A bet: I randomly select half of the class...

(I was targeting a group of around 70/2 people when I wrote this)

If any 2 people in that group have the same birthday, you give me a dollar.

If no birthday match is found, I give you a dollar.
Another bet:

I randomly select 10 people born in the same month.

Same deal as before.
Another bet:

I randomly select 10 people born in the same month

Same deal as before

My chances of winning: >80%
One last bet?

I randomly select 7 people

If any 2 people in that group have birthdays within a week of each other...
One last bet?

I randomly select 7 people

If any 2 people in that group have birthdays within a week of each other...

I win 60% of the time. (52% if within 6 days)
DISCRETE PROBABILITY
Roll a die ... Possible outcomes: \( \{1, 2, 3, 4, 5, 6\} \)
DISCRETE PROBABILITY

Roll a die ... Possible outcomes: \{1, 2, 3, 4, 5, 6\}

Each has a probability: \{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\}
DISCRETE PROBABILITY

Roll a die ... Possible outcomes: \{1, 2, 3, 4, 5, 6\}

Each has a probability: \{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\}

sum to 1
DISCRETE PROBABILITY

Roll a die ... Possible outcomes: \( \{1, 2, 3, 4, 5, 6\} \)

Each has a probability: \( \{\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\} \)

Sum to 1

Roll 2 dice ... sample space → ?
DISCRETE PROBABILITY

Roll a die ... Possible outcomes: \( \{1, 2, 3, 4, 5, 6\} \)
- Each has a probability: \( \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6} \)
- Sum to 1

Roll 2 dice ... Sample space \( \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \)
- \( (2,1), (2,2), \ldots \)
- \( \vdots \)
- \( (6,1), (6,2), \ldots \)
- \( \ldots (6,6) \)
Roll 2 indistinguishable dice...

Sample space $\mathcal{S} = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6), (4,4), (4,5), (4,6), (5,5), (5,6), (6,6)\}$
Roll 2 indistinguishable dice...

Sample space → \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6), (4,4), (4,5), (4,6), (5,5), (5,6), (6,6)\}

prob. = ?
Roll 2 indistinguishable dice...

Sample space \( \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,2), (2,3), (2,4), (2,5), (2,6),
(3,3), (3,4), (3,5), (3,6),
(4,4), (4,5), (4,6),
(5,5), (5,6),
(6,6) \} \)

\[
\text{prob. } \neq \frac{1}{36}
\]

if \((a,a)\) then \(\frac{1}{36}\)

if \((a,b)\) then \(\frac{2}{36}\)

\(a \neq b\)
Roll 2 indistinguishable dice...

Sample space \( \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\
(2,2), (2,3), (2,4), (2,5), (2,6), \\
(3,3), (3,4), (3,5), (3,6), \\
(4,4), (4,5), (4,6), \\
(5,5), (5,6), \\
(6,6)\} \)

\[
\text{prob. } \neq \frac{1}{36}
\]

if \((a,a)\) then \(\frac{1}{36}\)

\[
6 \cdot \frac{1}{36} = \frac{6}{36} \quad \text{\(\Rightarrow\)} \quad 1
\]

if \((a,b)\) then \(\frac{2}{36}\) if \(a \neq b\)

\[
15 \cdot \frac{2}{36} = \frac{30}{36} \]
Roll a die \[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]
Roll a die

\[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]

\[ P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2} \]
Roll a die

\[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]

\[ P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2} \]

Roll 2 dice...

\[ P(\text{sum = 7}) \ldots \]
Roll a die

\[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]

\[ P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2} \]

Roll 2 dice...

\[ P(\text{sum} = 7) = P(\{1,6\}, \{2,5\}, \{3,4\}, \{4,3\}, \{5,2\}, \{6,1\}) \]
Roll a die

\[
P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}
\]

\[
P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2}
\]

Roll 2 dice...

\[
P(\text{sum = 7}) = P(\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\})
\]

\[
= P(1,6) + P(2,5) + P(3,4) + P(4,3) + P(5,2) + P(6,1)
\]
Roll a die

\[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]

\[ P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2} \]

