Letters

Reduction of Inrush Currents in Toroidal Transformers by Sector Winding Design

Saeed Jazebi, *Member, IEEE*, Rasim Doğan, Baris Kovan, and Francisco de León, *Fellow, IEEE*

***Abstract*—In this letter, the principles for controlling the saturation inductance of toroidal transformers by leaving unwound sectors are presented. It is shown that inrush currents can be reduced substantially with this technique. This concept is supported by finite-element simulations, transient analyses, and laboratory experiments.**

***Index Terms*—Inrush currents, mitigation techniques, sector windings, toroidal transformers.**

I. Introduction

TOROIDAL transformers for low-frequency applications are made up of ring-shaped cores wound from a continuous tape of steel and are about 30%–40% lighter than E-I transformers. Toroidal technology results in more efficient designs when compared with stacked-core transformers because power loss is lower in both the iron core and the windings. The reasons are: 1) the magnetic flux runs along the orientation direction of the grains; 2) there are no gaps in the iron core; and 3) shorter wires are needed because the windings are distributed over the entire circumference. Toroidal transformers have better vibration and acoustic performance, since the core is mechanically contiguous. These transformers are popular for power electronic-based commercial products where the volume and weight of the component is an important factor. Some applications are uninterruptable power supplies, audio amplifiers, isolation transformers for electronic medical devices, test and measurement equipment, audio–video power conditioners, and airborne and marine applications [1]–[3]. However, the main drawback is that toroidal transformers draw larger energizing inrush currents [4].

There are various inrush current mitigation techniques that do not involve modifications of the transformer design. For example, application of the preinsertion resistors [5], negative temperature coefficient thermistors [6], switching-in at a specific instant of the terminal voltage [7], demagnetization of the core [8], energization of different phases in a particular sequence [9], compensation of the voltage sags [10], [11], and series dc reactors [12], [13].

In the experience of the authors, transformer-based solutions are more effective and reliable. Some of these methods include the use of air gaps [14] and virtual air gaps [15], [16]. Air gaps are capable of demagnetizing the transformer after it has been disconnected from the source. Therefore, gapped transformers reduce inrush currents to the level when the residual flux in the core is zero [4]. The manufacture of transformers with air gaps is expensive and some of the benefits of standard toroidal cores described above (acoustic noise and core losses) are lost. The alternative solutions are to use low permeability iron core, reduce the design flux density, or a combination of both [4]. These methods increase size, weight, and cost of transformers which may not be appropriate for space-constrained applications [4].

The least expensive and most effective solution is to solve the problem from its origins and change the design characteristics. It is well-known and confirmed with numerous experiments and computer simulations that there are two major design parameters affecting inrush currents: winding resistance and saturation inductance. Terminal saturation inductance is the inductance seen from a terminal when the transformer is driven into deep saturation. This parameter is different when measured/calculated for different windings.

Higher winding resistance and saturation inductance significantly reduce inrush currents. At the design stage, the total winding losses are usually kept as low as possible to maintain the target efficiency. The transformer designer can, within a narrow range, increase the resistance of the primary winding and compensate for the added losses with a reduction of the secondary winding resistance.

The electromagnetic design of the toroidal transformers, including electrostatic shielding, leakage inductance, as well as thermal behavior and insulation design for transients, have been presented in previous publications of our team [17]–[21]. In this letter, the capability of sector windings to increase the saturation inductance, and, consequently, reduce inrush currents is revealed. Laboratory measurements and computer simulations are carried out to support this idea. It is shown that inrush currents could be reduced to less than half without any reduction in efficiency, or increase of acoustic noise, and at no added cost.

II. Sensitivity Analysis of Inrush Currents With Respect to Resistance and Saturation Inductance

In this section, a parametric study is carried out to investigate the effects of saturation inductance and winding resistance on maximum inrush currents (see Table I). The terminal resistance (R) and the saturation inductance (L_s) of a 1 kVA, 120:120 V transformer are varied from their nominal values $R_n = 292$ mΩ and $L_{sn} = 259$ μH, over a wide range. The

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The authors are with the Department of Electrical and Computer Engineering, New York University, Five Metrotech Center, Brooklyn, NY 11201 USA (e-mail: jazebi@ieee.org; rd1224@nyu.edu; bk924@nyu.edu; fdeleon@nyu.edu).

