the claim that, according to the QECC interpretation, "spacetime is a QECC"? This claim has appeared in talks (e.g., Preskill, 2017) and in a recent popularized account of Almheiri et al. (2015) and Pastawski et al. (2015), in which these and other authors make comments to this effect (Wolchover, 2019). Note first that in the AdS/CFT correspondence, there are two spacetimes: the boundary spacetime (assumedly the conformal Lorentzian spacetime of the boundary CFT), and the bulk AdS spacetime. The general view of the AdS/CFT correspondence is that the spacetime of experience is the bulk, and it and bulk gravity can be understood in terms of the duality between the inexact bulk theory and the exact boundary CFT. Thus, assumedly, the spacetime identified with a QECC is the bulk AdS spacetime. But this seems problematic: As suggested above, under the QECC interpretation, the bulk theory does not describe a legitimate actual world candidate, and neither, in particular, does the bulk spacetime. The only other candidate spacetime that might be identified with a QECC is the boundary spacetime (a conformal Lorentzian manifold). But it still is unclear how a spacetime, regardless of which one, could literally be a quantum error-correcting code. The latter is a finite-dimensional Hilbert space of qudits characterized by a subspace that forms a codespace, and an encoding procedure that encodes qudits in the subspace in a way that protects them against errors. No reference to spatiotemporal properties is needed in this abstract description. Of course, a QECC can be realized as a concrete physical system that instantiates qudits, and a concrete physical system can possess spatiotemporal properties and hence be associated with a spacetime. I take it this is the intended sense of the claim that spacetime is a QECC; namely, the bulk spacetime in the AdS/CFT correspondence encodes the spatiotemporal properties of physical systems (bulk fields) whose states form a code subspace of an erasure-protection QECC of boundary states. But lingering concerns remain:

(i) The bulk spacetime itself is not a QECC; rather, the states of some physical systems that possess bulk spatiotemporal properties form a codespace for a QECC.
(ii) Moreover, the spacetime abstracted from those physical systems that realize a QECC is not identical to the entire bulk AdS spacetime; rather, it is the spacetime region defined by the entanglement wedge $W[R]$ in which the relevant physical systems are localized. So it is not the case that the entire bulk spacetime is a QECC. Rather, only certain regions in the bulk spacetime (entanglement wedges) can be associated with physical systems whose states form the codespace of a QECC.

Given the informal nature of the claim "spacetime is a QECC" (i.e., the informal venues in which it has been made), perhaps these concerns are not that pressing. On the other hand, there is a more precise way of understanding the meaning of this claim; namely, in terms of the entanglement structure of a QECC, on the one hand, and the entanglement structure of boundary CFT states, as encoded in the RT formula (9), on the other. I will examine this in Section 5 below, but before I do so, I'd like to consider a concrete realization of the QECC interpretation, as embodied in Pastawski et al.'s (2015) "HaPPY code". Ultimately, I will claim that a literal interpretation of the HaPPY code is the best route towards understanding the meaning of "spacetime is a QECC".

[19] Section 5 will argue that this is not the case for a variant of the QECC interpretation I refer to as the HaPPY code interpretation.

4. The HaPPY code realization

The HaPPY code is a realization of the QECC interpretation of the AdS/CFT correspondence discussed in Section 3.[20] In this section I outline its key features and how they relate to the QECC dictionary by explaining the entries in the second column of Table 1. I leave it to Section 5 to discuss how the HaPPY code might underwrite an understanding of the claim "spacetime is a QECC".

The HaPPY code makes use of yet another equivalent way of representing the encoding map of an erasure-protection QECC, this time in terms of what is called a "perfect" tensor. Form a k-indexed state $|\widetilde{j_1 \ldots j_k}\rangle$ as the tensor product of k codespace basis states defined by the encoding (7). Now take its inner product with the n-qudit basis state $|i_1 \ldots i_n\rangle$. This defines an $(n+k)$-indexed tensor:

$$\langle i_1 \ldots i_n | \widetilde{j_1 \ldots j_k}\rangle = T_{i_1 \ldots i_n j_1 \ldots j_k} \tag{11}$$

One can show that (11) encodes the unitary transformation U_R in (7). The general claim is that any erasure-protection encoding (7) defines a tensor (11); and *vice-versa*: any "perfect" tensor defines an erasure-protection encoding, where a perfect tensor is an even-indexed tensor such that any bipartition of its indices defines a unitary transformation.