Roll 2 dice...

\[ P(\text{sum} = 7) = P(\{1, 6\}, (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}) \]

\[ = P(1, 6) + P(2, 5) + P(3, 4) + P(4, 3) + P(5, 2) + P(6, 1) \]

\[ = 6 \cdot \frac{1}{36} = \frac{1}{6} \]
Roll a die

\[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]

\[ P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2} \]

Roll 2 dice...

\[ P(\text{sum} = 7) = P(\{1, 6\}, (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}) \]

\[ = P(1, 6) + P(2, 5) + P(3, 4) + P(4, 3) + P(5, 2) + P(6, 1) \]

\[ = 6 \cdot \frac{1}{36} = \frac{1}{6} \]

\[ P(\text{sum} = 7) = ? \]
Roll a die

\[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]

\[ P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2} \]

Roll 2 dice...

\[ P(\text{sum} = 7) = P(\{1,6\}, \{2,5\}, \{3,4\}, \{4,3\}, \{5,2\}, \{6,1\}) \]

\[ = P(1,6) + P(2,5) + P(3,4) + P(4,3) + P(5,2) + P(6,1) \]

\[ = 6 \cdot \frac{1}{36} = \frac{1}{6} \]
Roll a die

\[ P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6} \]

\[ P(\text{roll even}) = P(\{2, 4, 6\}) = P(2) + P(4) + P(6) = \frac{1}{2} \]

Pipeline

Roll 2 dice...

\[ P(\text{sum} = 7) = P(\{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}) \]

\[ = P(1,6) + P(2,5) + P(3,4) + P(4,3) + P(5,2) + P(6,1) \]

\[ = 6 \cdot \frac{1}{36} = \frac{1}{6} \]

\[ P(\text{sum} = 7) = P(\{(1,6), (2,5), (3,4)\}) = 3 \cdot \frac{2}{36} = \frac{1}{6} \]
Roll 10 dice (or 1 die 10 times)
Roll 10 dice (or 1 die 10 times)

Sample space size: ?
Roll 10 dice (or 1 die 10 times)

Sample space size: \(6^{10}\) > 60 million
Roll 10 dice (or 1 die 10 times)

Sample space size: $6^{10} > 60$ million

$P(\text{observe no 1's})$ ?
Roll 10 dice (or 1 die 10 times)

Sample space size: $6^{10} > 60$ million

$P($observe no 1's$)$?

How many outcomes have no 1's?
Roll 10 dice (or 1 die 10 times)

Sample space size: $6^{10}$ > 60 million

\[ P(\text{observe no 1's}) \]?

How many outcomes have no 1's? \[
\rightarrow 5^{10}
\]
Roll 10 dice (or 1 die 10 times)

Sample space size: $6^{10} > 60$ million

$P($observe no 1's$)$?

How many outcomes have no 1's? $\rightarrow 5^{10}$
Roll 10 dice (or 1 die 10 times)

Sample space size: $6^{10} > 60$ million

$P(\text{observe no 1's?})$

How many outcomes have no 1's? $5^{10}$

Or, say that each roll/die is independent
so for each roll, $P(\text{no 1}) = \frac{5}{6}$ $\Rightarrow \left(\frac{5}{6}\right)^{10}$

\{ to be discussed further \}
Poker: 52 cards (4 x 13 types); select 5.
Poker: 52 cards (4x13 types); select 5.

\[ P(4 \text{ of a kind}) = P(4 \text{ of 5 are of same type}) \]

\[ \text{e.g. } 3,3,3,3,7 \quad \text{or} \quad 8,8,8,J,8 \]

\[ = ? \]
Poker: 52 cards (4 × 13 types); select 5.

\[ P(4 \text{ of a kind}) = P(4 \text{ of } 5 \text{ are of same type}) \]

\[ \text{e.g. } 3,3,3,3,7 \quad \text{or} \quad 8,8,8,J,8 \]

\[ \text{ans: } \frac{\# 4\text{-of-a-kinds}}{\# \text{possible outcomes}} \]
Poker: 52 cards (4 x 13 types); select 5.

