MINIMUM (weight) SPANNING TREES

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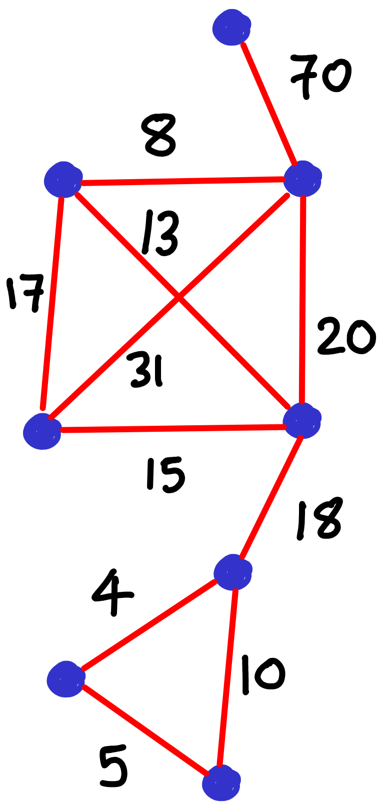

Input: graph w/ edge weights

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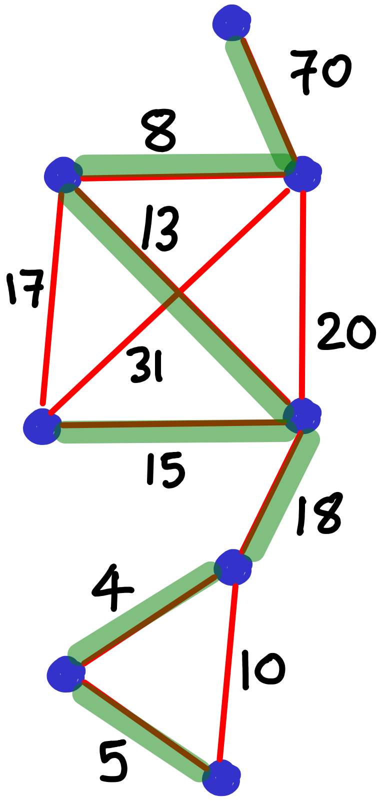

Input: graph w/ edge weights

Output:

- ✓ tree
- ✓ span (reach) all vertices
- ✓ minimize sum of weights

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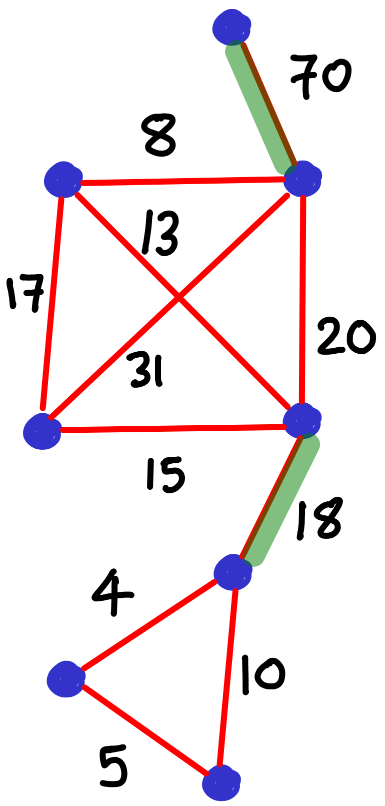

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Observations:

- Any critical edge (in terms of graph connectivity) must be in the MST (e.g. 70, 18)

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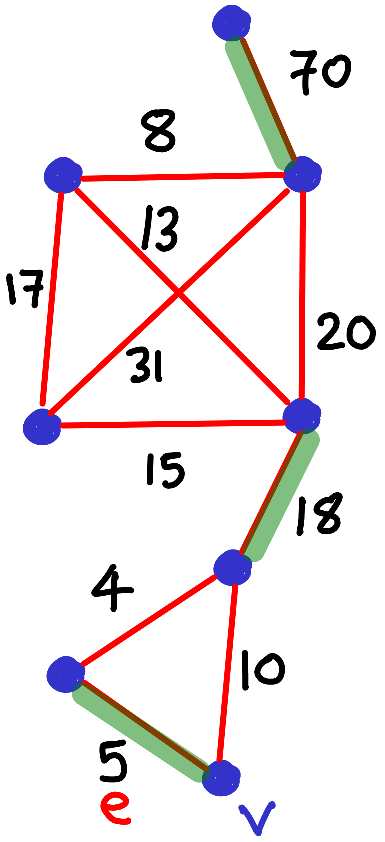

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- Any critical edge (in terms of graph connectivity) must be in the MST (e.g. 70, 18)

- For any vertex v with 2 incident edges, the smaller edge e must be in the MST WHY?

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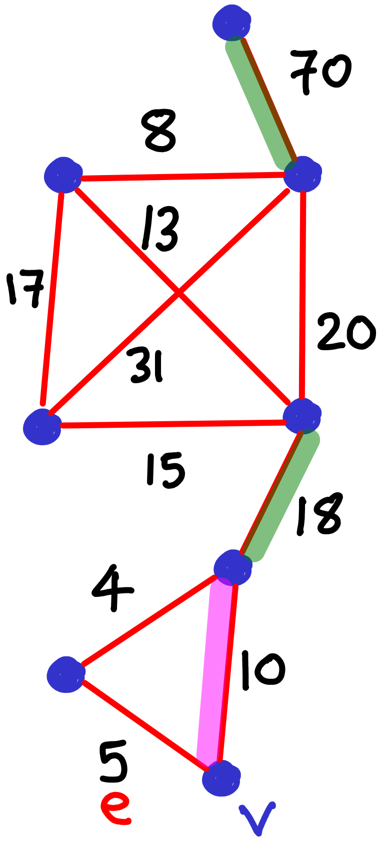

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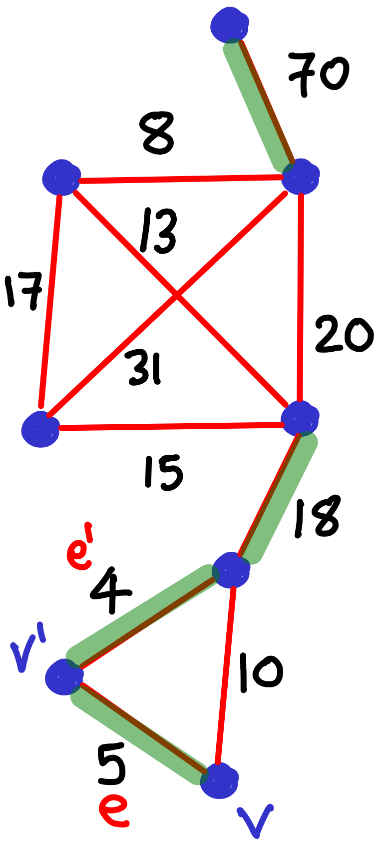
MINIMUM (weight) SPANNING TREES



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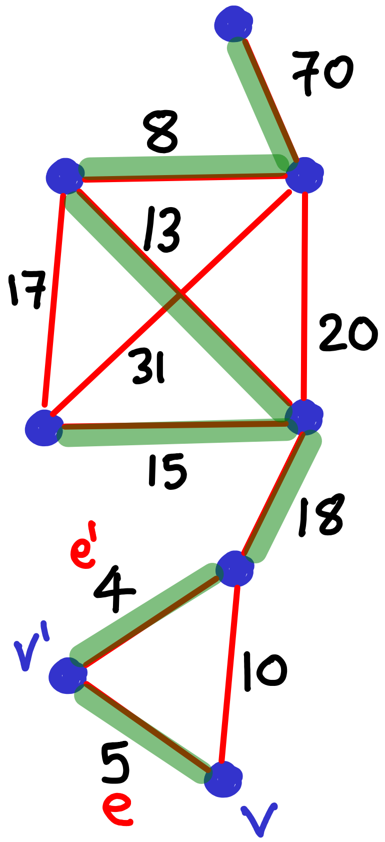

Input: graph w/ edge weights

Output:
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Observations, that we will generalize:

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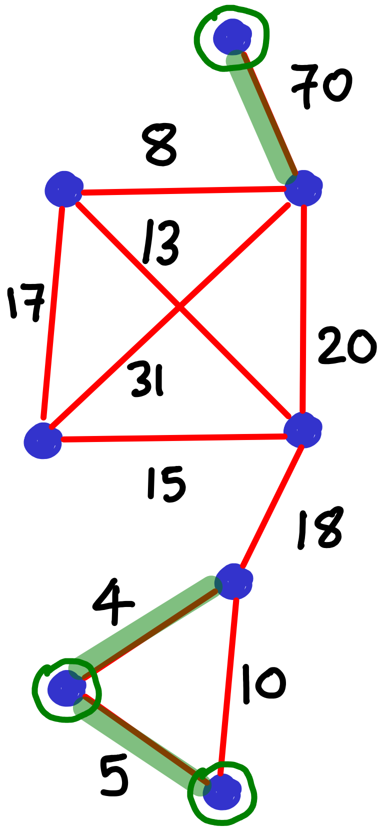
So far, we know that for degree-1 & degree-2 vertices
the lightest incident edge must be in MST



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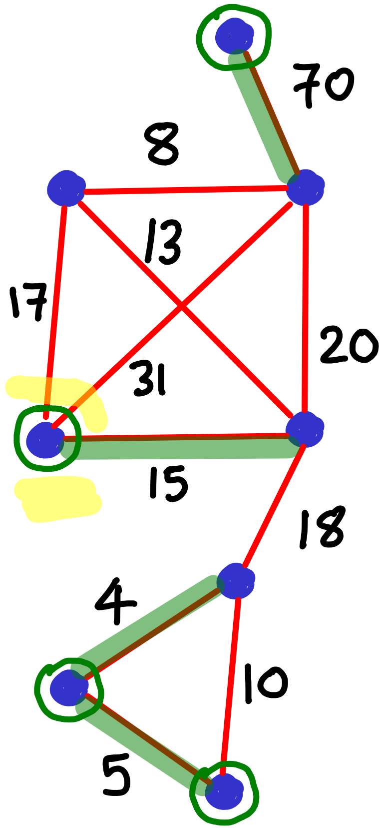
This holds for all vertices



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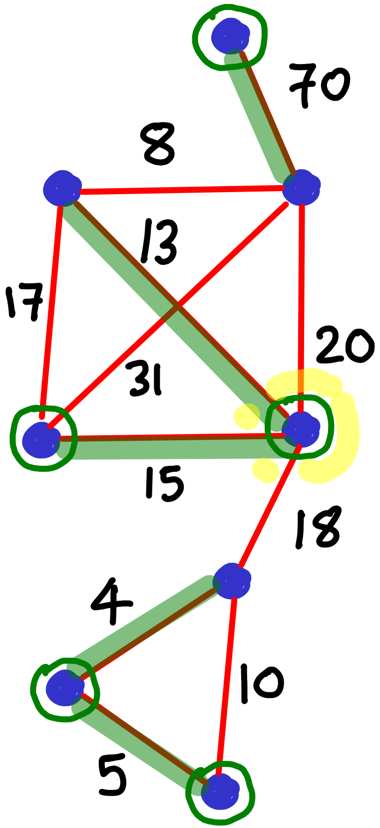

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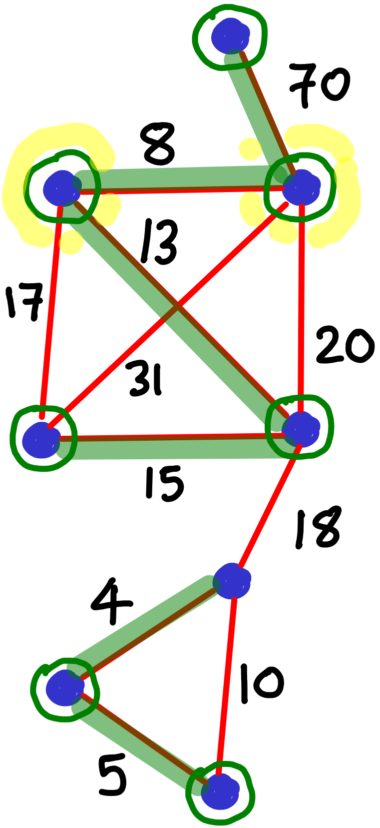

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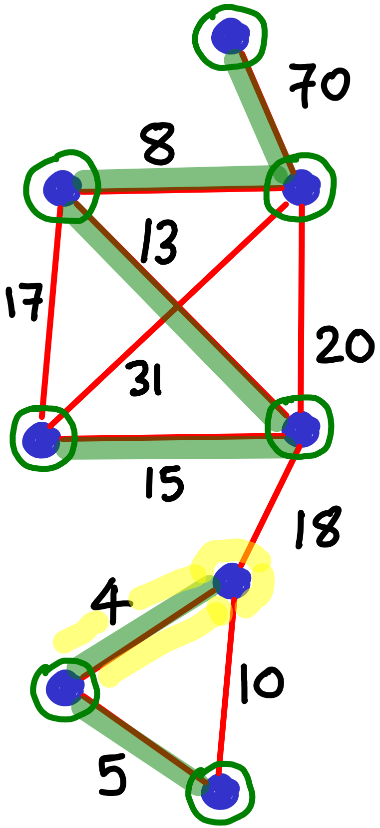

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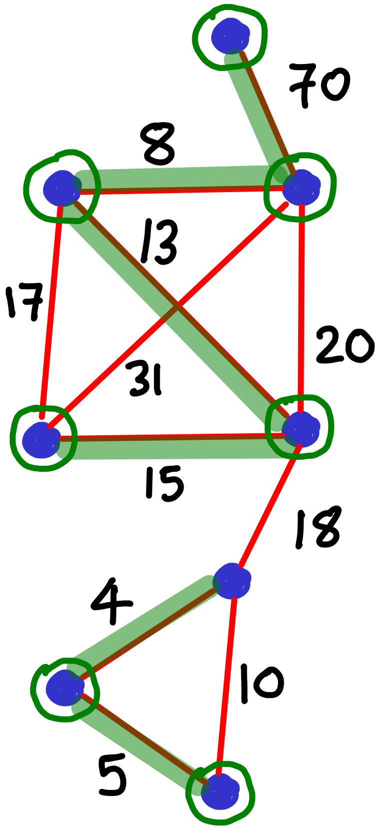