The HaPPY code is a realization of an erasure-protection QECC that encodes one logical qudit in five physical qudits in such a way to protect the former against the erasure of any two of the latter. The corresponding perfect tensor thus has six indices. The realization takes the form of a uniform tiling of the 2-dim hyperbolic plane by pentagons, with four pentagons per vertex. On this negatively curved lattice, each face (pentagon) is associated with a 6-index perfect tensor with five indices contracted with indices of neighboring tensors and the sixth index free. The resulting tensor network is thus characterized by some number of uncontracted bulk indices, representing logical qudits, and some number of uncontracted boundary indices, representing physical qudits.[21] Pastawski et al. (2015) showed that this lattice system has the following properties:

(a) For any lattice boundary subregion R, there is a discrete version of the entanglement wedge of R in the lattice bulk, called the "greedy entanglement wedge" $\mathscr{E}[R]$.[22] Any bulk operator with support on $\mathscr{E}[R]$ can be pushed through the tensor network to the boundary subregion R.
(b) For any lattice boundary subregion R, there is a minimal length bulk cut γ_R, called a "greedy geodesic", with the same boundary as R.[23] A lattice RT formula holds for R and γ_R; namely, $S_R = |\gamma_R|$ where $|\gamma_R|$ is the length of γ_R.

The codespace of the HaPPY code QECC is provided by the greedy entanglement wedge $\mathscr{E}[R]$ of property (a), which also provides a lattice version of the Entanglement Wedge Reconstruction Conjecture. Property (b) establishes a lattice version of the RT formula. These relations are summarized in the second column of Table 1.

5. Spacetime as a QECC?

I'd now like to return to the suggestion, considered at the end of Section 3, that the QECC interpretation of the AdS/CFT correspondence entails that "spacetime is a QECC". On the surface, it's not clear what this could mean. In the QECC interpretation, the focus is on the state space of the boundary CFT. The claim is that this state space instantiates the structure of a QECC, with boundary states comprising physical states, and bulk states localized in the entanglement wedge of a given boundary subregion comprising encoded logical states in the codespace. Spacetime, either in the bulk or at the boundary, plays no explicit role in this structure. In this section, I will consider two options for understanding how spacetime might be associated with the QECC interpretation. The first involves a proposal made by Van Raamsdonk (2010) in which the degree of entanglement of boundary states tracks the degree of connectedness of the bulk spacetime. The link to the QECC interpretation is provided by the fact that elements of the codespace of a QECC are maximally entangled states with respect to a relevant decomposition of the physical state space. The second option is to interpret the HaPPY code literally as describing a fundamental discrete system of spins, from which both the boundary and the bulk emerge in an appropriate continuum limit. Ultimately, it is the latter option that I think best makes sense of the claim "spacetime is a QECC".

5.1. Boundary entanglement as bulk connectivity

Van Raamsdonk (2010) has suggested there is a relation between entanglement on the boundary and spacetime connectedness in the bulk. This is based on interpreting the RT formula (9) as encoding the connectivity property of a bulk spacetime in the entanglement entropy of a boundary state. Suppose the CFT Hilbert space can be decomposed as $\mathscr{H} = \mathscr{H}_R \otimes \mathscr{H}_{\overline{R}}$, for boundary spatial subregion R and complement $\overline{R}$. A CFT boundary state entangled over these regions can then be expressed as

$$|\Psi\rangle = \sum_{i,j} p_{ij} |\psi_i^R\rangle \otimes |\psi_j^{\overline{R}}\rangle \tag{12}$$

where $|\psi_i^R\rangle$ and $|\psi_j^{\overline{R}}\rangle$ are bases for $\mathscr{H}_R$ and $\mathscr{H}_{\overline{R}}$, and the p_{ij} are constants. Consider the connected bulk time slice that has boundary $R \cup \overline{R}$ and that consists of the union of the region H_R with its complement $\overline{H_R}$, separated by the minimal surface γ_R.[24] The RT formula (9) entails that as the entanglement entropy of the state (12) decreases, so does the area of γ_R, and in the limit as $S(\rho_R)$ goes

[20] The HaPPY code is named after its authors: Harlow, Pastawski, Preskill, and Yoshida (Pastawski et al. 2015).