\[ P(4 \text{ of a kind}) = P(4 \text{ of 5 are of same type}) \]

e.g. 3,3,3,3,7 or 8,8,8,J,8

\[ \text{ans: } \frac{\# \text{ 4-of-a-kinds}}{\# \text{ possible outcomes}} \rightarrow ? \] (order doesn't matter)
Poker: 52 cards \((4 \times 13\) types\); select 5.

\[
P(4 \text{ of a kind}) = P(4 \text{ of 5 are of same type})
\]

\[
e.g. \quad 3,3,3,3,7 \quad \text{or} \quad 8,8,8,J,8
\]

\[
\text{ans:} \quad \frac{\# \text{ 4-of-a-kinds}}{\# \text{possible outcomes}} \quad \rightarrow \quad \binom{52}{5} \quad \text{if order doesn't matter}
\]

\[
\text{what if it did?} \rightarrow \text{hw}
\]
Poker: 52 cards (4 x 13 types); select 5.

\[ P(4 \text{ of a kind}) = P(4 \text{ of 5 are of same type}) \]

e.g. 3,3,3,3,7 or 8,8,8,J,8

ans: \[ \frac{\# \text{4-of-a-kinds}}{\# \text{possible outcomes}} \rightarrow \frac{52}{5} \] order doesn't matter
Poker: 52 cards (4 \times 13 \text{ types}) \,; \, \text{select 5.}

\[ P(4 \text{ of a kind}) = P(4 \text{ of 5 are of same type}) \]

e.g. \, 3,3,3,3,7 \, or \, 8,8,8,8,J,8

\[ \text{ans: } \frac{\text{# 4-of-a-kinds}}{\text{# possible outcomes}} \rightarrow \frac{\text{AAAAX}}{\text{KKKK}} \rightarrow \begin{array}{c} \{13 \text{ types} \times 48 \text{ choices of} \\ \text{remaining card} \} \\
\binom{52}{5} \end{array} \]

order doesn't matter
Poker: 52 cards (4x13 types); select 5.

\[ P(4 \text{ of a kind}) = P(4 \text{ of 5 are of same type}) \]

e.g. 3,3,3,3,7 or 8,8,8,J,8

\[
\text{ans:} \quad \frac{\# \text{4-of-a-kinds}}{\# \text{possible outcomes}} \quad \text{13 types \times 48 choices of remaining card} \quad (\begin{pmatrix} 52 \\ 5 \end{pmatrix}) \quad \text{order doesn't matter}
\]

\[
\frac{13 \cdot 48}{\begin{pmatrix} 52 \\ 5 \end{pmatrix}} = \frac{1}{4165} \approx 0.00024
\]
Probability visualization

Sample space $\rightarrow$ area $= 1$
Probability Visualization

Sample space $\rightarrow$ area $= 1$
Probability Visualization

Sample space $\rightarrow$ area $= 1$

$P(\text{outcome } i) = \text{area}(i)$

Example: roll one die $\rightarrow$ all areas equal $= \frac{1}{6}$
Probability Visualization

- Sample space $\rightarrow$ area = 1
  \[ P(\text{outcome } i) = \text{area}(i) \]

- Example: roll one die $\rightarrow$ all areas equal $= \frac{1}{6}$

- \[ P(\text{event}) = \text{sum of appropriate areas} \]
  - e.g. $P(\text{roll prime \# OR even \#})$

- Which outcomes?
**Probability Visualization**

- Sample space $\rightarrow$ area = 1
  \[ P(\text{outcome } i) = \text{area}(i) \]
  
  Example: roll one die $\rightarrow$ all areas equal $= \frac{1}{6}$

- $P(\text{event}) = \text{sum of appropriate areas}$
  
  E.g. $P(\text{roll prime\# OR even\#})$
  
  $\underbrace{2, 3, 5} \quad \underbrace{2, 4, 6}$
  
  $P(\text{prime})=? \quad P(\text{even})=?$
**Probability Visualization**

Sample space \( \rightarrow \) area = 1

\[ \text{P(outcome } i \text{)} = \text{area}(i) \]

