Starting at top-left of $n \times m$ grid, moving only down or right, how many ways to reach bottom-right?

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**repetitive subproblems**

want to avoid repetition

$A[r,c] = \min\{A[r-1,c], A[r,c-1]\}$

$\Omega(2^n)$ for $n \times n$
Starting at top-left of $n \times m$ grid, moving only down or right, how many ways to reach bottom-right?
How many times will we recurse in a unique way?

\[ A[r, c] \rightarrow r \cdot c \text{ distinct subproblems} \]

- \( A[r-1, c] \)
- \( A[r, c-1] \)
- \( [r-2, c] \)
- \( [r-2, c-1] \)
- \( [r-3, c] \)
- \( [r-3, c-1] \)
- \( [r-3, c-2] \)
- \( [r-3, c-3] \)
- \( [r-2, c-2] \)
- \( [r-2, c-3] \)
- \( [r-1, c-2] \)
- \( [r-1, c-3] \)
- \( [r, c-2] \)
- \( [r, c-3] \)

how many times will we realize that we have seen a subproblem before?
MEMOIZATION (making memos)

For this problem, m x n table


Recursion:
first find \( A[r-1,c] \)
then find \( A[r,c-1] \)
MEMOIZATION (making memos)

For this problem, m x n table


Recursion:
- first find \( A[r-1,c] \)
- then find \( A[r,c-1] \)
MEMOIZATION (making memos)

For this problem, \( m \times n \) table

\[
\]

Recursion:
- first find \( A[r-1,c] \) ↑
- then find \( A[r,c-1] \) ←

\( \Theta(n,m) \) time & space
Starting at top-left of $n \times m$ grid, moving only down or right, how many ways to reach bottom-right?

**Dynamic Programming** (bottom-up: base cases first)


Fill any cell as long as what it depends on is full.
Starting at top-left of \( nxm \) grid, moving only down or right, how many ways to reach bottom-right?

**Dynamic Programming** (bottom-up: base cases first)

\[
\begin{array}{cccccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\hline
1 & 2 \\
1 \\
1 \\
1 \\
1 \\
\end{array}
\]

\[
\]

Fill any cell as long as what it depends on is full.
Starting at top-left of \(n \times m\) grid, moving only down or right, how many ways to reach bottom-right?

DYNAMIC PROGRAMMING (bottom-up: base cases first)

\[
\]

Fill any cell as long as what it depends on is full.
Starting at top-left of nxm grid, moving only down or right, how many ways to reach bottom-right?

**Dynamic Programming (bottom-up: base cases first)**


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Fill any cell as long as what it depends on is full.
Starting at the top-left of an nxm grid, moving only down or right, how many ways to reach the bottom-right? ... with obstacles

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- Each empty cell represents a possible path to reach the bottom-right.
- Obstacles are marked with 'O'.
Starting at top-left of $n \times m$ grid, moving only down or right, how many ways to reach bottom-right? ... with obstacles

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"... with obstacles" refers to the cells marked with "0" which are considered obstacles.
Starting at top-left of nxm grid, moving only down or right, how many ways to reach bottom-right? ... with obstacles

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