[21] To demonstrate that this tensor network defines a 6-index perfect tensor, Pastawski et al. (2015, pg. 6) argue in the following way: The tensors can be arranged in layers, starting from the center, with each tensor having at most 2 legs contracted with tensors in the previous layer. Thus each tensor has at most 3 inputs (including the logical input); hence each tensor can be regarded as having 3 inputs and 3 outputs, which defines a unitary isometry, and the product of all these isometries is itself an isometry.

[22] $\mathscr{E}[R]$ is defined as the set of lattice bulk points reached by applying the following "greedy" algorithm to all connected components of R simultaneously (Pastawski et al. 2015, p. 9): Consider a sequence of cuts $\{c_\alpha\}$ through the tensor network, each with the same boundary as R, and a corresponding sequence of perfect tensors $\{P_\alpha\}$, such that each cut in the sequence is obtained from the previous one by the following: Begin with the "trivial" cut identified with R; then identify a perfect tensor with at least half of its legs contracted with P_α and construct $P_{\alpha+1}$ by adding this perfect tensor to P_α. Continue until the length of the cut is minimum in the sense that no single tensor can be added or removed which reduces the cut's length.

[23] γ_R is obtained by means of the greedy algorithm of the previous footnote.

[24] Recall from Section 3 that γ_R is the minimal bulk spatial surface with the same boundary as R, and H_R is the bulk surface bounded by $\gamma_R \cup R$.

to zero, γ_R becomes a point.[25] In this limit, the bulk spatial region $H_R \cup \overline{H_R}$ splits into two disconnected pieces, H_R and $\overline{H_R}$.[26] In this way, the degree of entanglement of the boundary state (12), as encoded in its entanglement entropy, tracks the connectedness of the corresponding bulk region $H_R \cup \overline{H_R}$.

For the purposes of this essay, van Raamsdonk's interpretation of the RT formula raises two questions:

(i) For given boundary subregions R and $\overline{R}$, under what conditions should we expect a CFT boundary state to be entangled with respect to the decomposition $\mathscr{H} = \mathscr{H}_R \otimes \mathscr{H}_{\overline{R}}$?
(ii) What does this relation between boundary entanglement and bulk connectedness suggest about the nature of spacetime in general, and the claim that "spacetime is a QECC", in particular?

Arguably, the QECC interpretation provides a precise answer to the first question, and a vague hint at the second. The answer to the first question rests on the fact that, in an erasure-protection QECC characterized by an n-qudit physical Hilbert space that protects against the erasure of $(n-m)$ qudits, a state in the corresponding codespace is maximally entangled with respect to the decomposition $\mathscr{H}^n = \mathscr{H}^{(m)} \otimes \mathscr{H}^{(n-m)}$.[27] In the QECC interpretation of the AdS/CFT correspondence, the CFT Hilbert space is assumed to admit a decomposition $\mathscr{H}^{(n)} = \mathscr{H}_R \otimes \mathscr{H}_{\overline{R}}$ in which the set R is interpreted as containing m qudits and the set $\overline{R}$ is interpreted as containing $(n-m)$ qudits. Hence a state in the codespace is maximally entangled with respect to $\mathscr{H}_R \otimes \mathscr{H}_{\overline{R}}$; i.e., it takes the general form (12). Thus the QECC answer to question (i) is that we should expect CFT boundary states to be entangled with respect to $\mathscr{H}_R \otimes \mathscr{H}_{\overline{R}}$, for any given boundary spatial subregion R, since states associated with such an R form the codespace of a QECC.