Example: roll one die \( \rightarrow \) all areas equal \( = \frac{1}{6} \)

\[ \text{P(event)} = \text{sum of appropriate areas} \]

E.G. \( \text{P(roll prime \# OR even \#)} \)

\[ \text{P(prime)} = \frac{3}{6}, \quad \text{P(even)} = \frac{3}{6} \]
**Probability Visualization**

Sample space → area = 1

\[ P(\text{outcome } i) = \text{area}(i) \]

example: roll one die → all areas equal = \( \frac{1}{6} \)

\[ P(\text{event}) = \text{sum of appropriate areas} \]

\[ \begin{align*}
\text{e.g. } P(\text{roll prime }\# \text{ or even }\#) & = \left\{ \frac{2,3,5}{2,4,6} \right\} = \frac{5}{6} \\
P(\text{prime}) & = \frac{3}{6} \\
P(\text{even}) & = \frac{3}{6}
\end{align*} \]

avoid doublecounting
 NOT \( \frac{3}{6} + \frac{3}{6} \)
roll prime #: 2, 3, 5
roll even #: 2, 4, 6
Roll prime #: 2, 3, 5
Even #: 2, 4, 6
\[ P(\text{roll prime \# OR even \#}) \rightarrow P(A \cup B) \]

- \(2, 3, 5\)
- \(2, 4, 6\)
\[ P(\text{roll prime} \# \text{ or even } \#) = P(A \cup B) \]
\[ P(\text{roll prime # or even #}) \rightarrow P(A \cup B) \]

The diagram illustrates the probability space of rolling a die, with the events A and B represented by the circles within the rectangle. The outcomes 2, 3, 5 are prime numbers, and 2, 4, 6 are even numbers. The union of these events is denoted by \( P(A \cup B) \).
\[ P(\text{roll prime number or even number}) \rightarrow P(A \cup B) \]

- Roll numbers 2, 3, 5
- Roll numbers 2, 4, 6

Venn diagram:
- All probability space
- \( P(A) \)
- \( P(B) \)
- \( P(A \cap B) \)

Set operations:
- "OR" for \( P(A \cup B) \)
- "AND" for \( P(A \cap B) \)
all probability space

\[ P(A) \quad P(B) \quad \text{VS} \quad P(A \cup B) \quad P(A \cap B) \quad ? \]
$P(A)$  $P(B)$  $P(A \cup B)$  $P(A \cap B)$
\[ P(A) + P(B) = P(A \cup B) + P(A \cap B) \]

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

Avoid double counting.
\[ P(\text{someone in class was born on Feb. 29}) \]
\[ P(\text{someone in class was born on Feb. 29}) = P\left( \text{student 1 born on Feb. 29} \right) U \left( \text{student 2 born on Feb. 29} \right) \ldots \\
\ldots U \left( \text{student 3 born on Feb. 29} \right) \ldots U \ldots \left( \text{student } k \text{ born on Feb. 29} \right) \]
\[ P(\text{someone in class was born on Feb. 29}) = P[\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \cup \ldots \cup (\text{student } k \text{ born on Feb. 29})] \approx \frac{1}{365 \cdot 4 + 1} \sim 0.07\% \]
$P(\text{someone in class was born on Feb. 29})$

$= P\left[ (\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \ldots \cup (\text{student } k \text{ born on Feb. 29}) \right]$

$\leq \frac{1}{365 \cdot 4 + 1} \sim 0.07\%$
\[ P(\text{someone in class was born on Feb. 29}) = P\left[ (\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \cup \cdots \cup (\text{student k born on Feb. 29}) \right] \]

\[ \approx \frac{1}{365 \cdot 4 + 1} \approx 0.07\% \]