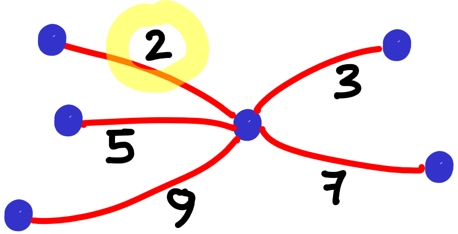

Suppose not true

So far, we know that for degree-1 & degree-2 vertices
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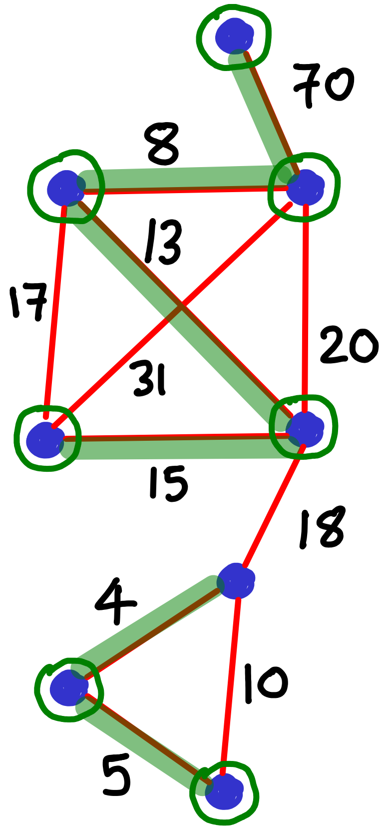
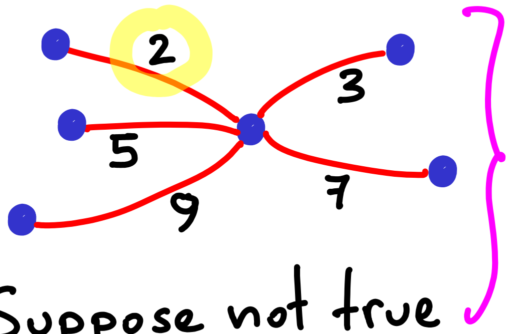
This holds for all vertices





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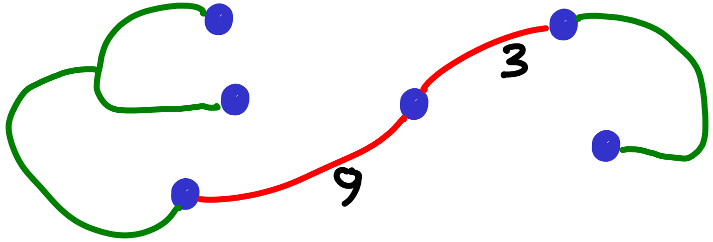

"MST" without 2

So far, we know that for degree-1 & degree-2 vertices the lightest incident edge must be in MST

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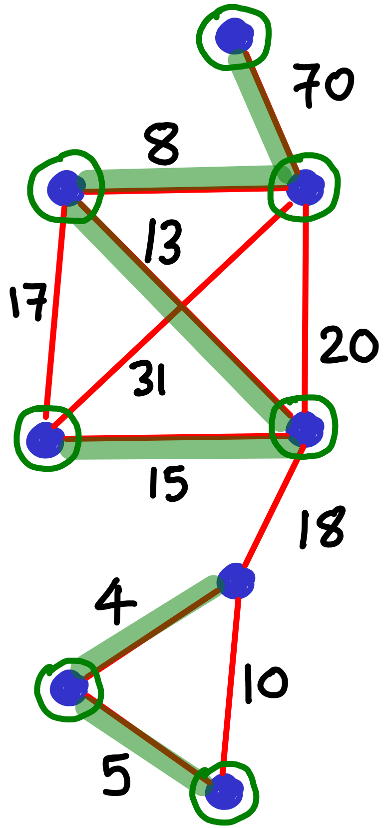

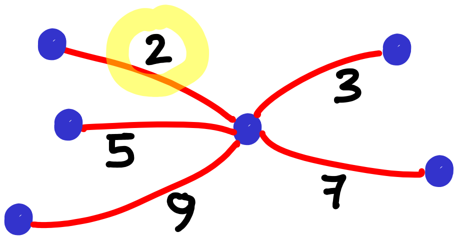

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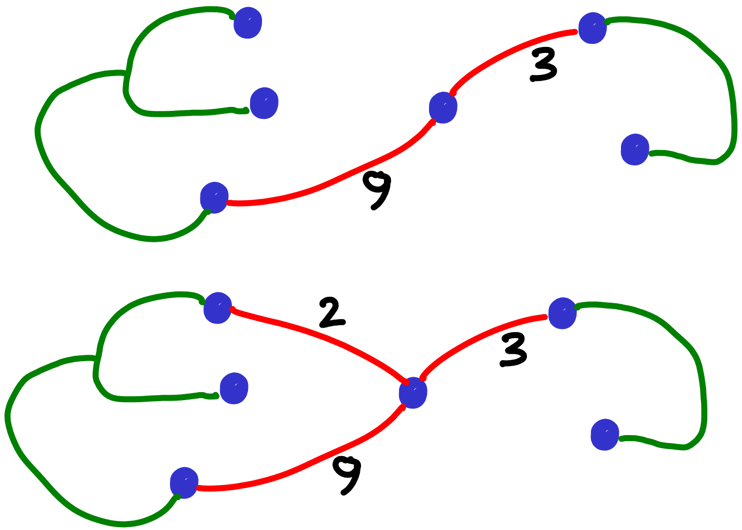


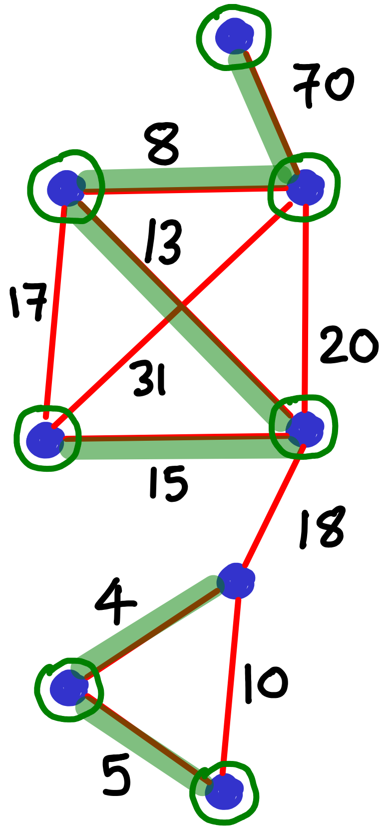
"MST" without 2

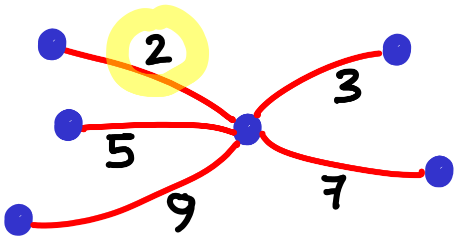
↓

Put 2 in.

So far, we know that for degree-1 & degree-2 vertices the lightest incident edge must be in MST

This holds for all vertices





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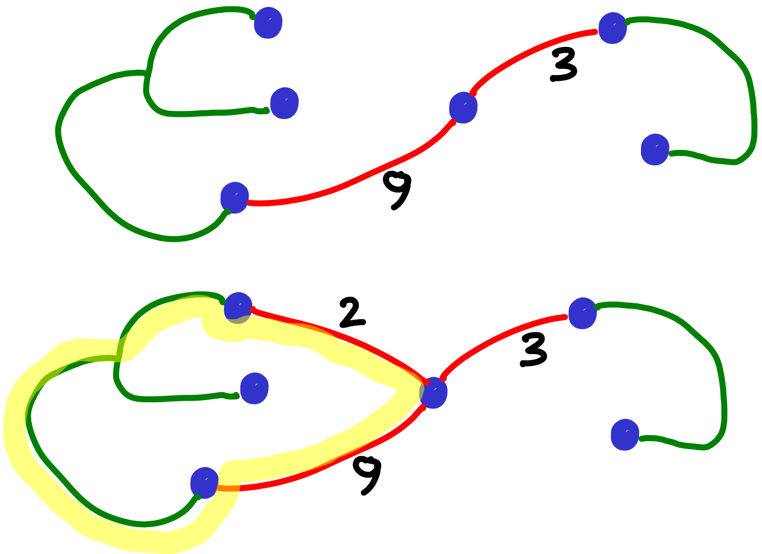


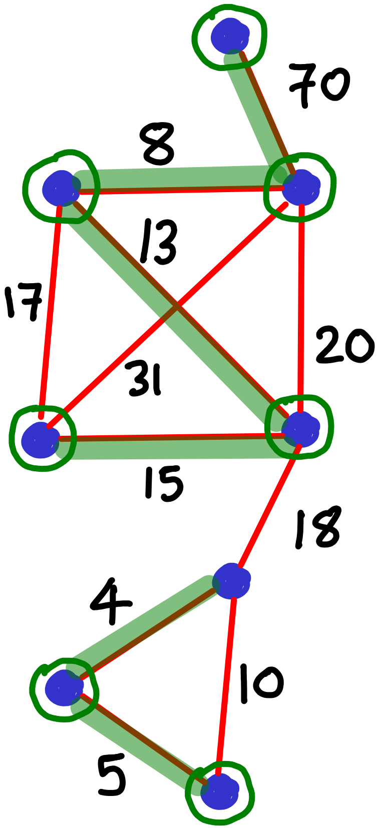
"MST" without 2

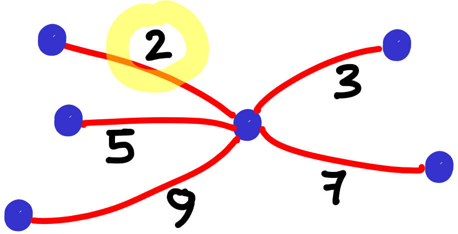
↓

Put 2 in.
Create cycle

So far, we know that for degree-1 & degree-2 vertices the lightest incident edge must be in MST

This holds for all vertices





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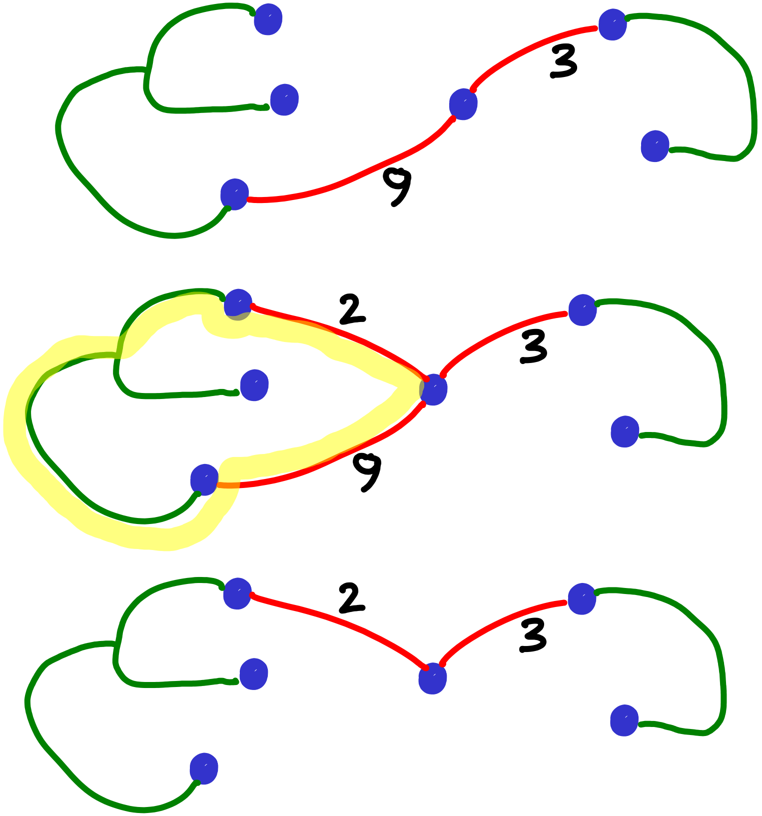


"MST" without 2

↓

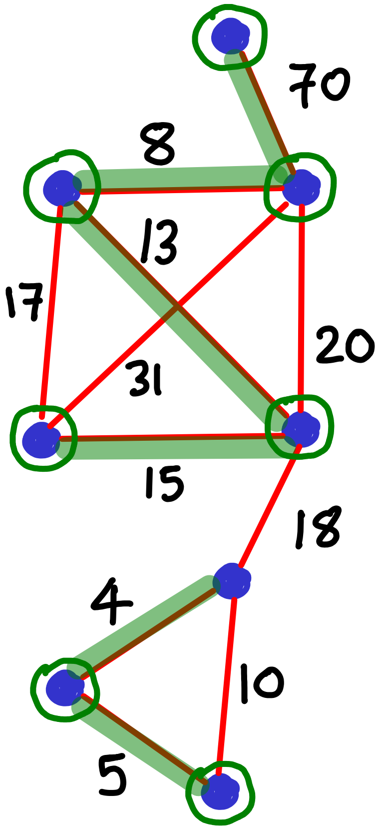
Put 2 in. Create cycle

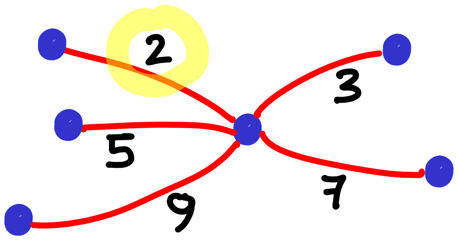
↓

Remove last edge on cycle

So far, we know that for degree-1 & degree-2 vertices the lightest incident edge must be in MST

This holds for all vertices





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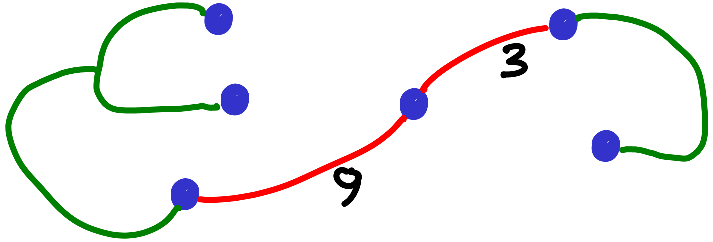