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TABLE I
MAXIMUM INRUSH CURRENTS [A] VARYING THE TERMINAL RESISTANCE (R) AND SATURATION INDUCTANCE (L_s)

		L_s						
		L_{sn}	$1.2L_{sn}$	$1.6L_{sn}$	$2L_{sn}$	$3L_{sn}$	$4L_{sn}$	$6L_{sn}$
R	$5R_n$	98.3	96.9	94.3	91.8	86.2	81.3	73.2
	$2R_n$	201.8	196.2	186.0	176.9	157.8	142.5	119.6
	R_n	**310.5**	297.7	275.2	256.0	218.2	190.4	152.1
	$0.5R_n$	425.2	401.9	362.3	330.1	270.4	229.5	176.2
	$0.1R_n$	605.2	559.7	486.8	431	335.5	274.6	202.2

source resistance is measured ($R_s = 100$ mΩ). The maximum peak of inrush currents is calculated with transient simulations using the electromagnetic transient program (EMTP) [22] with an experimentally validated model [23]. The measured peak of inrush current for the transformer is 310 A. The corresponding calculated value with EMTP is 310.5 A; this number is bold in Table I to highlight the base case. Simulation results indicate that the inrush currents decrease by about 66% when the saturation inductance is $L_s = 6\ L_{sn}$ for the case of $R = 0.1R_n$. However, for $R = 5R_n$, the inrush currents reduce only by about 25% for the same case. The results indicate that increasing saturation inductance of transformers with higher power capacity (thicker windings) is more effective in mitigating inrush currents, where the dominant factor is not the winding resistance. This fact can be validated analytically.

Jazebi *et al.* [24] propose (1), shown at the bottom of the page, for the calculation of inrush currents. In this expression, $L_s, V_m, \omega, R, I_1, t_1, t_{\rm pk}$ are saturation inductance, peak of voltage, angular frequency, total resistance of system, current at saturation point, corresponding time to reach saturation point, and peak time, respectively. One can see from (1) that inrush currents are reduced when either the resistance or the saturation inductance is increased. However, increasing resistance makes the transformer less efficient. In this letter, the saturation inductance is increased with no added expense. This equation is highly dependent on the terminal resistance, as the important parameter $t_{\rm pk}$ is a direct function of R [24]. Therefore, inrush current calculations that ignore the terminal resistance are prone to large errors. Neglecting the effect of the resistance in (1) yields (2)

$$I_{\max} \approx I_1 + \frac{V_m}{\omega L_s}\left(1 + \frac{\lambda_0 - \lambda_s}{\lambda_n}\right) \approx \frac{k}{L_s}. \qquad (2)$$

Note that I_1 is usually very small when compared to the other terms, and, thus, it is neglected in (2). One can see that inrush currents decrease in inverse proportion with the increase of saturation inductance. The procedure to calculate inrush currents based on (1) consists of several formulas and calculation steps that are not repeated here because of the lack of space. For more information, see [24].

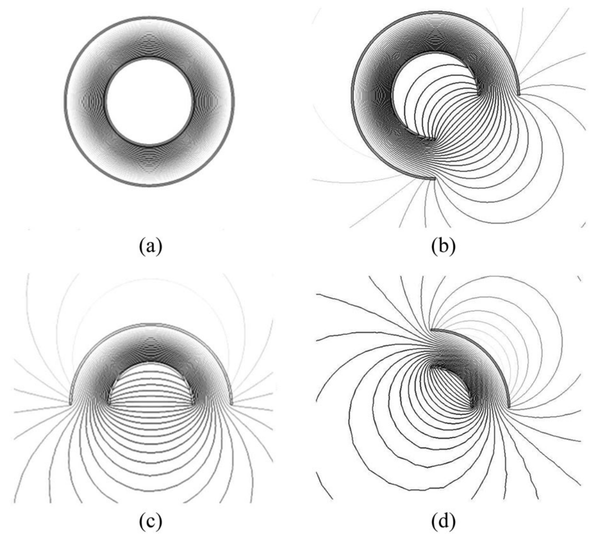

Fig. 1. Difference between magnetic flux patterns in open circuit (in deep saturation) for: (a) 360°; (b) 270°; (c) 180°; and (d) 90° wound transformers.