But we could say it is < \sum P(i)
\[ P(\text{someone in class was born on Feb. 29}) \]

\[ = P[(\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \ldots \cup (\text{student 3 born on Feb. 29}) \ldots \cup \ldots (\text{student k born on Feb. 29})] \]

\[ \leftarrow \text{awful but we could say it is } < \sum P(i) \sim 80 \cdot 0.07\% \sim 5.6\% \]

\[ \sim \frac{1}{365 \cdot 4 + 1} \sim 0.07\% \]
\[ P(\text{someone in class was born on Feb. 29}) \]

\[ = P\left[ (\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \cup \ldots \cup (\text{student k born on Feb. 29}) \right] \]

\[ \leftarrow \text{awful, but we could say it is } \sum P(i) \sim 80 \cdot 0.07\% \sim 5.6\% \]

\[ \sim \frac{1}{365 \cdot 4 + 1} \sim 0.07\% \]

\[ = 1 - P(\text{nobody in class was born on Feb. 29}) \]
\[ P(\text{someone in class was born on Feb. 29}) \]

\[ = P(\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \ldots \]

\[ \ldots \cup (\text{student 3 born on Feb. 29}) \ldots \cup \ldots (\text{student } k \text{ born on Feb. 29}) \]

\[ \leq \text{awful} \quad \text{but we could say it is} \quad \sum P(i) \approx 80 \cdot 0.07\% \]

\[ \approx \frac{1}{365 \cdot 4 + 1} \quad \approx 0.07\% \]

\[ = 1 - P(\text{nobody in class was born on Feb. 29}) \]

\[ = 1 - P(\text{student 1 not born on Feb. 29}) \cap (\text{student 2 not born on Feb. 29}) \]

\[ \ldots \cap (\text{student } k \text{ not born on Feb. 29}) \]
\[ P(\text{someone in class was born on Feb. 29}) \]
\[ = P\left[ (\text{student 1 born on Feb. 29}) \cup (\text{student 2 born on Feb. 29}) \cup \ldots \cup (\text{student k born on Feb. 29}) \right] \]
\[ \leq \text{ awful but we could say it is } < \sum P(i) \approx 0.07\% \approx 5.6\% \]
\[ = 1 - P(\text{nobody in class was born on Feb. 29}) \]
\[ = 1 - P\left[ (\text{student 1 not born on Feb. 29}) \cap (\text{student 2 not born on Feb. 29}) \cap \ldots \cap (\text{student k not born on Feb. 29}) \right] \]
\[ = 1 - \alpha^k \quad \alpha = P(\text{student i not born on Feb. 29}) \]
\[ P(\text{someone in class was born on Feb. 29}) \quad \text{(suppose k=80 students)} \]

\[ = P\left(\text{student 1 born on Feb. 29}\right) \cup \left(\text{student 2 born on Feb. 29}\right) \ldots \]

\[ \ldots \cup \left(\text{student 3 born on Feb. 29}\right) \ldots \cup \ldots \cup \left(\text{student k born on Feb. 29}\right) \]

\[ \leftarrow \text{awful but we could say it is } \sum P(i) \sim 80 \cdot 0.07\% \quad \frac{1}{365 \cdot 4 + 1} \sim 0.07\% \]

\[ = 1 - P(\text{nobody in class was born on Feb. 29}) \]

\[ = 1 - P\left(\text{student 1 NOT born on Feb. 29}\right) \cap \left(\text{student 2 NOT born on Feb. 29}\right) \]

\[ \ldots \cap \left(\text{student k NOT born on Feb. 29}\right) \]

\[ = 1 - \alpha^k = 1 - \left(\frac{365 \cdot 4}{365 \cdot 4 + 1}\right)^k \quad \text{exactly} \]

80 students \sim 5\%
\[ P_i + \overline{P_i} = 1 \]
\[ \text{area in circle } i + \text{area outside circle } i = 1 \]

\[ \bigcup P_i = 1 - \bigcap \overline{P_i} \]

- inside any circle
- outside every circle
\( P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \) no Feb.29 allowed
\( P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \)  

no Feb. 29 allowed

if \( k \geq 365 \) then?
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \quad \text{no Feb.29 allowed} \]