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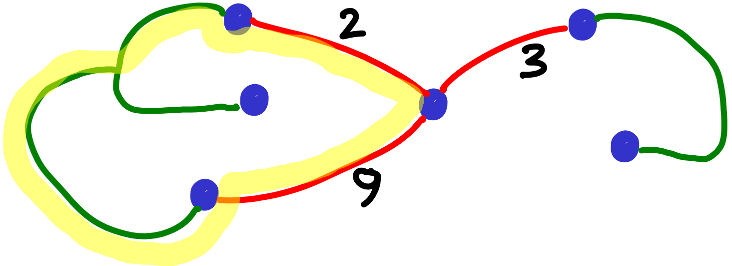
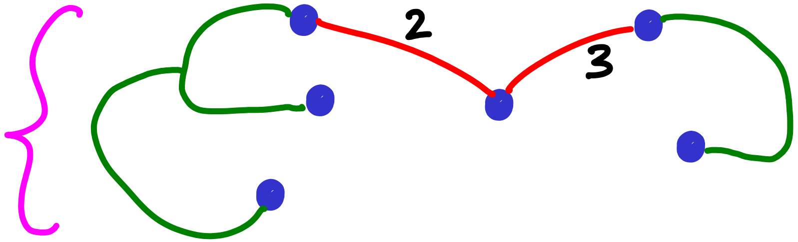


Put 2 in. Create cycle

Better spanning tree:



Remove last edge on cycle

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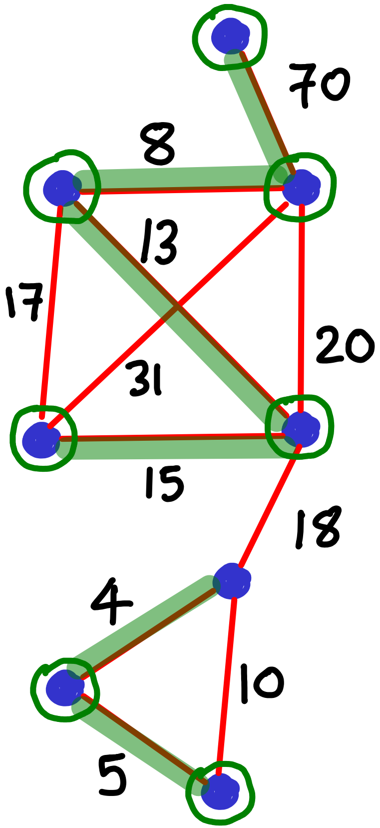

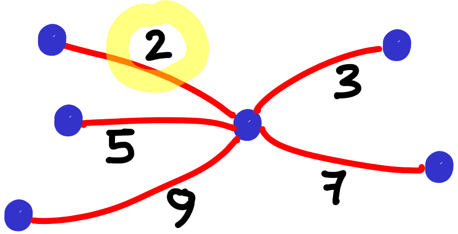

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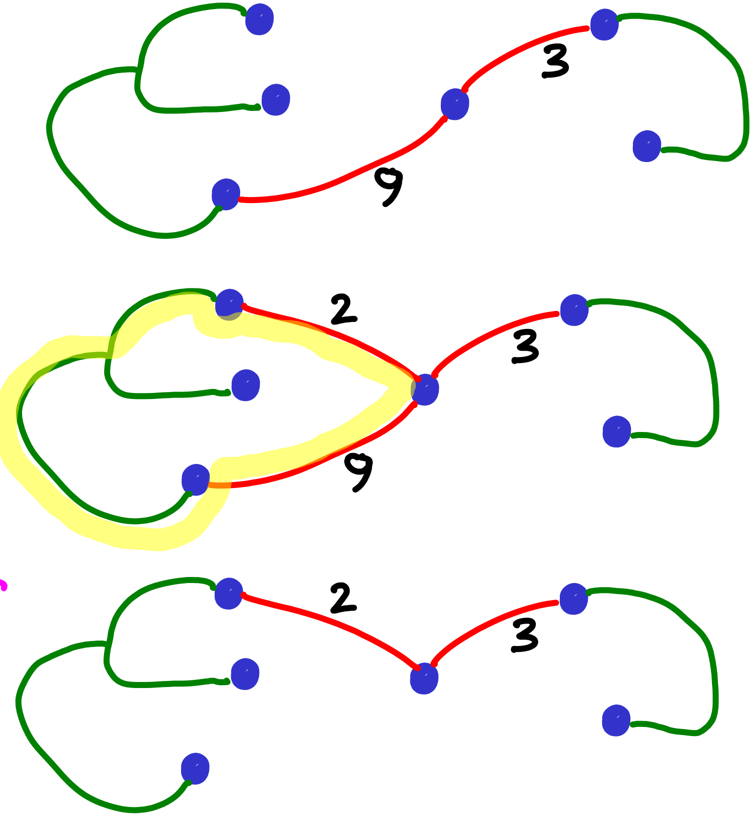


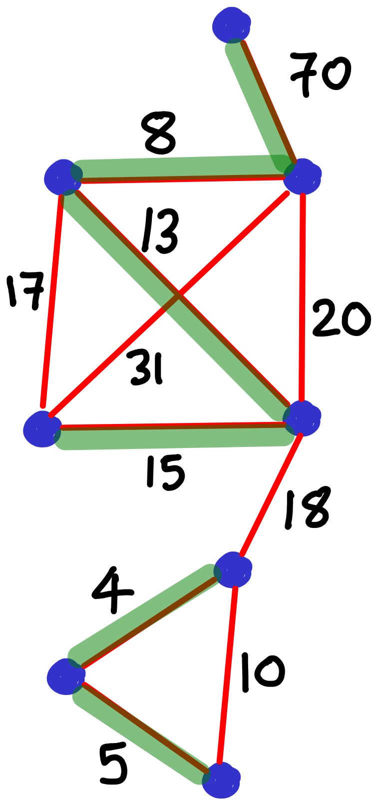
"MST" * without 2

Put 2 in. Create cycle

Remove last edge on cycle

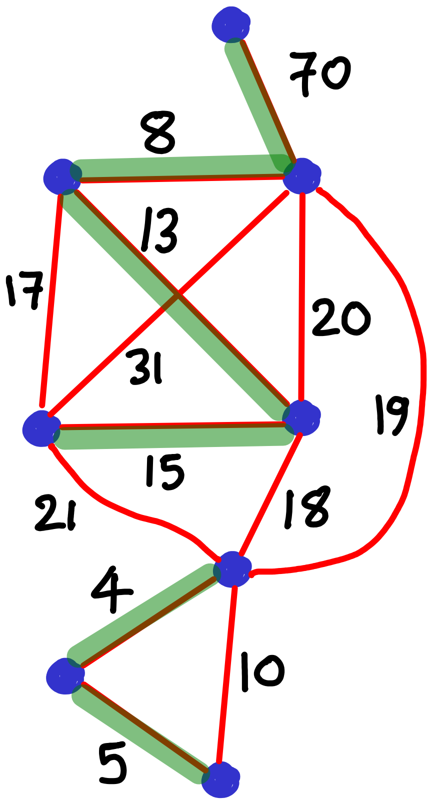
* Better spanning tree:
Contradiction

If every vertex votes for one edge, we might not get the entire MST.



If every vertex votes for one edge, we might not get the entire MST.

What should we do in this example?



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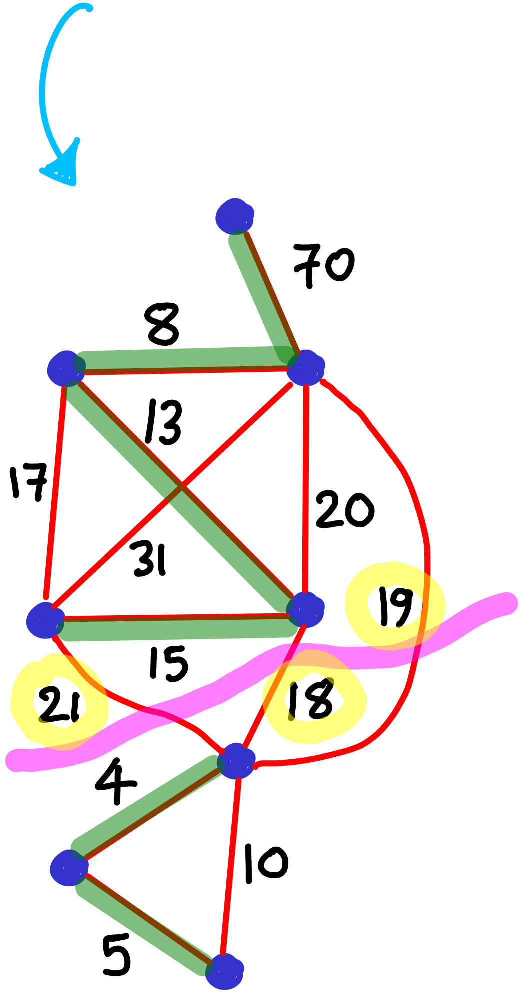
What should we do in this example? Best connection: min{21, 18, 19}



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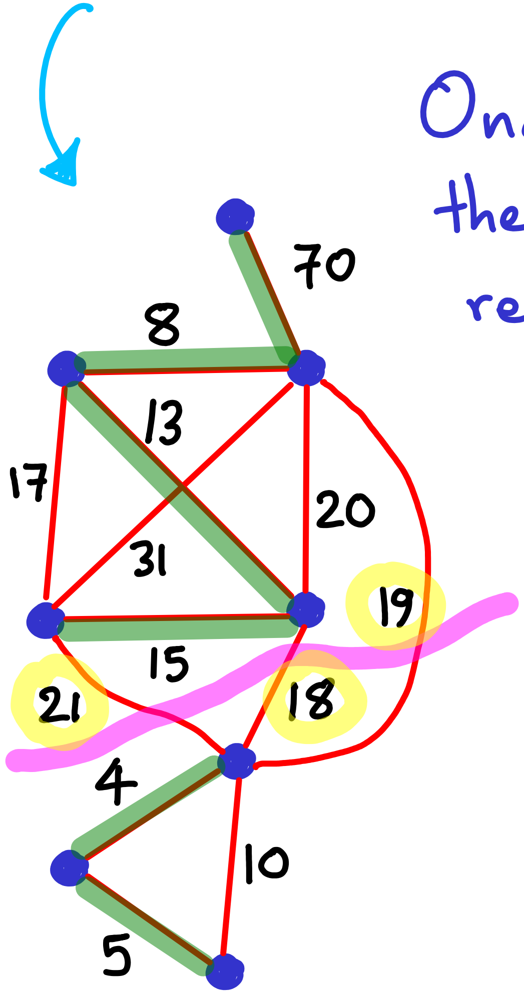
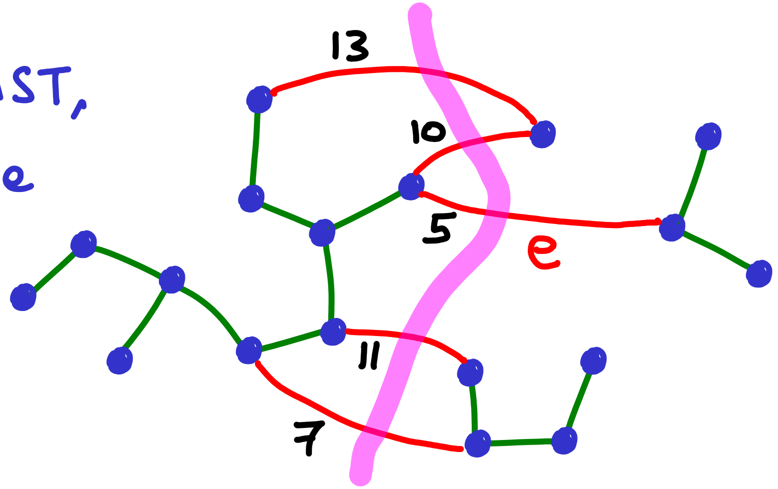


Once you know a component of MST, the lightest edge connecting it to the rest of the graph must be in MST.

WHY?



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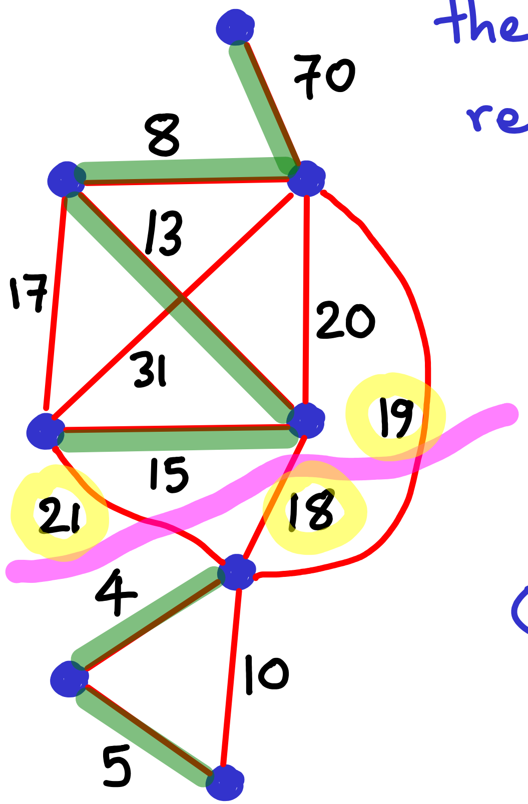

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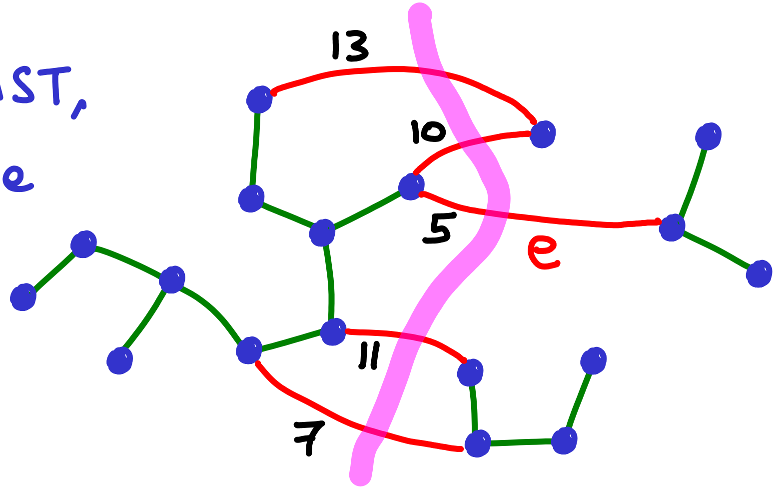
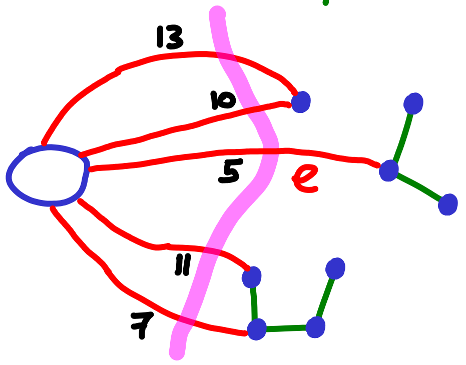


WHY?