III. EFFECT OF SECTOR WINDINGS ON SATURATION INDUCTANCE OF TOROIDAL TRANSFORMERS

In [21], sector windings were used to control the transformer leakage inductance, but the effects of sector windings on the saturation inductance were not studied. In this letter, it is revealed that the saturation inductance of a toroidal transformer can be controlled by leaving unwound sectors. The difference between the saturation inductance of a sector winding and a fully wound transformer is explained by comparing the magnetic field distribution in deep saturation for different sector windings (see Fig. 1). In the sectored wound transformers, i.e., when the windings do not cover the entire 360°, stray flux is pushed into the air. The amount and distribution of stray flux depends on the sector that is not wound.

A parametric study is carried out with 3-D finite-element simulations on the variation of the saturation inductance of toroidal transformers with respect to the angle of the winding. In this letter, the saturation inductance is equivalent to the air-core inductance since the transformers under study do not have any enclosure or accessories. The air-core inductance would be measured across a winding (with all other windings opened) as if the ferromagnetic core is completely saturated. In this condition, the incremental permeability of the iron core becomes very close to that of air $\mu_r = 1$. Finite-element simulations have been carried out using the same conditions for all windings with different angles. The sectored winding is energized with constant current

$$i\left(t_{\rm pk}\right) = I_1 e^{-\frac{R\left(t_{\rm pk}-t_1\right)}{L_s}} + \frac{V_m R\left[\sin\left(\omega t_{\rm pk}\right) - \sin\left(\omega t_1\right) e^{-\frac{R\left(t_{\rm pk}-t_1\right)}{L_s}}\right] + V_m \omega L_s\left[\cos\left(\omega t_1\right) e^{-\frac{R\left(t_{\rm pk}-t_1\right)}{L_s}} - \cos\left(\omega t_{\rm pk}\right)\right]}{R^2 + \left(\omega L_s\right)^2} \qquad (1)$$

TABLE II
SATURATION INDUCTANCE FOR VARIOUS ANGLES OF THE SECTOR WINDING OBTAINED FROM 3-D FINITE-ELEMENT SIMULATIONS

Wound sector	60°	90°	120°	180°	240°	300°	360°
L_s	1530 μH $\approx 6L_{sn}$	1066 μH $\approx 4L_{sn}$	801 μH $\approx 3L_{sn}$	524 μH $\approx 2L_{sn}$	417 μH $\approx 1.6L_{sn}$	306 μH $\approx 1.2L_{sn}$	259 μH $= L_{sn}$

(1 A). Therefore, saturation depth for all the transformers is kept constant. The core relative permeability is considered as $\mu_r = 1$ to emulate the completely saturated core condition (air-core inductance). The magnetic energy in the complete domain is used to calculate saturation inductance as follows:

$$L_s = \frac{2\int W_m dv}{I^2}, \quad (3)$$

where W_m is the magnetic energy and I is the current of the winding. The results of the parametric studies with 3-D finite-element simulations are presented in Table II. Note that in this letter, different from [21], the angle of the wound sector and not the unwound sector, is used. For example, for a 120° transformer, 120° of the core is covered with the winding and the remaining 240° is left unwound. It is assumed that the wire gauge, number of turns, and the length of the windings do not change with the variation of the wound angle. This means that the total volume of copper is kept approximately constant. The base case is a 1-kVA prototype with both primary and secondary wound at 360°, the so called 360°−360°, here after. This is the transformer that has been studied in the previous section. One can observe that the angles of the sector windings presented in Table II approximately match the parameters studied in Table I. A reciprocal function is fitted to the data presented in Table II

$$L_s(\mu H) \approx \frac{a}{x} \quad (4)$$

where $a = 95{,}630$ and x is the wound angle in degrees. The coefficient of determination is $r^2 = 0.9977$, indicating that (4) is a very good approximation. This equation is derived from a mathematical curve fitting solely for the 1-kVA torodial transformers studied in this letter. Substituting (4) into (1), yields (5) shown at the bottom of the page, that can be used to calculate maximum inrush currents as a function of sector angle. Similar equations could be derived for different sizes of toroidal transformers. However, the relative coefficients may vary for transformers with different capacities, voltage ratios, insulation thicknesses, etc.