Note that this QECC way of answering question (i) does not require one to view the relation between an entangled boundary state and a connected bulk region as a duality relation.[28] Indeed, the relevant duality relation in the context of the QECC interpretation is between the states of boundary subsystems localized in R and the states of bulk subsystems localized in $W[R]$ (i.e., Harlow's, 2018, pg. 38, "subregion duality"). In any event, suppose we allow that, in the QECC interpretation, one might say that the connectivity property of the bulk spacetime arises from the entanglement structure of a subset of physical qudits on the boundary; namely, those that form the codespace. At best this is vaguely suggestive of the claim "spacetime is a QECC". At worst, the latter claim seems a bit misleading. Rather than spacetime being a QECC, what van Raamsdonk's proposal suggests is that the connectivity property of spacetime in the bulk is related to the entanglement structure of the codespace of a boundary CFT, under the QECC interpretation of the latter.

5.2. The HaPPY code interpretation

As argued above, under the QECC interpretation of the AdS/CFT correspondence, spacetime (whether in the bulk or on the boundary) cannot literally be a QECC. Rather, a QECC can be realized by physical systems which can possess spatiotemporal properties. In the QECC interpretation, these physical systems are described by the boundary CFT, and there is a common claim in the physics literature that the bulk emerges from the boundary. So perhaps the claim that spacetime is a QECC is better understood as the claim that spacetime in the bulk emerges from boundary systems that realize the structure of a QECC. On the other hand, many authors have observed that the duality between the boundary and the bulk is exact and thus symmetrical; hence to the extent that a notion of emergence requires an asymmetrical relation between a fundamental physical system and an emergent physical system, it cannot be underwritten solely by duality.[29] This is where the HaPPY code realization of the QECC interpretation can help; namely, by supplying the missing asymmetrical relation. The resulting picture in which both the boundary CFT and the bulk AdS theory of gravity emerge from a more fundamental discrete system that instantiates the HaPPY code is what I will call the HaPPY code interpretation of the AdS/CFT correspondence.

The HaPPY code tensor network described in Section 4 can be realized as a system of spins, one "logical" spin for each free bulk index, and one "physical" spin for each free uncontracted boundary index. The result is a composite system on a negatively curved lattice made up of a finite number of spin subsystems, with each lattice cell containing one spin, and the rest sprinkled along the lattice boundary. For a given boundary region R, the states of bulk spins localized in the corresponding greedy entanglement wedge $\mathscr{E}[R]$ form an erasure-protection QECC codespace $\mathscr{H}_C$ such that any local boundary operator with support on $\overline{R}$ acts as a multiple of the identity on $\mathscr{H}_C$ (QECC condition), and any local bulk operator with support on $\mathscr{E}[R]$ can be expressed as a boundary operator with support on R (Erasure-Protection condition). One can now imagine taking a continuum limit of this discrete lattice system (e.g., take the lattice cell area $A \to 0$ and the number of spins $N \to \infty$ while keeping N/A constant). Provided this limit preserves the relevant QECC structure, it should produce the AdS/CFT correspondence. To establish this, one would have to show that the essential conditions (a), (b), (c) of Section 3 that define QECC structure are preserved. That they should be preserved is suggested by the HaPPY code dictionary. (One would have to show in particular that a discrete greedy entanglement wedge is mapped to a continuum entanglement wedge, and a discrete cut γ_R is mapped to a minimal spatial surface.) Under such a limit, the degrees of freedom of the lattice bulk become AdS degrees of freedom, with the hyperbolic lattice becoming AdS spacetime. The degrees of freedom of the lattice boundary become CFT degrees of freedom, including spatiotemporal degrees of freedom. In this scenario, both the CFT boundary spacetime and the AdS bulk spacetime emerge in the continuum limit of a fundamental discrete HaPPY code lattice system.

Table 2 indicates how the boundary CFT and the bulk AdS theory can both be considered to emerge in the continuum limit of a more fundamental HaPPY code lattice system of spins. According to this HaPPY code interpretation, the asymmetrical relation of emergence is underwritten by a continuum limit that preserves the relevant aspects of the underlying QECC structure. This QECC structure is characterized by a physical qudit state space that is realized in the

[25] Van Raamsdonk (2010) uses Ryu and Takayanagi's (2006) original formula, which does not include Faulkner et al.'s (2013) "quantum correction"; i.e., the second term on the right in expression (9).

