set $S$ of intervals

7———10
5———11
4———8
15———18
17———19
21———23
**INTERVAL TREES**

Set $S$ of intervals:

- $4 \ldots 8$
- $5 \ldots 11$
- $7 \ldots 10$
- $15 \ldots 18$
- $17 \ldots 19$
- $21 \ldots 23$

**Query:** Given an interval $\times$, return any interval in the set $S$ that partially overlaps $\times$ (if one exists).
types of overlap:
i) "smaller"
types of overlap:
1) "smaller"
2) "bigger"
types of overlap:

1) "smaller"
2) "bigger"
3) "left" & "right"
types of overlap:
1) "smaller"
2) "bigger"
3) "left" & "right"

First comparison: $lo(s_i)$ vs $L$
types of overlap:
1) "smaller"
2) "bigger"
3) "left" & "right"

First comparison: $lo[s_i]$ vs $L$

is there some large enough $hi[s_i]$?
types of overlap:
1) "smaller"
2) "bigger"
3) "left" & "right"

First comparison: $l_0[s_i] \text{ vs } L$

is there some large enough $h_i[s_i]$?

If $l_0[s_i] \leq L$ and $h_i[s_i] \geq L$ then overlap
types of overlap:
1) "smaller"
2) "bigger"
3) "left" & "right"

First comparison: $l_0[s_i]$ vs L

is there some small enough $l_0[s_i]$?

If $l_0[s_i] > L$ AND $\leq R$ then overlap
types of overlap:
1) "smaller"
2) "bigger"
3) "left" & "right"

First comparison: $lo[s_i]$ vs $L$
- is there some large enough $hi[s_i]$?
- is there some small enough $lo[s_i]$?

If $lo[s_i] \leq L$ AND $hi[s_i] \geq L$ then overlap
If $lo[s_i] > L$ AND $< R$ then overlap
SEARCHING FOR OVERLAPPING INTERVALS

BST w/ LEFT ENDS as KEYS
SEARCHING FOR OVERLAPPING INTERVALS

ID:

BST w/ LEFT ENDS as KEYS

Augment: MAX RIGHT END of SUBTREE
SEARCHING FOR OVERLAPPING INTERVALS

ID:

compare w/ root first

query segment
SEARCHING FOR OVERLAPPING INTERVALS

ID:

CASE 1

IF NO OVERLAP

L → R

X

R < X
SEARCHING FOR OVERLAPPING INTERVALS

IF NO OVERLAP

right subtree can't overlap

R < x < W

case 1

L

R

x

W
SEARCHING FOR OVERLAPPING INTERVALS

1D:

IF NO OVERLAP

right subtree can't overlap

keep searching LEFT

R < x < W
SEARCHING FOR OVERLAPPING INTERVALS

IF NO OVERLAP

L  case 2  R
SEARCHING FOR OVERLAPPING INTERVALS

IF Z > L

IF NO OVERLAP

L case 2 R

Z
SEARCHING FOR OVERLAPPING INTERVALS

1D:

IF $z \geq L$

search left

IF NO OVERLAP

L

case 2

R

$\exists y, z'$

s.t.

$y < L < z'$

\{guaranteed overlap\}
SEARCHING FOR OVERLAPPING INTERVALS

1D:

IF $z \geq L$
- search left

ELSE 

$\exists y z' \ $ s.t. $y < L < z'$

$\{ \text{guaranteed overlap} \}$

ELSE $(z < L)$

IF NO OVERLAP
SEARCHING FOR OVERLAPPING INTERVALS

IF $Z \geq L$
- search left

IF NO OVERLAP

Z

IF $y < L < z'$
- guaranteed overlap

else $(Z < L)$
- NO overlap to left
- search right
How can we update MAX RIGHT END of SUBTREE?

BST w/ LEFT ENDS as KEYS
BST on $\text{lo}[s_i]$
BST

4 → 8

5 → 11

7 → 10

15 → 18

17 → 19

21 → 23
augmented BST

$max(t) = \max \left\{ h_i(t), \max(t_L), \max(t_R) \right\}$
max1, max2, max3: unchanged by rotation
max(A) & max(B): trivial to update

we can maintain a balanced BST augmented w/ max value of subtrees