If \( k > 365 \) use pigeonhole
\[ P( \geq 2 \text{ people in a group of } k \text{ have same birthday}) \]

\[ \sum_{i=1}^{k-1} \frac{P[(1,2) \cup (1,3) \cup (1,4) \cdots \cup (1,k) \cup (2,3) \cup (2,4) \cdots \cup (2,k) \cdots \cdots \cup (k-1,k)]}{365} \]

if \( k > 365 \) use pigeonhole
P(> 2 people in a group of k have same birthday) 

\[ P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup (1,k) \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)] \]

awful

\[ \frac{1}{365} \]

if \( k > 365 \) use pigeonhole

\[ = 1 - P(\text{all k have distinct birthdays}) \]
P(\( \geq 2 \) people in a group of k have same birthday) \( \text{no Feb.29 allowed} \)

\[ \sum_{\{1,2\} \cup \{1,3\} \cup \{1,4\} \ldots \cup \{1,k\} \cup \{2,3\} \cup \{2,4\} \ldots \cup \{2,k\} \ldots \ldots \cup \{k-1,k\}} \]

awful

\[ \frac{1}{365} \]

if \( k > 365 \) use pigeonhole

\[ = 1 - P(\text{all } k \text{ have distinct birthdays}) \]

\[ = P(2\text{nd person has different bday than 1st}) = \frac{364}{365} = P(A) \]
\[ P( \geq 2 \text{ people in a group of } k \text{ have same birthday}) \text{ no Feb.29 allowed} \]

\[ \geq P[ (1,2) U (1,3) U (1,4) \ldots U (1,k) U (2,3) U (2,4) \ldots U (2,k) \ldots \ldots \ldots U (k-1,k) ] \]

awful

\[ \leq \frac{1}{365} \]

if \( k > 365 \) use pigeonhole

\[ = 1 - P(\text{all } k \text{ have distinct birthdays}) \]

\[ P(2\text{nd person has different bday than 1st}) = \frac{364}{365} = P(A) \]

\[ P(3\text{rd} \ldots \ldots 1\text{st & 2nd}) = \frac{363}{365} = P(B) \]
\[ P(\text{\(\geq\) 2 people in a group of k have same birthday}) \quad \text{no Feb.29 allowed} \]

\[ \Downarrow P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup (1,k) \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)] \]

awful

\[ \frac{1}{365} \]

\[ = 1 - P(\text{all k have distinct birthdays}) \]

\[ \Downarrow P(\text{2nd person has different bday than 1st}) = \frac{364}{365} = P(A) \]

\[ P(\text{3rd \ldots \ldots 1st & 2nd}) = \frac{363}{365} = P(B) \]

assuming?
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \]

\[
\leq P\left[ \bigcup_{i=1}^{k} U(i,1) \cup U(1,i) \cup U(i,k) \cup U(2,i) \cup U(i,2) \cup U(2,k) \cup \cdots \cup U(k-1,k) \right]
\]

\[ \approx \frac{1}{365} \]

if \( k > 365 \) use pigeonhole principle.

awful

\[ 1 - P(\text{all } k \text{ have distinct birthdays}) \]

\[ P(2\text{nd person has different bday than 1st}) = \frac{364}{365} = P(A) \]

\[ P(3\text{rd } \cdots \cdots \cdots \text{ } 1\text{st } \& \text{ 2nd}) = \frac{363}{365} = P(B) \]

assuming 1st & 2nd differ

"conditional probability"

I’m abusing notation a bit, it should be \( P(B|A) \), meaning \( P(B \text{ given } A) \)
$P(\geq 2 \text{ people in a group of } k \text{ have same birthday})$  \hspace{1cm} \text{no Feb.29 allowed}

$\Rightarrow P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup (1,k) \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)]$

awful

$\sim \frac{1}{365}$

if $k \geq 365$ use pigeonhole

$= 1 - P(\text{all } k \text{ have distinct birthdays})$

$\Rightarrow P(\text{2nd person has different bday than 1st}) = \frac{364}{365} = P(A)$