Same proof by contradiction as before



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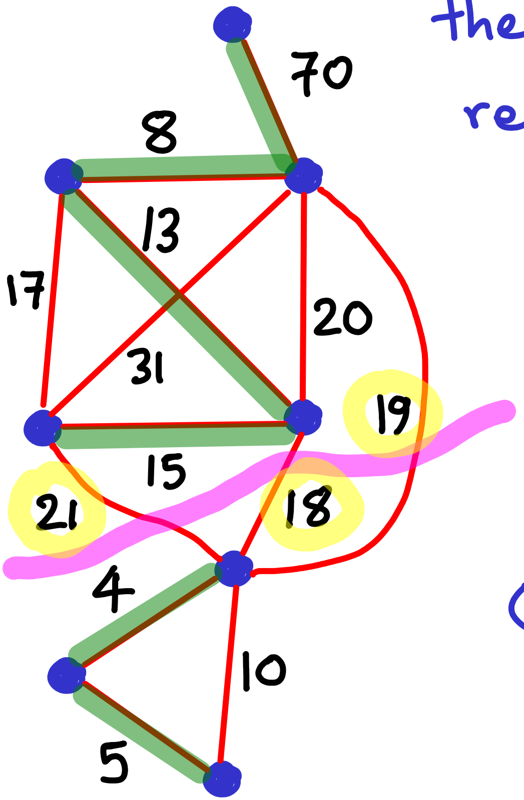

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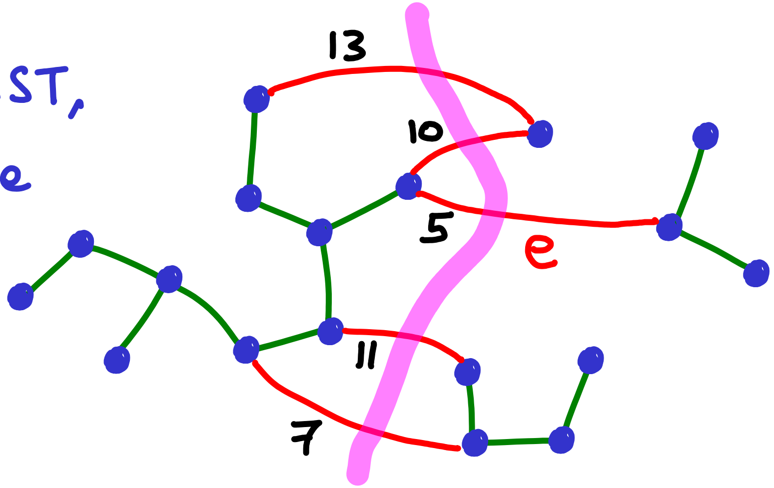

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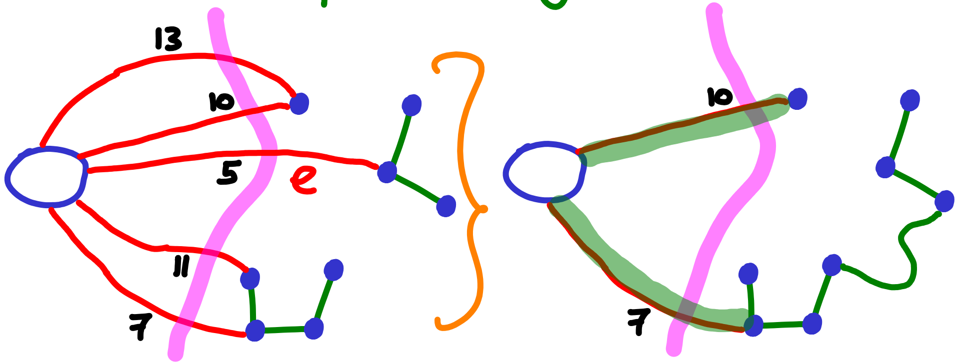

Whatever MST you get,

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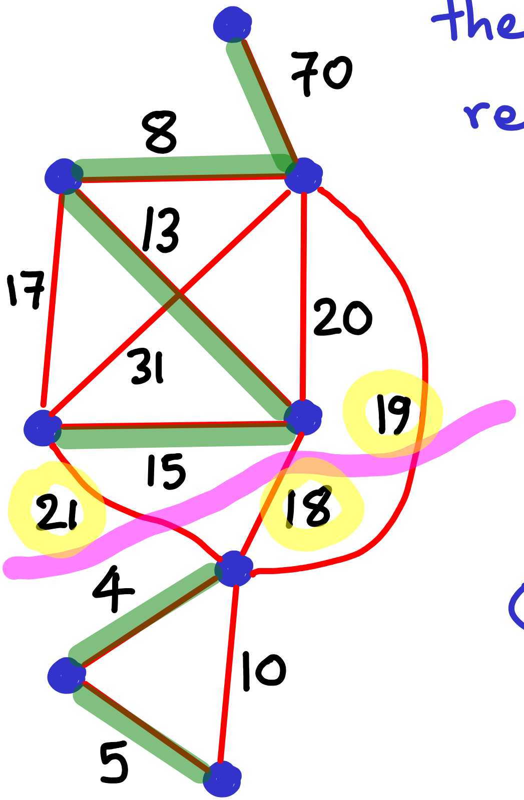

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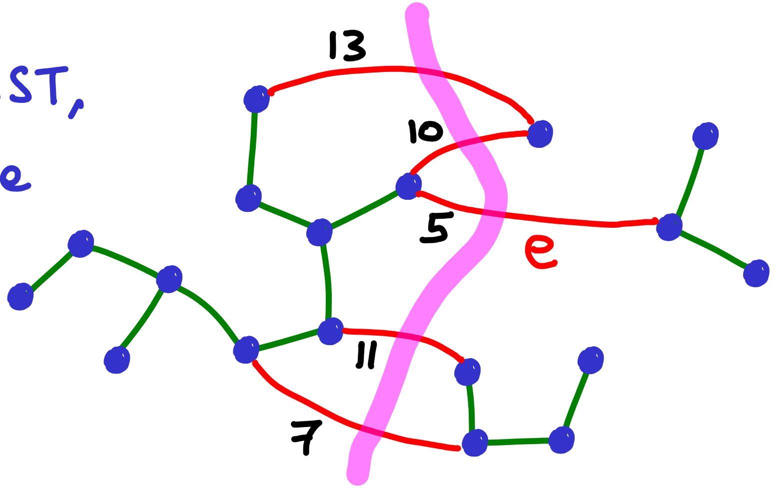

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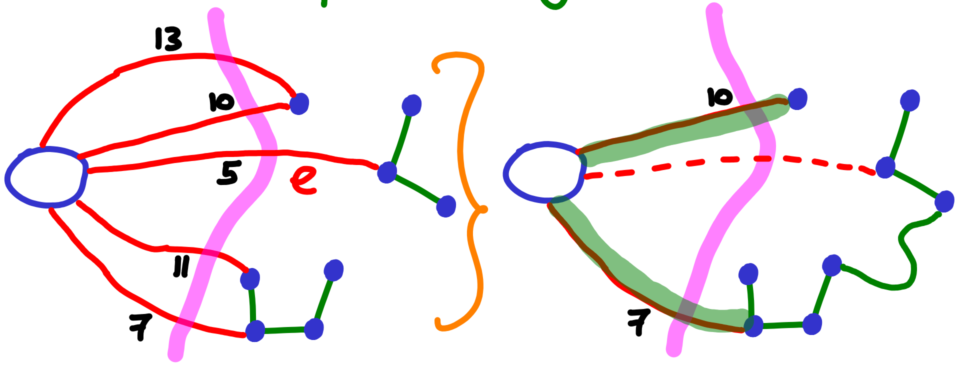

Whatever MST you get, insert e, get cycle

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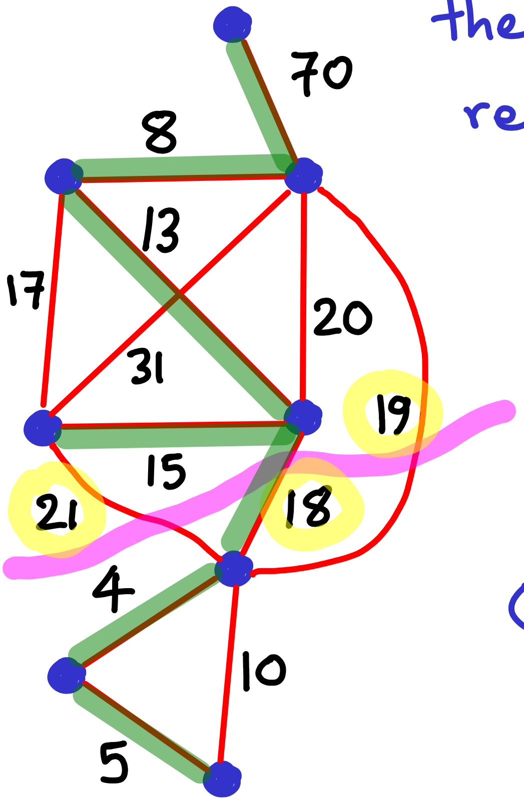

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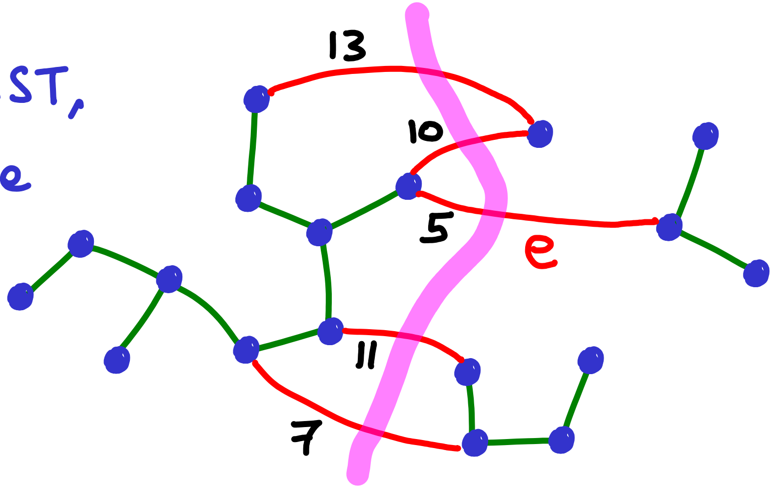

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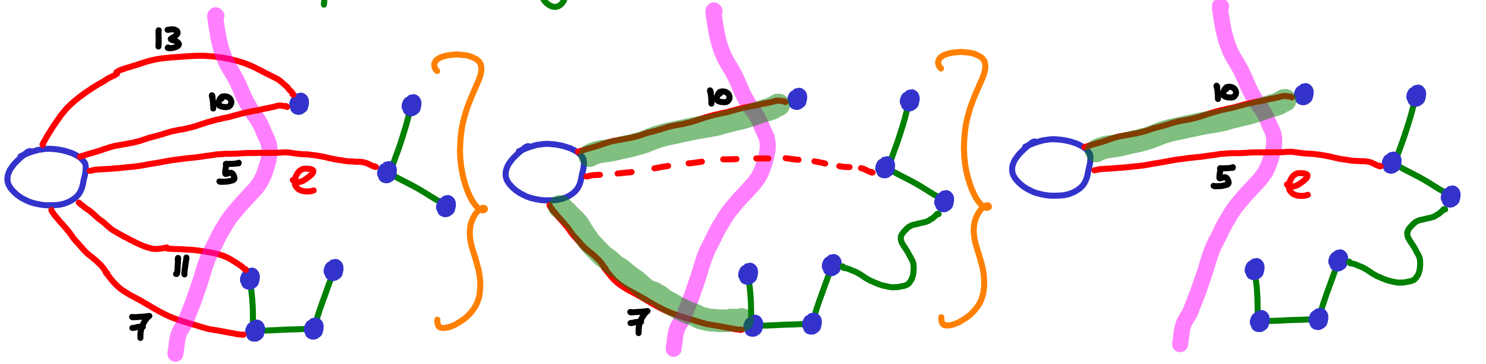
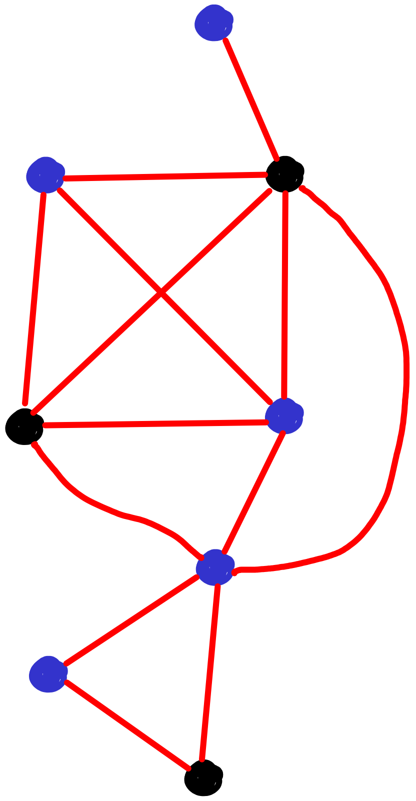