[26] The partition of the bulk spacetime into H_R and $\overline{H_R}$ separated by γ_R is a partition into the union of a closed set H_R and an open set $\overline{H_R}$; hence, under this partition, the bulk spacetime is connected. In the limit in which the area of γ_R is zero, the sets H_R and $\overline{H_R}$ are disjoint open sets; hence their union is a disconnected space.

[27] This is a consequence of a result derived by Preskill (1999, pp. 1516).

[28] Van Raamsdonk (2010, pg. 2324) suggests that doing so generalizes a result due to Maldacena (2003) who showed that an entangled CFT thermofield double state is dual to the (connected) maximally extended AdS-Schwarzschild black hole spacetime.

[29] See, e.g., Teh (2013), Dieks et al. (2015), De Haro et al. (2016a) and De Haro (2017). Some authors have suggested that emergence can be supported in those cases in which only an approximate duality holds between boundary and bulk.

Table 2
The HaPPY code lattice interpretation of AdS/CFT.

CFT	AdS	AdS/CFT
↑	↑	↑
continuum limit	*continuum limit*	*emergence of spacetime*
|	|	|
boundary spins	bulk spins	HaPPY code lattice
↑	↑	↑
realization	*realization*	*realization*
|	|	|
physical state space	codespace	QECC structure

HaPPY code by the collection of boundary spin states, and a codespace that is realized in the HaPPY code by a collection of bulk spin states. In the QECC structure-preserving continuum limit, a boundary CFT emerges from the boundary spins, and a bulk AdS theory of gravity emerges from the bulk spins. Spacetime, either at the boundary or in the bulk, can be viewed as emerging from the fundamental discrete spin system. Moreover, the AdS/CFT duality relation can be considered an exact symmetrical relation between the boundary CFT and the bulk AdS theory; a relation that emerges from the fundamental lattice system that itself does not exhibit it.

5.3. Common core or overarching structure?

In Section 3.2, I suggested that the QECC interpretation is best understood in terms of Le Bihan and Read's (2018, pg. 3) "discrimination" realist interpretive option, under which, when faced with dual theories, the realist claims that only one describes an actual world candidate. Under the QECC interpretation, the boundary CFT is the primary system, with bulk states consisting of a subset of boundary states. On the other hand, the HaPPY code interpretation is at odds with "discrimination" insofar as it claims that both boundary and bulk theories describe legitimate, *al beit* emergent, actual world candidates. Le Bihan and Read (2018, pp. 2, 8) describe two additional realist alternatives to "discrimination", what they refer to as "common core", which claims that dual theories share a common mathematical core, and "overarching theory", which claims that dual theories can be embedded in an overarching theory. The HaPPY code interpretation is at odds with these options, too, as I shall now argue.

5.3.1. Common core

Under Le Bihan and Read's (2018, pg. 3) "common core" interpretative option, one identifies the mathematical structure common to solutions to two dual theories and interprets it as representing a legitimate actual world candidate. In the HaPPY code interpretation, there is a mathematical structure that underlies both dual theories, but it is not common to both; rather, both emerge from different aspects of it. Again, this underlying structure consists of a Hilbert space of qudits, and a codespace subspace spanned by a set of maximally entangled qudits. The role of the HaPPY code is to provide a realization of this structure, and to serve as a fundamental physical system from which emerge, in the continuum limit, both the boundary CFT and the bulk AdS theory of gravity.

The "common core" interpretative option has recently been advocated by De Haro (2019) and De Haro and Butterfield (2018, 2019). These authors claim that duality in general is an isomorphism between two theories (or "models" in their sense) that share a common core "bare theory":

> … there is a bare theory—the common core of the two given theories—which has various models, among which are the two given theories. The duality then consists in the fact that these two models are isomorphic as regards the structure and notions given in the bare theory (De Haro & Butterfield, 2018; pg. 372). The two theories share a common core; the common core itself is a theory, which we call the bare theory … The two given theories are isomorphic models of this common core, the bare theory (De Haro & Butterfield, 2019; pg. 2).

De Haro (2019, pg. 18) identifies the common core of the AdS/CFT duality as a triple $(\mathscr{H}, \mathscr{C}, \mathscr{D})$ of states, quantities, and dynamics, informed by the differentiate AdS/CFT dictionary (1). This account is eminently reasonable, insofar as the differentiate dictionary suggests a commonality of observables (correlation functions), based on a commonality of states and dynamics. For instance, Harlow observes that.