$P(\text{3rd \ldots \ldots \ldots\ldots 1st \& 2nd}) = \frac{363}{365} = P(B)$

$P(\text{4th \ldots \ldots \ldots (1-3)}) = \frac{362}{365} = P(C)$

etc
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \quad \text{no Feb.29 allowed} \]

\[ \subseteq P[(1,2) \cup (1,3) \cup (1,4) \ldots \cup (1,k) \cup (2,3) \cup (2,4) \ldots \cup (2,k) \ldots \ldots \cup (k-1,k)] \]

awful \[ \frac{1}{365} \]

\[ = 1 - P(\text{all } k \text{ have distinct birthdays}) \]

\[ = 1 - P(\text{2nd person has different bday than 1st}) \]

\[ = \frac{364}{365} = P(A) \]

\[ = 1 - \frac{363}{365} = P(B) \]

\[ = \frac{362}{365} = P(C) \]

\[ = 1 - [P(A) \land P(B) \land P(C) \ldots] \]

\[ \text{if } k \geq 365 \text{ use pigeonhole} \]
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \]

\[ P(\geq 2) = 1 - P(\text{all } k \text{ have distinct birthdays}) \]

\[ = 1 - P(\text{2nd person has different bday than 1st}) \]

\[ = 1 - P(\text{2nd person has different bday than 1st and 2nd}) \]

\[ = 1 - \frac{365 \times 364 \times 363 \times \ldots \times (365-k+1)}{365^k} \]

\[ = 1 - \frac{365 \times 364 \times 363 \times \ldots \times 365-k+1}{365^k} \]

\[ \approx 1 - \frac{365!}{365^k (365-k)!} \]

if \( k > 365 \) use pigeonhole

\[ P(\text{not at least 2 people}) \]

\[ P(\text{no Feb 29 allowed}) = \frac{362}{365} \]

\[ P(\text{leaves of } k) = \frac{363}{365} \]

\[ P(\text{B}) = \frac{364}{365} \]

\[ P(\text{C}) = \frac{365}{365} \]

\[ P(\text{A}) = \frac{365-1}{365} \]

\[ P(\text{A} \cap \text{B} \cap \text{C}) = \frac{365-1}{365} \]

awful
$P(\geq 2 \text{ people in a group of } k \text{ have same birthday})$

$$= 1 - \frac{365!}{(365-k)!} \cdot \frac{365^k}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (365-k+1)}{365^k}$$
P( \geq 2 \text{ people in a group of } k \text{ have same birthday})

\[ = 1 - \frac{365!}{(365-k)!} \cdot \frac{1}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (365-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k} \]

$(365)_k$ is notation for $365 \cdot 364 \cdots (365-k+1)$
$P(\geq 2 \text{ people in a group of } k \text{ have same birthday})$

$$= 1 - \frac{365!}{(365-k)!} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (365-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$k = 2 \quad \rightarrow \quad P \approx 0.27\% \left(\frac{1}{365}\right)$
P(\geq 2 \text{ people in a group of } k \text{ have same birthday})

= 1 - \frac{365!}{(365-k)!} \cdot \frac{1}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (365-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}

k = 2 \quad \implies \quad P \sim 0.27\% \quad \left(\frac{1}{365}\right)

k = 4 \quad \implies \quad P \sim 1.64\%
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \]

\[
= 1 - \frac{365!}{(365-k)!} \cdot \frac{1}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (365-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}
\]

\[ k = 2 \quad \rightarrow \quad P \sim 0.27\% \quad \left( \frac{1}{365} \right) \]

\[ k = 4 \quad \rightarrow \quad P \sim 1.64\% \]

\[ k = 23 \quad \rightarrow \quad P \sim 50.73\% \]
\[ P(> 2 \text{ people in a group of } k \text{ have same birthday}) \]

\[ = 1 - \frac{365!}{(365-k)!} \cdot \frac{365 \cdot 364 \cdot 363 \cdots (365-k+1)}{365^k} \]