Whatever MST you get, insert e, get cycle, improve MST, contradiction

$A \subseteq V$

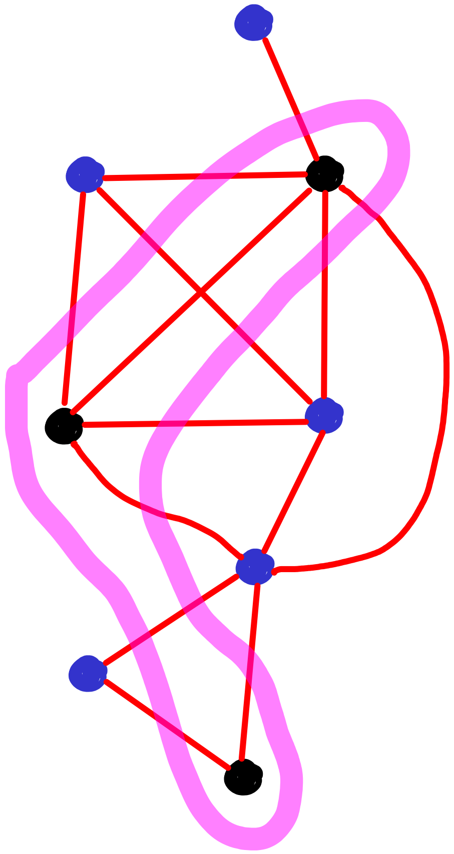
$B : V - A$



$A \subseteq V$

$B: V - A$

Cut separates A, B



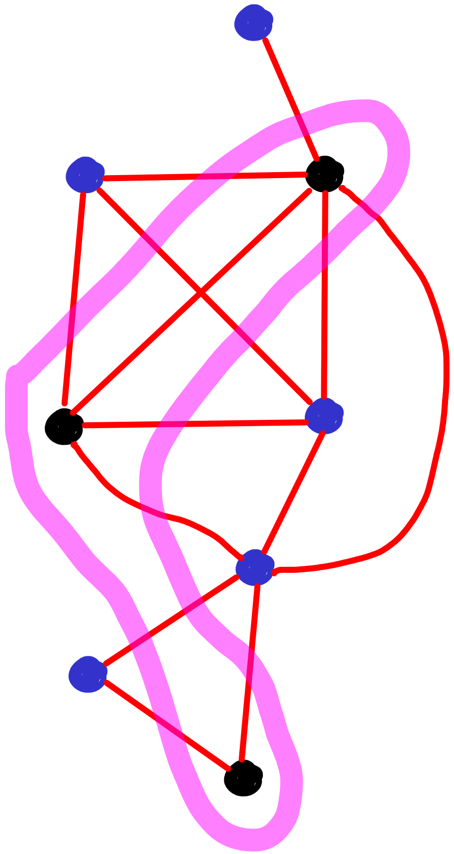
$A \subseteq V$

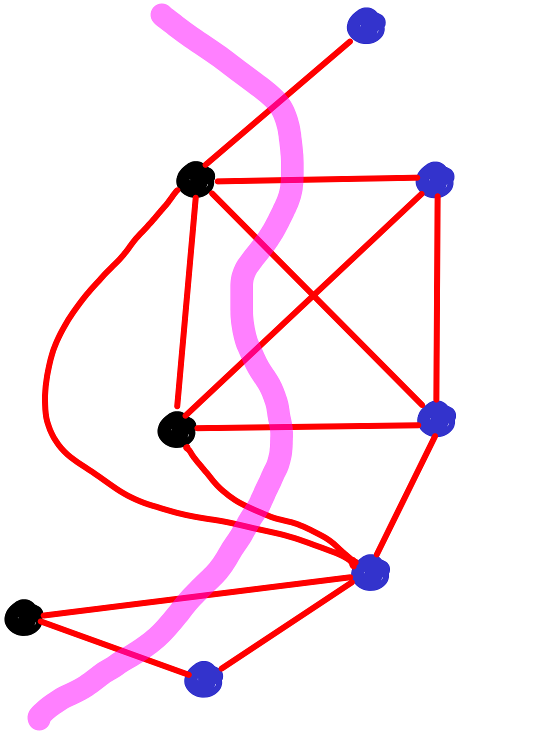
$B: V - A$

Cut separates A, B

Redraw G

Cut crosses all 





This is an abstract concept.
(independent of drawing)

A cut identifies all edges between A, B

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Cut separates A, B

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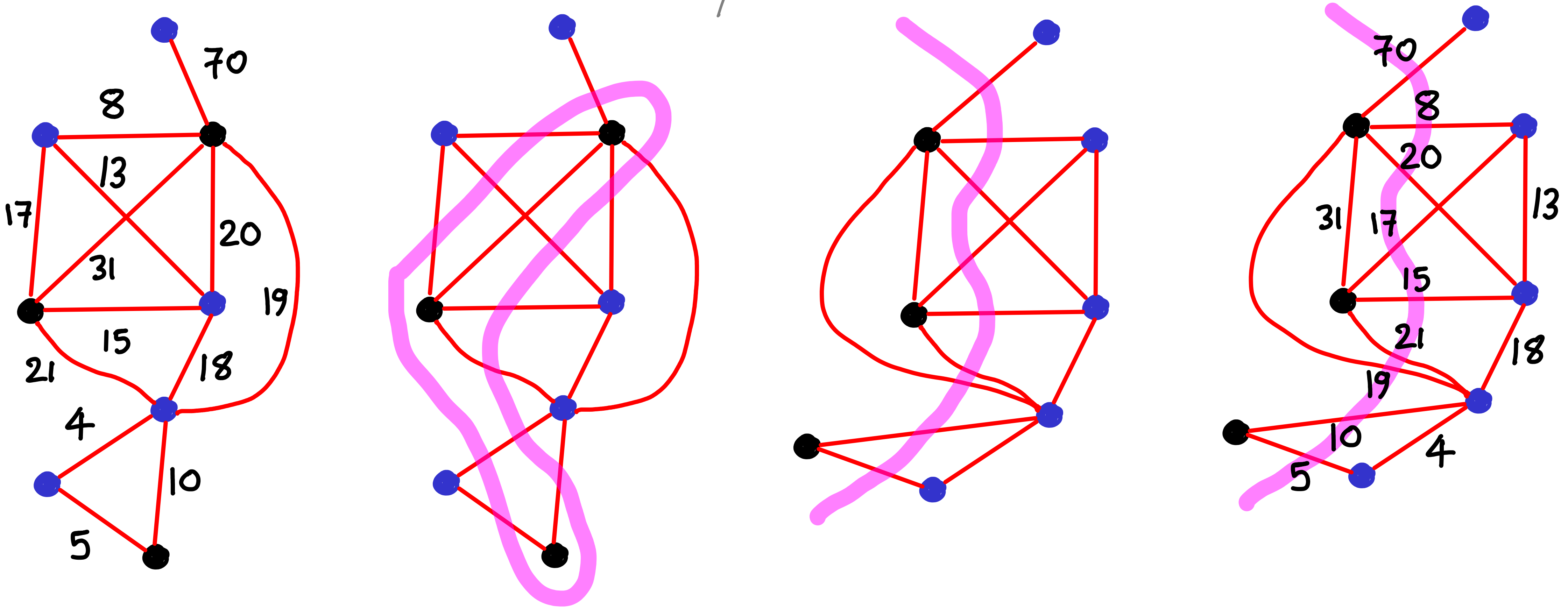

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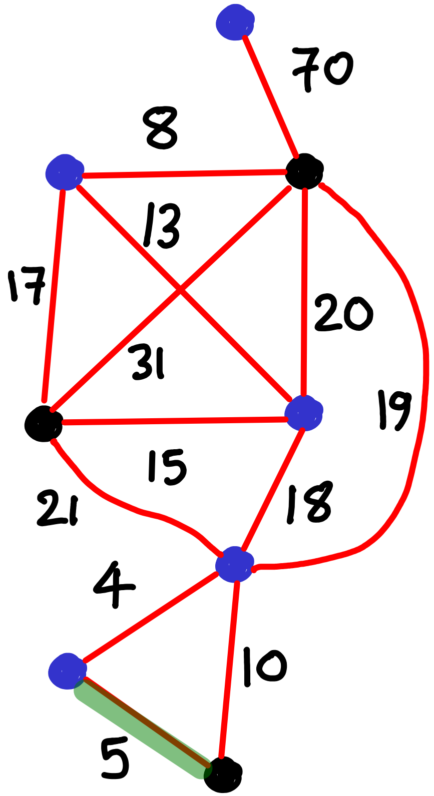
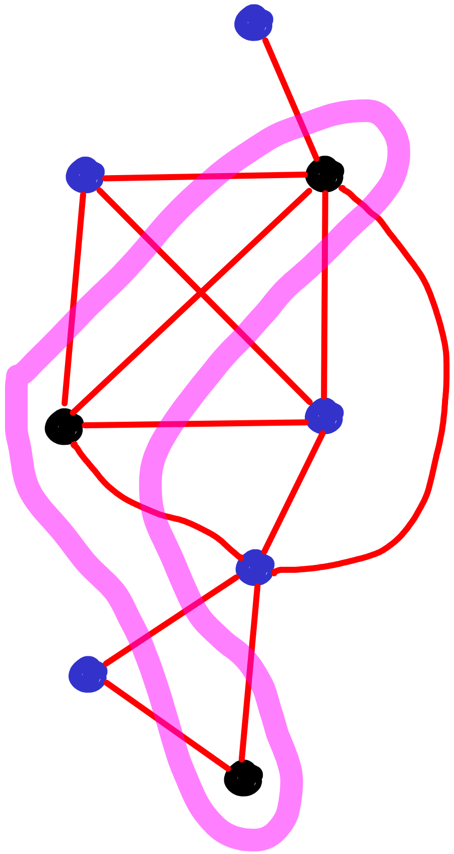
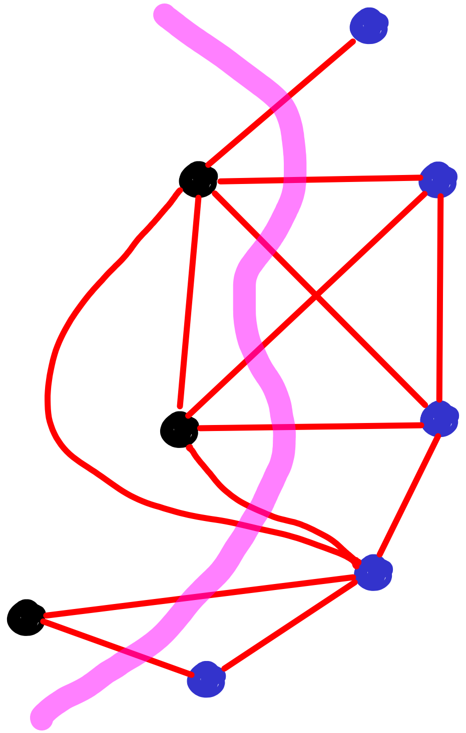
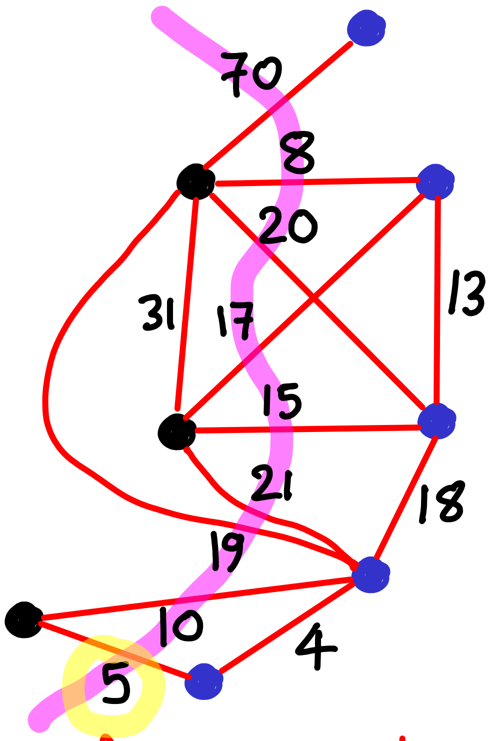








This is an abstract concept.
(independent of drawing)