> … the Hilbert space of physical states of the bulk is by definition identical to the CFT Hilbert space. Moreover, symmetry generators of $SO(d, 2)$ in the CFT are identical with the corresponding bulk symmetry generators of asymptotically AdS space. In particular the Hamiltonian is the same on both sides. Quantities which only depend on the space of states and the Hamiltonian, for example the thermal partition function or the free energy at finite temperatures, are thus computed by their CFT expressions by definition (Harlow, 2016, p. 33).

However, there are two concerns with identifying the common core of AdS/CFT in this way. The first is with the limitations of the differentiate dictionary mentioned above in Section 2; namely, it cannot account for all interesting bulk observables. These limitations motivate the "reconstruction" program and the AdS-Rindler representation of a bulk field, which, in turn, motivate the QECC and HaPPY code interpretations. To be sure, some authors who adopt the common core interpretation are aware of the limitations of the differentiate dictionary. De Haro, Teh, and Butterfield (2017, pp. 75), for instance, stress that the differentiate dictionary is not in the business of "… calculating correlation functions of bulk operators"; rather, it defines "boundary correlation functions … as functional derivatives with respect to the asymptotic boundary values of (boundary conditions on) the bulk fields". I take it these are the same concerns indicated in Section 2 above: the differentiate dictionary only establishes a correspondence between boundary quantities and bulk quantities near the boundary. In particular, it does not address "interesting" bulk observables deep in the bulk, or non-local bulk observables like Wilson loops, as De Haro (2019, pg. 20) also acknowledges: "There is no claim here that my description contains all of the information about the CFT. Nonlocal operators, such as Wilson loops (and perhaps additional states), also need to be compared." The question for advocates of a common core interpretation then is, are the limitations of the differentiate dictionary significant enough to resist interpreting it in terms of a common core? QECC and HaPPY code advocates will say "yes".

A second more minor concern is with identifying the common core as a theory with a dynamics. The QECC and HaPPY code interpretations suggest there is a mathematical core at the basis of AdS/CFT, but, under a common understanding of the distinction between kinematics and dynamics, this core does not appear to be dynamical. Suppose the kinematics of a theory is encoded in the space of possible states that a physical system described by the

theory can be in; or, alternatively, in the set of possible observables that the system can exhibit. Suppose further that the dynamics of a theory is encoded in the form of a map that time-evolves states and/or observables (perhaps in the explicit form of a dynamical equation of motion, or in the implicit form of a Hamiltonian). Then, to the extent that the entries in Table 1 encode constraints on states and observables, and make no reference to a dynamical map or a Hamiltonian, these entries encode kinematical, as opposed to dynamical, constraints. Thus to the extent that these entries encode the mathematical core common to a QECC, the HaPPY code, and AdS/CFT, one might claim that this common core is not dynamical. The claim here is that the structure of an erasure-protection QECC, as given by a finite-dimensional Hilbert space, characterized by a codespace subspace and an encoding map, is purely kinematical; i.e., it is associated with kinematically possible, as opposed to dynamically possible, states of a physical system.[30]

5.3.2. Overarching structure

In addition to "common core", Le Bihan and Read (2018, pg. 8) also identify another realist interpretative option they refer to as "overarching theory". Under this option, given two dual theories, one identifies the solution space of an overarching theory in which the solution spaces of the dual theories can be embedded; and one then interprets the solutions of the overarching theory as representing legitimate actual world candidates. The problem with understanding the HaPPY code interpretation in terms of this view involves identifying the "overarching theory". One option is to identify it with the theory that describes a system of spins on a HaPPY code lattice: one might claim that solutions to the boundary CFT and the bulk AdS theory of gravity can be embedded in the solution space of the lattice spin theory. What makes this problematic is that it is not clear how a dynamics for a system of finite spins can be mapped onto either the dynamics of a conformal field theory or the dynamics of a bulk theory of gravity. Recall that the HaPPY code interpretation relies on the existence of a continuum limit that maps a HaPPY code lattice theory onto AdS/CFT, but all that is required of this map is that it preserve relevant kinematical, as opposed to dynamical, features of the former.