\[ = 1 - \frac{(365)_k}{365^k} \]

\[
\begin{align*}
  k = 2 & \quad \rightarrow \quad P \sim 0.27 \% \quad \left( \frac{1}{365} \right) \\
  k = 4 & \quad \rightarrow \quad P \sim 1.64 \% \\
  k = 23 & \quad \rightarrow \quad P \sim 50.73 \% \\
  k = 30 & \quad \rightarrow \quad P \sim 70.6 \%
\end{align*}
\]
$P(\geq 2$ people in a group of $k$ have same birthday$)$

$$= 1 - \frac{365!}{(365-k)!} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (365-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$k = 2 \quad \rightarrow \quad P \approx 0.27\% \quad \left(\frac{1}{365}\right)$

$k = 4 \quad \rightarrow \quad P \approx 1.64\%$

$k = 23 \quad \rightarrow \quad P \approx 50.73\%$

$k = 30 \quad \rightarrow \quad P \approx 70.6\%$

$k = 70 \quad \rightarrow \quad P \approx 99.9\%$
$P(\text{> 2 people in a group of } k \text{ have same birthday})$

\[
= 1 - \frac{365!}{(365-k)!} \cdot \frac{1}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (365-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}
\]

$k = 2 \quad \rightarrow \quad P \sim 0.27\% \quad \left(\frac{1}{365}\right)$

$k = 4 \quad \rightarrow \quad P \sim 1.64\%$

$k = 23 \quad \rightarrow \quad P \sim 50.73\%$

$k = 30 \quad \rightarrow \quad P \sim 70.6\%$

$k = 70 \quad \rightarrow \quad P \sim 99.9\%$

$k \sim 116 \quad \rightarrow \quad P \sim 1 - \frac{1}{109}$
$P(\geq 2 \text{ people in a group of } k \text{ have same birthday})$

$$= 1 - \frac{365!}{(365-k)!} \cdot \frac{1}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (365-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$k = 2 \quad \Rightarrow \quad P \approx 0.27\% \quad \left( \frac{1}{365} \right)$

$k = 4 \quad \Rightarrow \quad P \approx 1.64\%$

$k = 23 \quad \Rightarrow \quad P \approx 50.73\%$

$k = 30 \quad \Rightarrow \quad P \approx 70.6\%$

$k = 70 \quad \Rightarrow \quad P \approx 99.9\%$

$k \approx 116 \quad \Rightarrow \quad P \approx 1 - \frac{1}{109}$

$k = 300 \quad \Rightarrow \quad P \approx 1 - \frac{1}{1080}$
$P(\geq 2 \text{ people in a group of } k \text{ have same birthday})$

$$= 1 - \frac{365!}{(365-k)!} \frac{1}{365^k} = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (365-k+1)}{365^k} = 1 - \frac{(365)_k}{365^k}$$

$k=2 \quad \rightarrow \quad P \sim 0.27\% \ (\frac{1}{365})$

$k=4 \quad \rightarrow \quad P \sim 1.64\%$

$k=23 \quad \rightarrow \quad P \sim 50.73\%$

$k=30 \quad \rightarrow \quad P \sim 70.6\%$

$k=70 \quad \rightarrow \quad P \sim 99.9\%$

$k \sim 116 \quad \rightarrow \quad P \sim 1 - \frac{1}{10^9}$

$k=300 \quad \rightarrow \quad P \sim 1 - \frac{1}{10^{80}}$

($10^{80} \sim \#\text{atoms in universe}$)

($k > 365 \quad \rightarrow \quad P = 1$)
\[ P(\geq 2 \text{ people in a group of } k \text{ have same birthday}) \]

\[ = 1 - \frac{365 \cdot 364 \cdot 363 \cdots (365-k+1)}{365^k} \]

For the bet involving \( k \) people born in a month w/ 30 days substitute \( 365 \rightarrow 30 \)

\[ (k=10) \quad 1 - \frac{30 \cdot 29 \cdot 28 \cdots 23 \cdot 22 \cdot 21}{30^{10}} \approx 0.815 \]