A cut identifies all edges between A, B

CLAIM: for any cut, the min-weight edge crossing the cut must be in MST

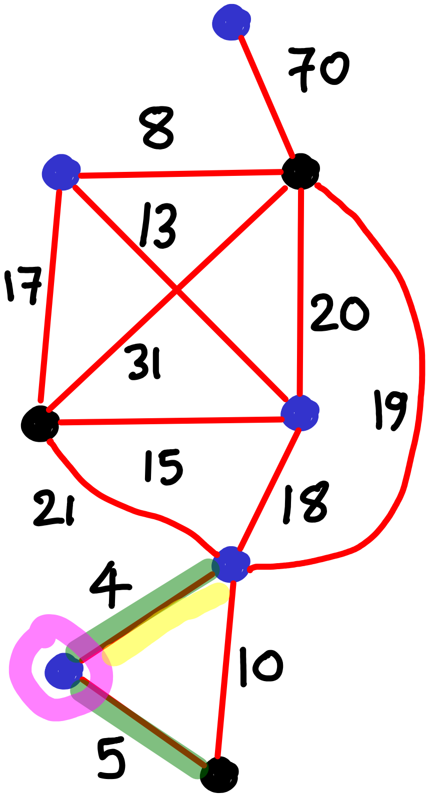
$A \subseteq V$

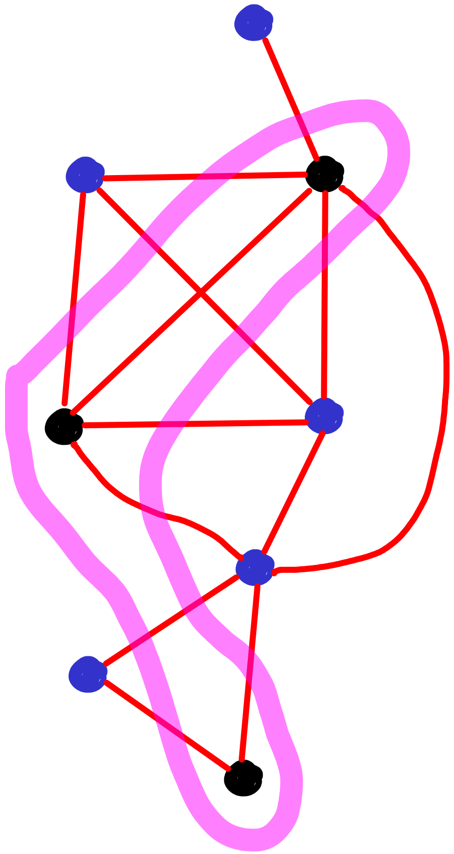
$B: V - A$

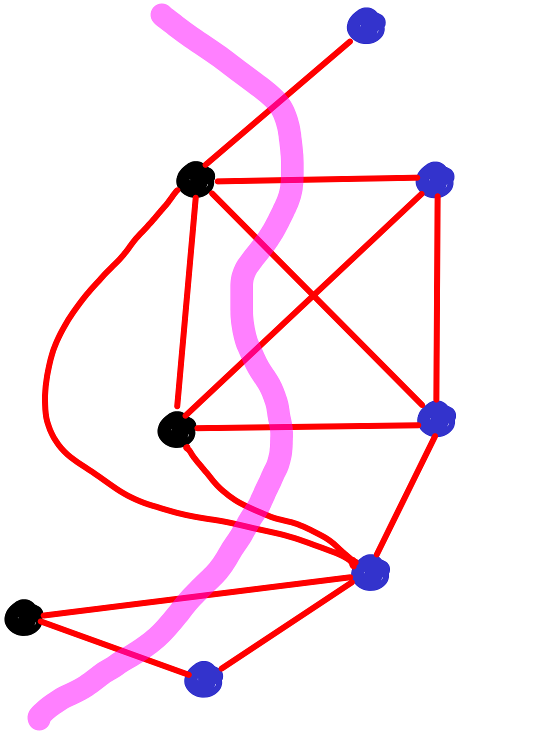
Cut separates A, B

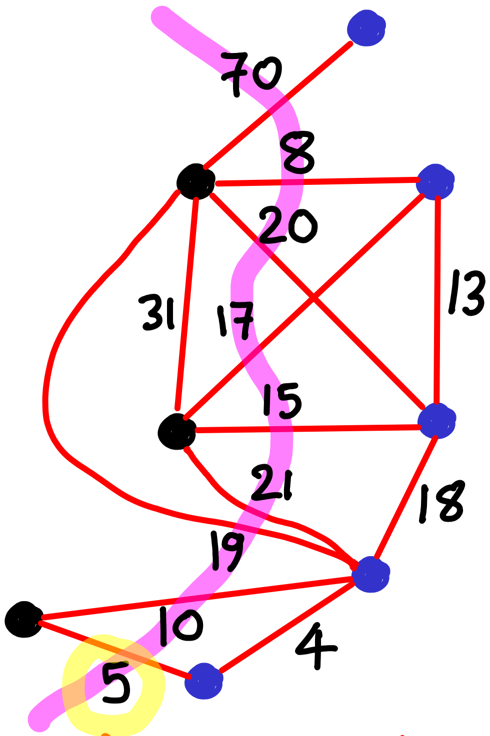
Redraw G

Cut crosses all









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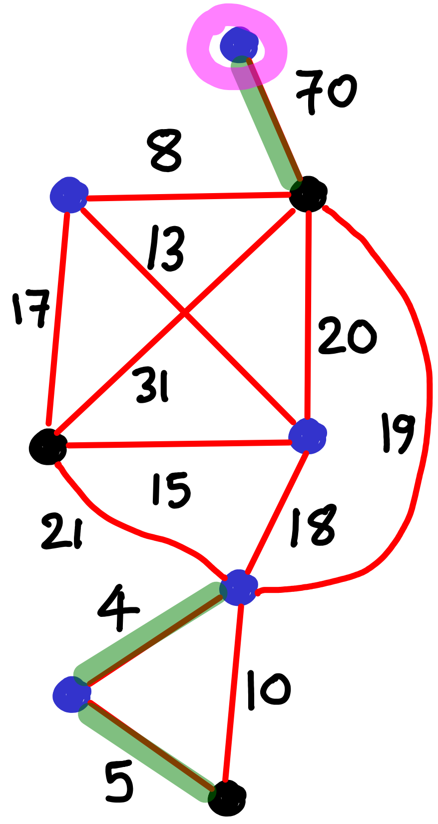
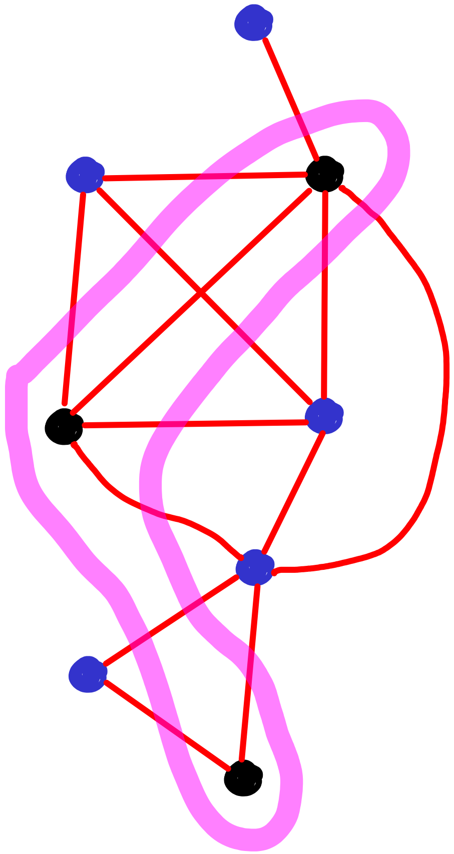
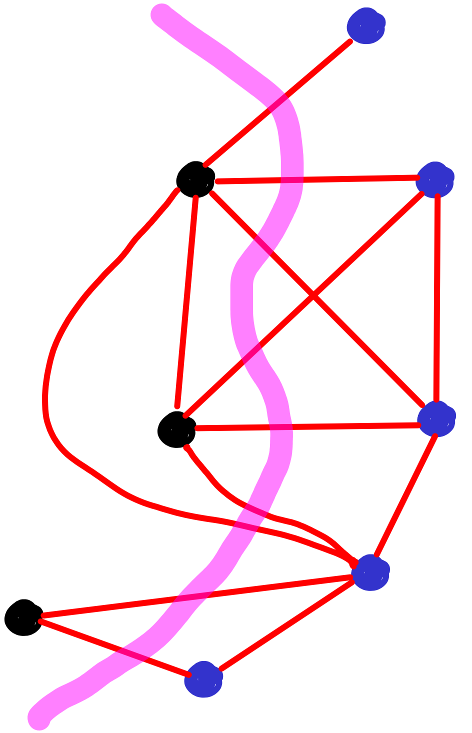
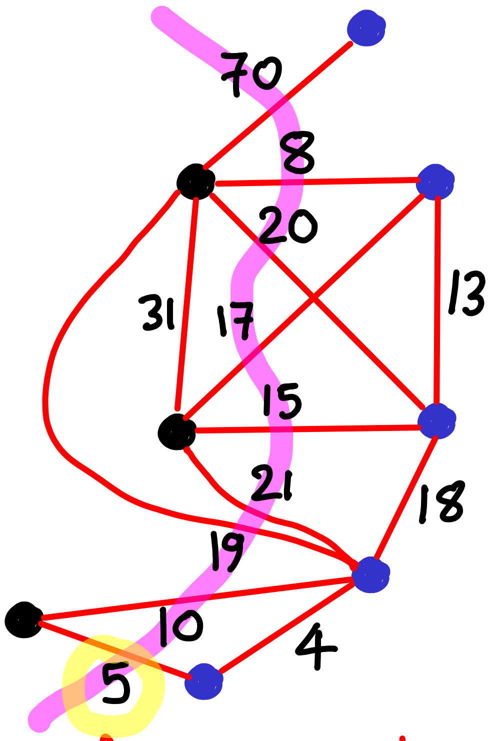




Redraw G

Cut crosses all [edge]





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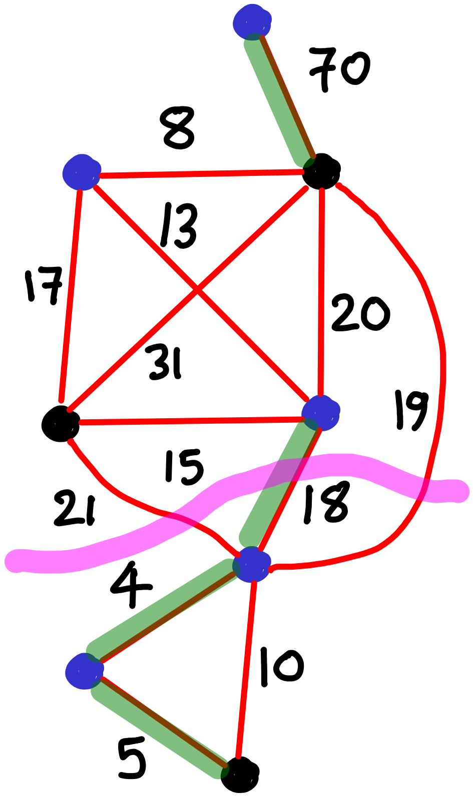
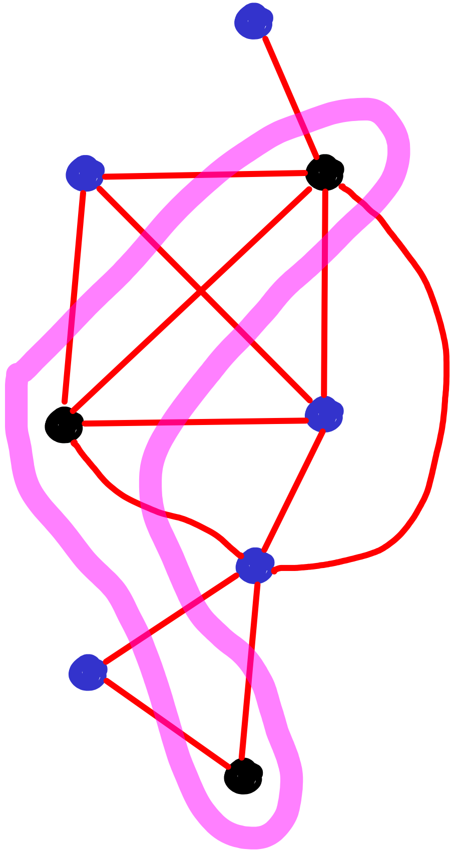
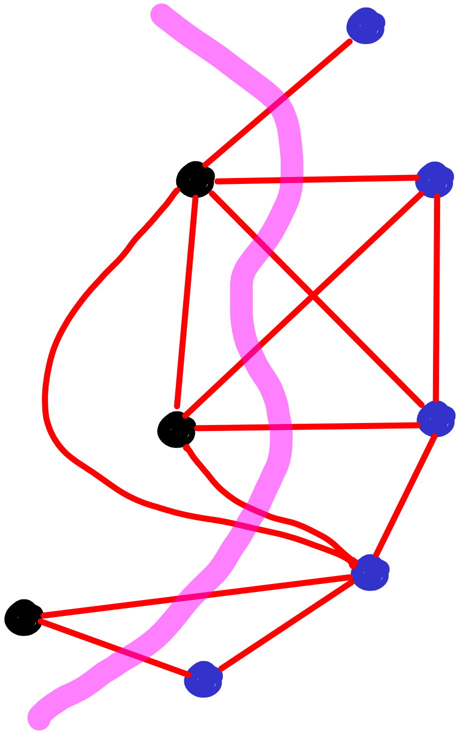
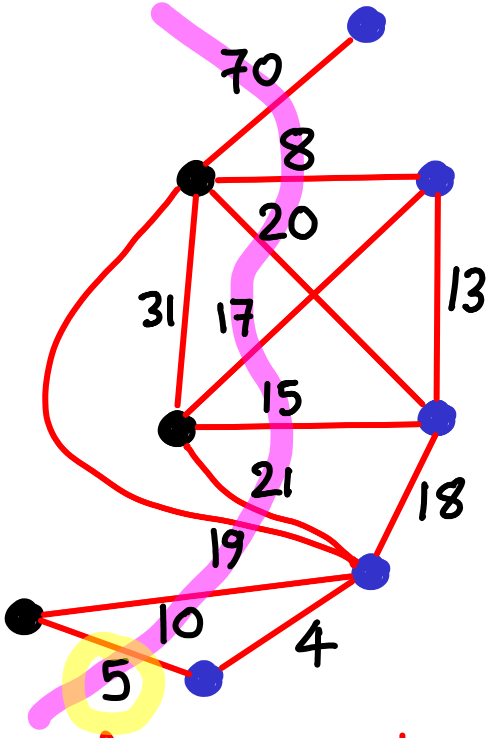
Redraw G

Cut crosses all 









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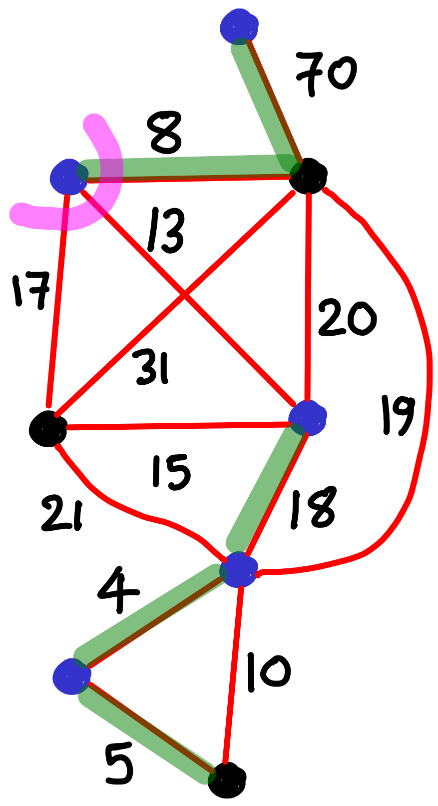