Another option is to identify the overarching theory with a QECC, but this also seems problematic. As emphasized above, a QECC is not a dynamical theory; in particular, it does not possess a dynamics with an associated space of solutions. A QECC, rather, is a specification of a space of kinematically, as opposed to dynamically, possible states. The underlying QECC structure in the HaPPY code interpretation ultimately imposes a constraint on the kinematically possible states of both the boundary CFT and the bulk AdS theory of gravity; it does not impose constraints on their dynamics.

These problems suggest a modification of the "overarching theory" interpretation to an "overarching structure" interpretation, in which the overarching structure need not be dynamical. Under an "overarching structure" interpretation, the spaces of kinematically possible states of the dual theories are embeddable in a larger space of kinematically possible states. But note that under the HaPPY code interpretation, it is not the case that the kinematically possible states of the boundary CFT and the bulk gravity theory are embedded in a larger space of kinematically possible states. Rather, both the kinematically possible states of the boundary and bulk theories emerge in a continuum limit from kinematically possible states that characterize a HaPPY code discrete system, and these latter are embeddable in the structure of a QECC. Thus the fundamental overarching structure is that of a QECC, but the boundary CFT and the bulk AdS gravity are not embedded in it; rather, they emerge as different aspects of a realization of it.

Again, under this understanding of the HaPPY code interpretation, one might claim that the kinematical degrees of freedom of the AdS/CFT correspondence emerge in an appropriate continuum limit from a HaPPY code lattice system that realizes the (kinematical) structure of a QECC. In particular, under the HaPPY code interpretation of AdS/CFT, the fundamental physical system is a finite dimensional system of spins on a lattice that possesses both kinematical and dynamical degrees of freedom; and the physical systems described by AdS/CFT emerge from this fundamental lattice system in an appropriate continuum limit under which the kinematical degrees of freedom of the latter emerge from the kinematical degrees of freedom of the former. But, as noted above, it is less clear how the dynamical degrees of freedom of the physical systems described by AdS/CFT arise (i.e., certainly not simply from the dynamics of a system of spins on a lattice). Of course this understanding of the HaPPY code interpretation assumes that a clear distinction between kinematics and dynamics can be made, and this is complicated by the fact that, ultimately, the theory of gravity that the AdS bulk theory is supposed to encode is general relativity, in which such a distinction is hard to establish. But one might claim that this may be asking too much of the HaPPY code interpretation: it's function is simply to explain, in the first instance, how the spatiotemporal degrees of freedom of the physical systems described by AdS/CFT arise from the structure of a QECC (i.e., it explains the sense in which "spacetime is a QECC′"); and faulting it for being unable to provide an account of the dynamical degrees of freedom of these systems may be asking too much.[31]

6. Conclusion

Is spacetime a quantum error-correcting code? The QECC interpretation of the AdS/CFT correspondence suggests that local bulk degrees of freedom encode boundary degrees of freedom in a redundant way, but this by itself does not say anything about the nature of the bulk spacetime and its relation to the boundary. In this essay I've argued that one way of connecting spacetime to the QECC interpretation is through its realization by the HaPPY code, what I've called the HaPPY code interpretation of the AdS/CFT correspondence. According to this interpretation, spacetime in the bulk emerges in a continuum limit from a discrete system of spins that realizes the kinematical structure of a QECC. Moreover, this structure is not common to the boundary and the bulk theories, nor are they embeddable in it; rather, the physical systems that the boundary and bulk theories describe emerge from a physical system that realizes it.

CRediT authorship contribution statement

Jonathan Bain: Conceptualization, Writing - original draft, Writing - review & editing.

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[30] De Haro and Butterfield (2019, pg. 10) allow that the common core associated with a duality need not be a dynamical theory, but they claim that the interesting cases of duality are associated with a dynamical theory.

[31] One way the dynamical degrees of freedom of AdS/CFT might emerge from a physical system that realizes the structure of a QECC is by a dynamical realization of the QECC encoding map; but speculation along these lines is perhaps best left to another essay.

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