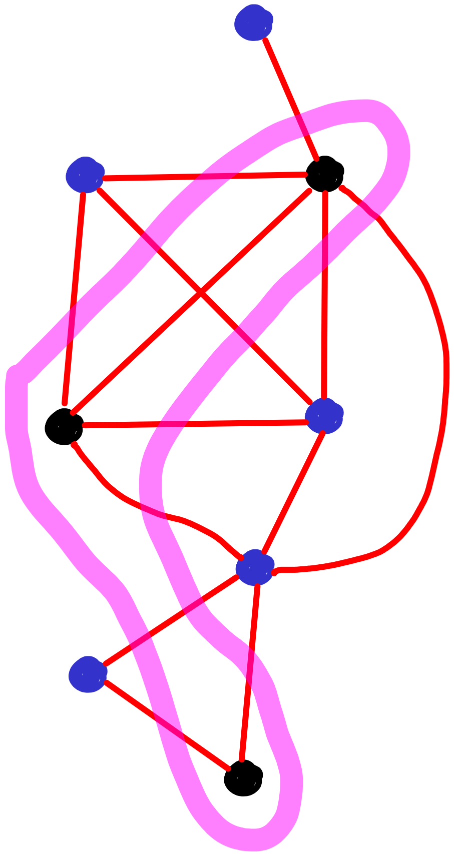

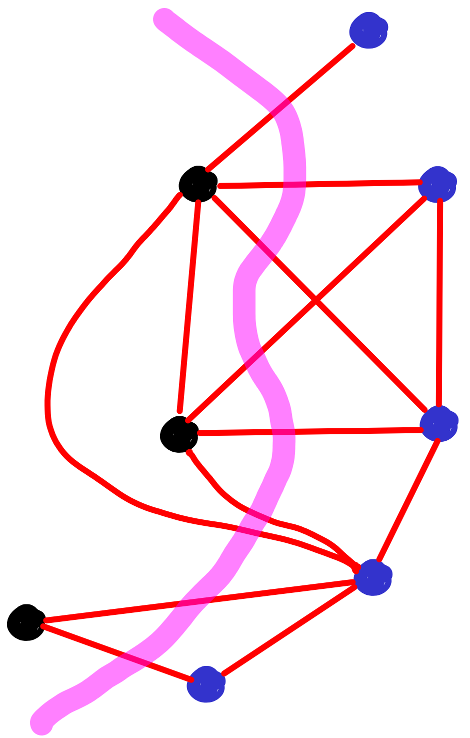

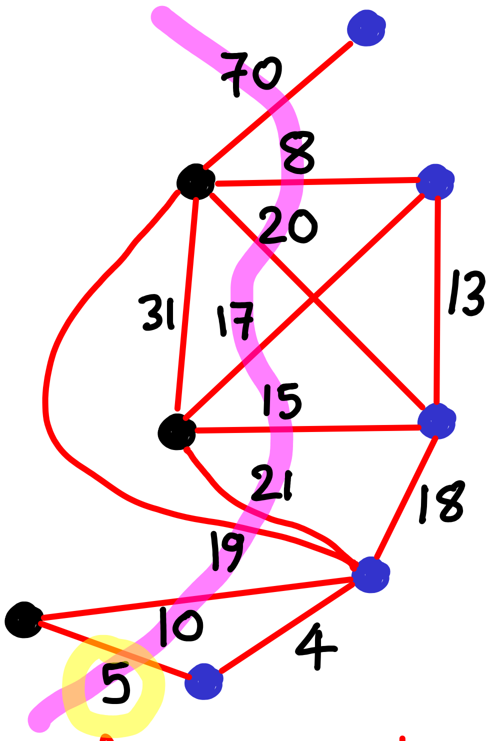

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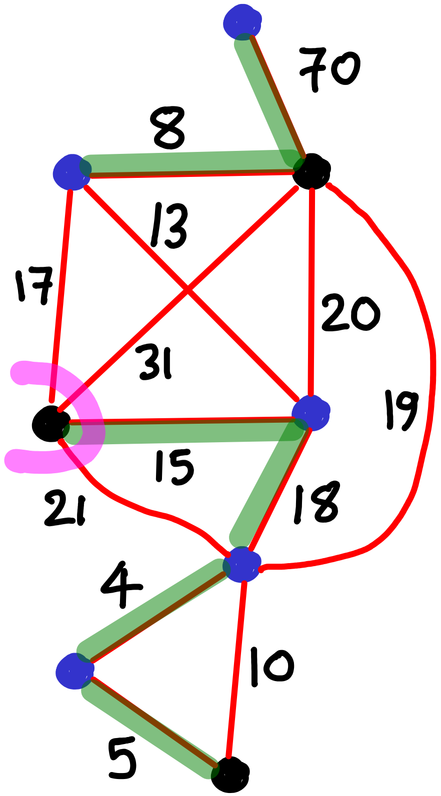
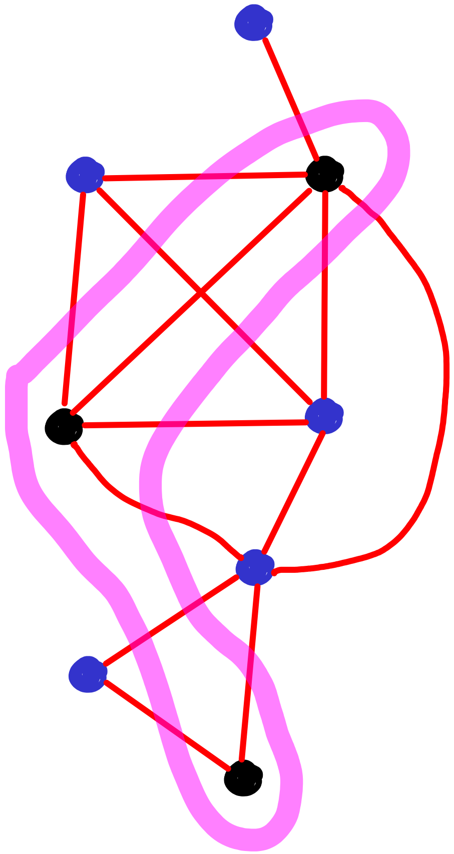
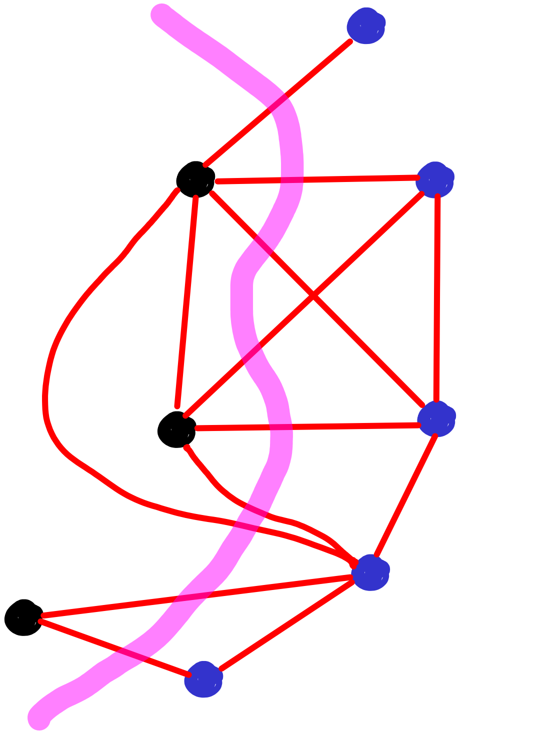
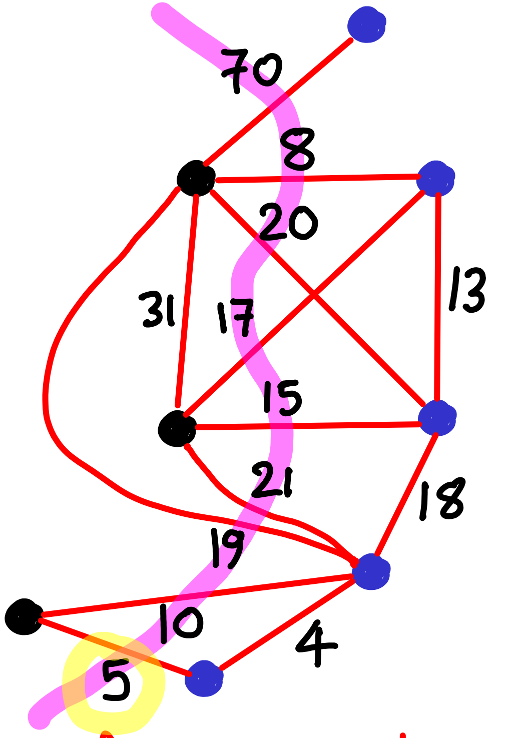
Redraw G

Cut crosses all









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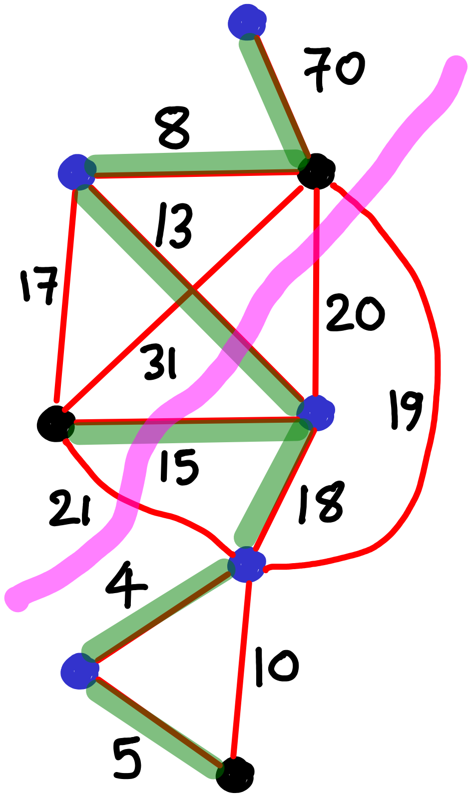
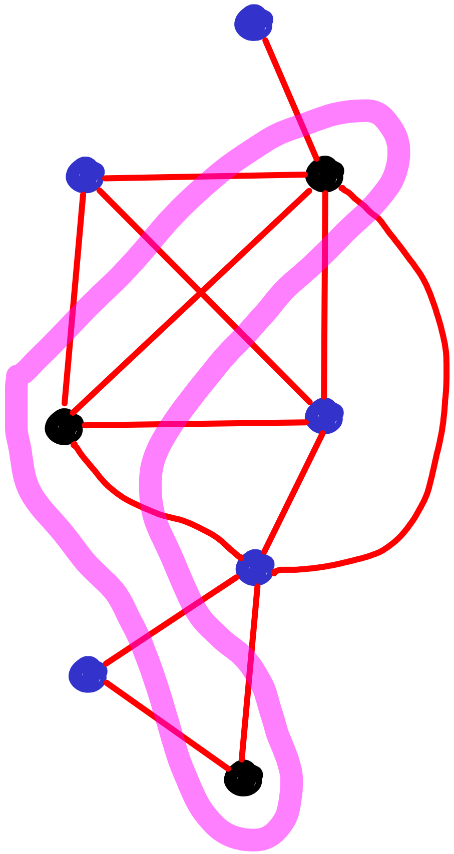
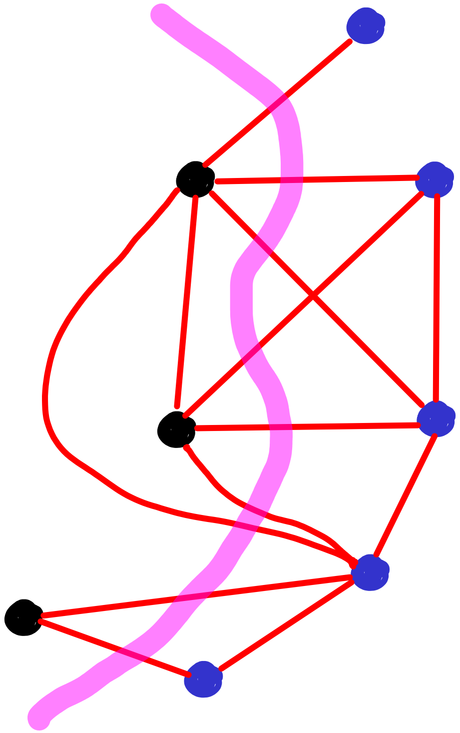
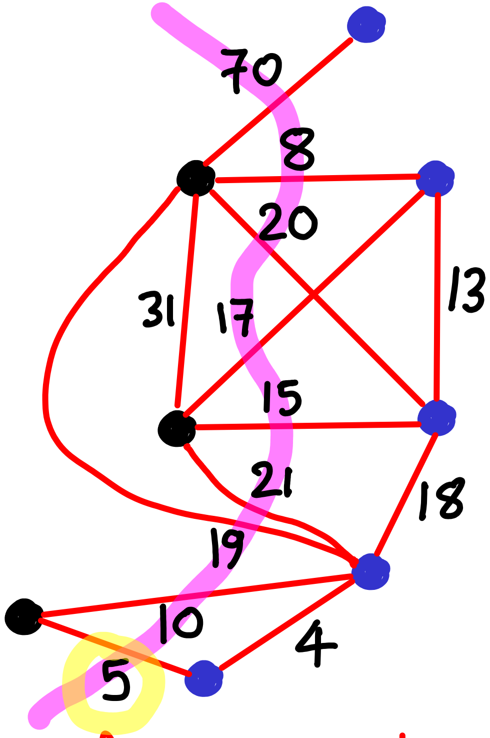




Redraw G

Cut crosses all 





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CLAIM: for any cut, the min-weight edge crossing the cut must be in MST

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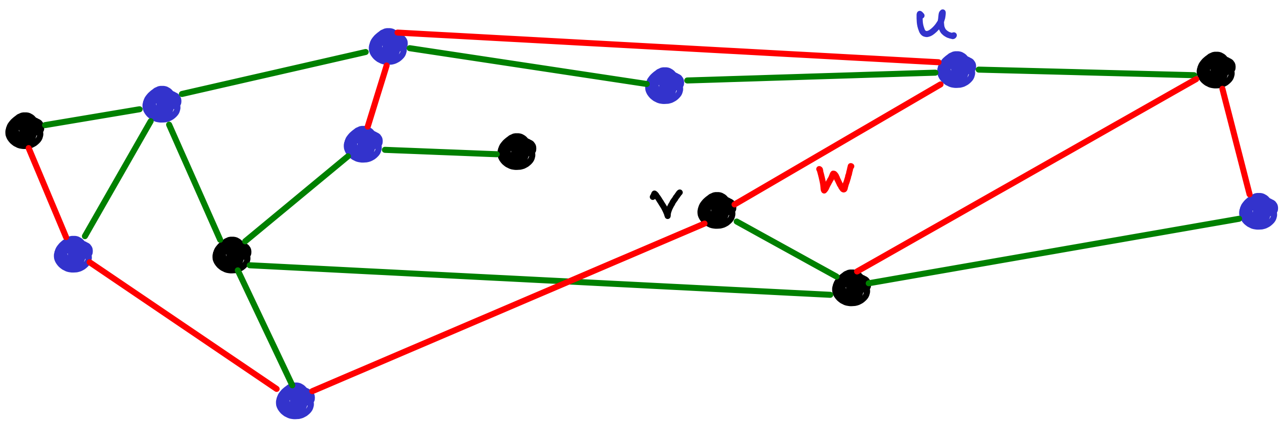
Proof: let u, v be the min-weight edge. Suppose it is not in MST.

w



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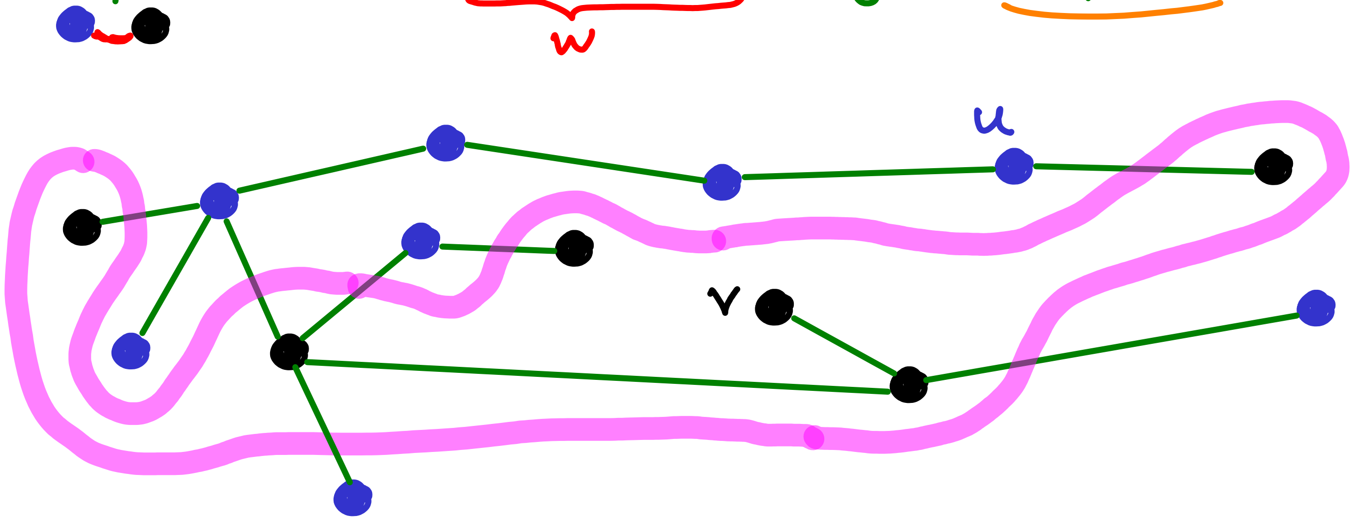

- Focus on MST and the given cut

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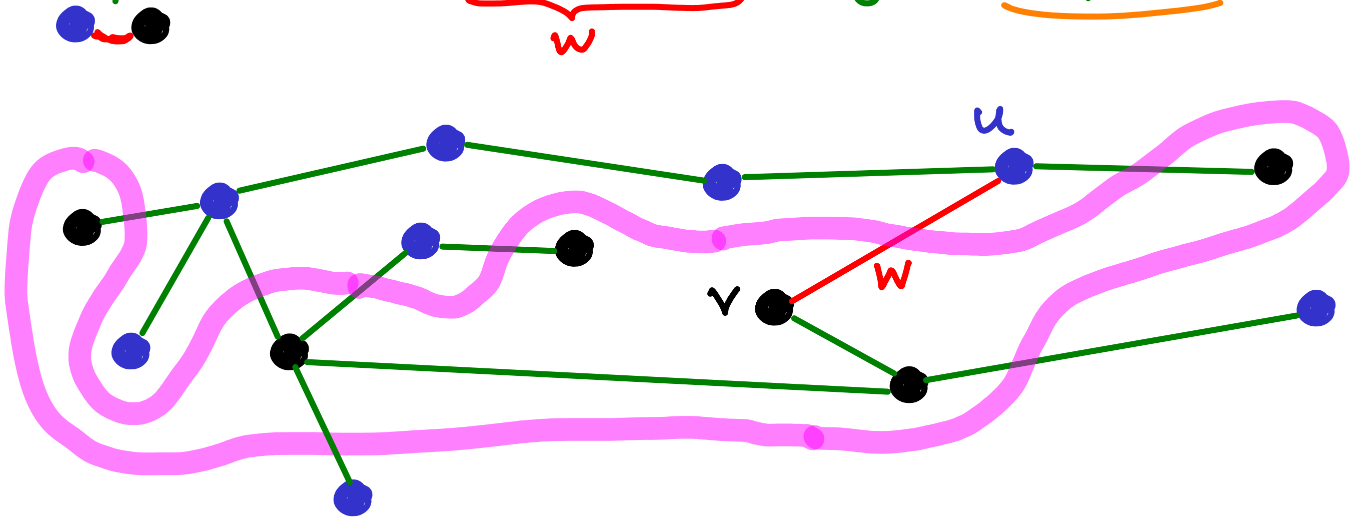
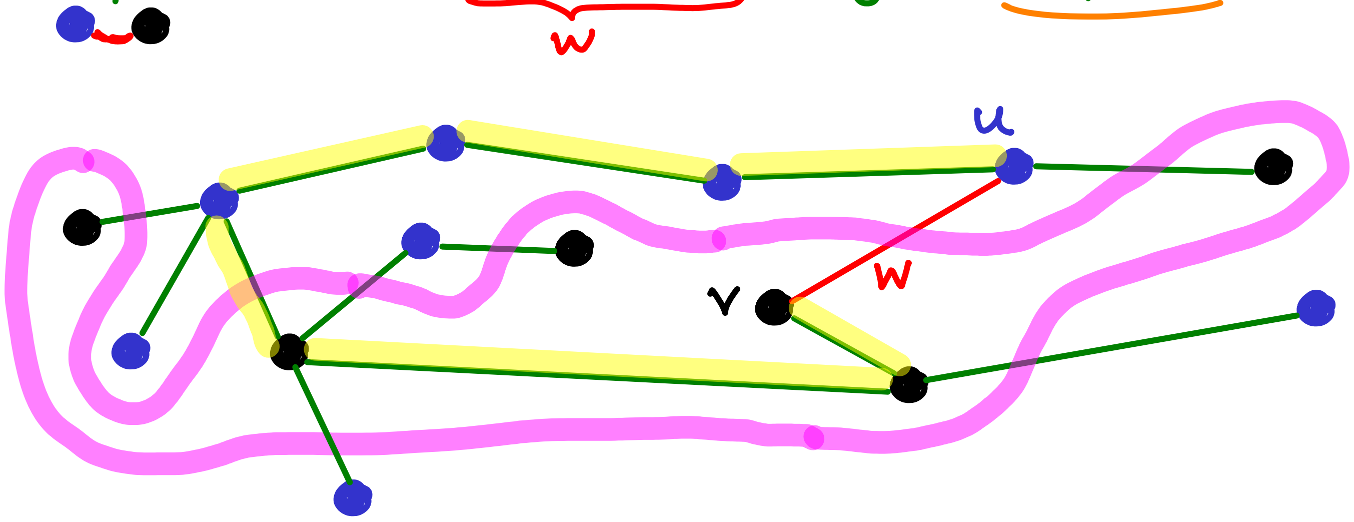


- Focus on MST and the given cut
- Insert u,v

CLAIM: for any cut, the min-weight edge crossing the cut must be in MST

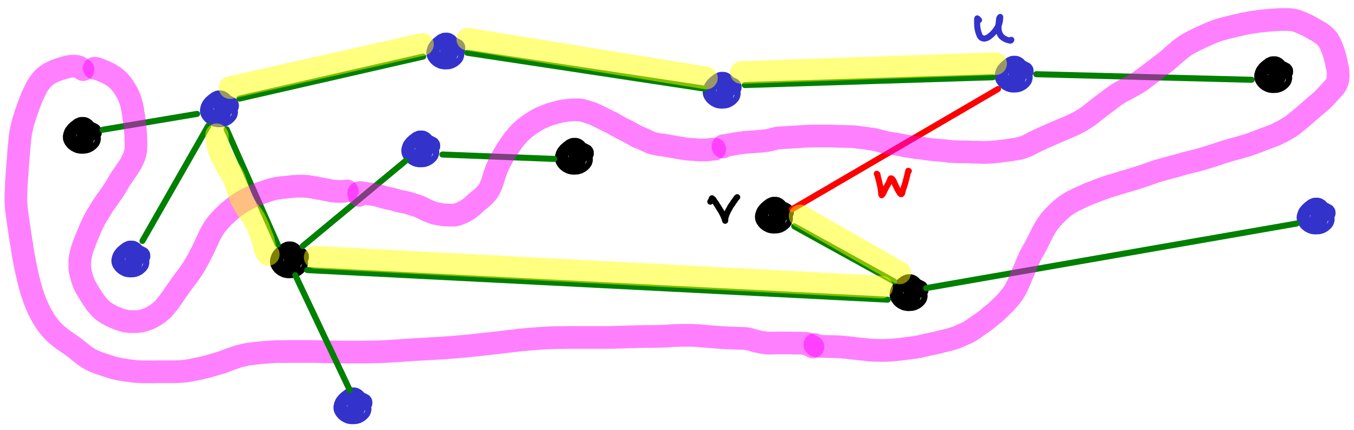
Proof: let u,v be the min-weight edge. Suppose it is not in MST.



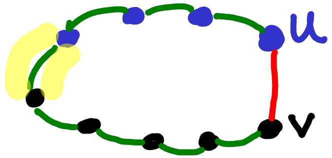
- Focus on MST and the given cut
- Insert u,v : create cycle

CLAIM: for any cut, the min-weight edge crossing the cut must be in MST

Proof: let u,v be the min-weight edge. Suppose it is not in MST.
w



- Focus on MST and the given cut
- Insert u,v : create cycle
  ↳ must contain another edge that crosses cut



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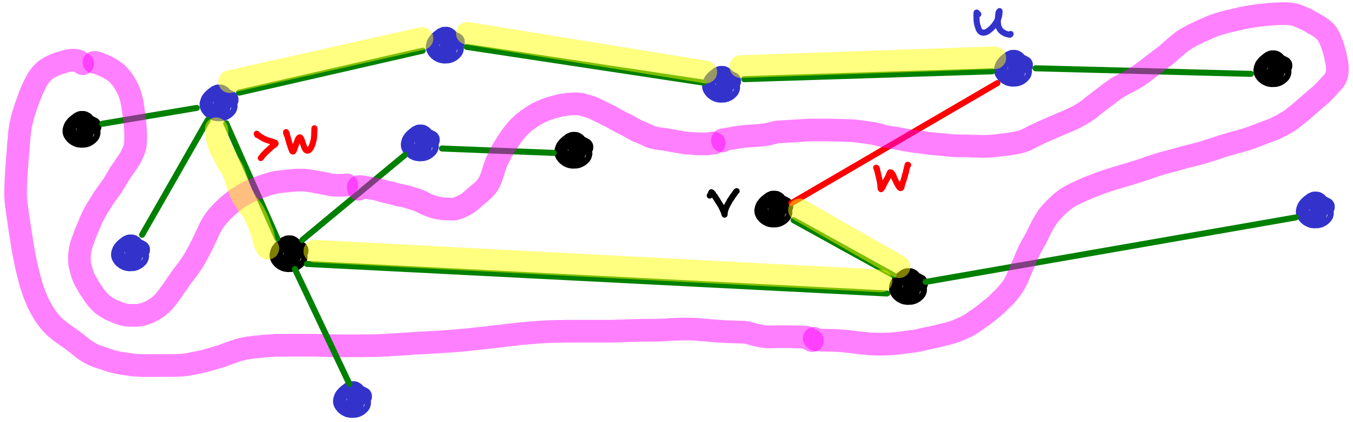
Proof: let u,v be the min-weight edge. Suppose it is not in MST.

(w)



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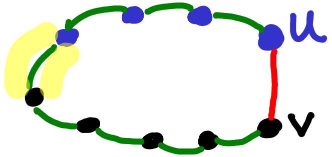

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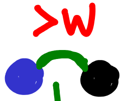

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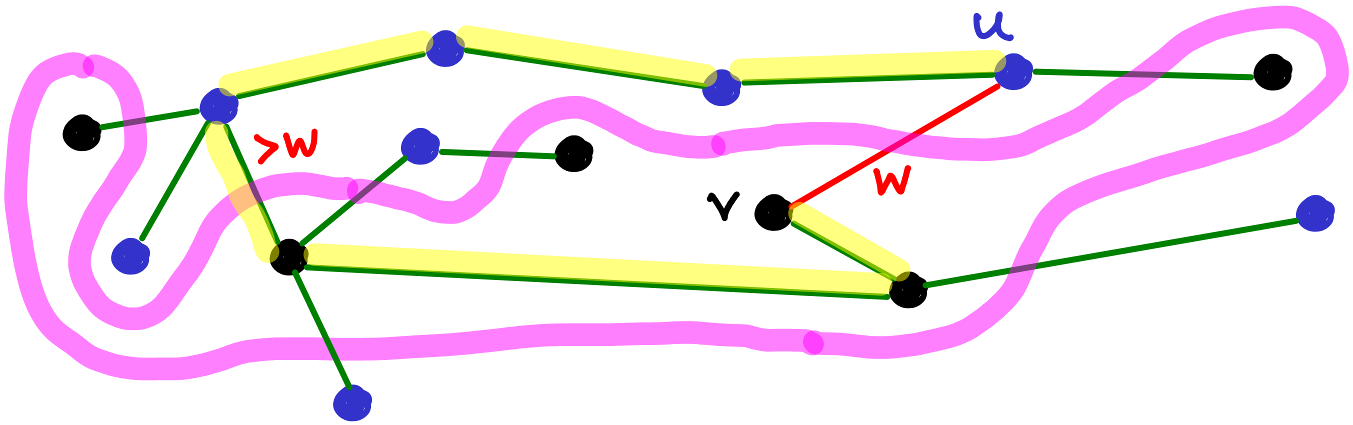
w



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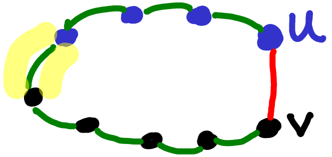

- Insert u,v : create cycle

↳ must contain another edge that crosses cut

- Remove that edge : improve tree: CONTRADICTION