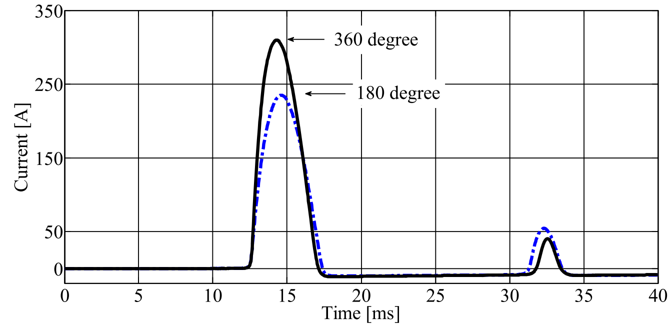

Fig. 2. Comparison between experimental results of 180° and 360° wound transformers for inrush currents.

TABLE III
PARAMETERS FOR THE TESTED TRANSFORMERS

Wound sector	R_1 [mΩ]	$R_2{}'$ [mΩ]	L_{leakage} [mH]	R_m [Ω]	L_m [mH]	L_s [μH]
180°–180°	304	305	8.75	1136	1840	524
360°-360°	292	314	0.303	1375	1210	259

All parameters are obtained from measurements except the saturation inductances that are calculated from 3-D finite-element simulations.

IV. EXPERIMENTAL RESULTS

Measurements of inrush currents have been performed in the laboratory on two prototypes: 180°-180° and 360°-360° wound transformers (see Fig. 2). For the 360°-360° transformer, both windings cover the full 360°. For the 180°-180° transformer, each winding covers only 180° of the core without overlapping the other winding. Both transformers have been completely demagnetized before inrush current tests and the switch was closed at the zero crossing of the voltage waveform. The peaks of the inrush currents are 310 A, and 235 A for the 360°-360° and the 180°-180° transformers, respectively. This means that the 180° sector winding reduces the inrush current by 24.2%. Since all other parameters of the two transformers are very similar, one can conclude that the reduction is caused by the higher saturation inductance (see Table III). In Table III, R_1 and $R_2{}'$ are the primary and secondary winding resistances, and R_m and L_m are the magnetizing resistance and magnetizing inductance computed at rated voltage, respectively. The calculated inrush current peak values obtained from EMTP simulations and the analytical expression (5) for the 180° and 360° transformers are compared with experiments in Table IV.

V. DISCUSSION

It has been demonstrated that sector windings can significantly reduce inrush currents by controlling saturation

$$i(t_{\text{pk}}) = I_1 e^{-\frac{Rx(t_{\text{pk}}-t_1)}{a}} + \frac{V_m R x^2 \left[\sin(\omega t_{\text{pk}}) - \sin(\omega t_1)\, e^{-\frac{Rx(t_{\text{pk}}-t_1)}{a}}\right] + V_m \omega a x \left[\cos(\omega t_1)\, e^{-\frac{Rx(t_{\text{pk}}-t_1)}{a}} - \cos(\omega t_{\text{pk}})\right]}{(Rx)^2 + (a\omega)^2} \quad (5)$$

TABLE IV
COMPARISON OF TEST, EMTP SIMULATIONS, AND ANALYTICAL CALCULATIONS USING (5)

Wound sector	Test [A]	EMTP [A]	Difference [%]	(5) [A]	Difference [%]
180°–180°	235	256	8.9	258	9.7
360°-360°	310	310.5	0.1	320.2	3.3

TABLE V
LEAKAGE INDUCTANCE, REGULATION, AND INRUSH FOR DIFFERENT SECTORS

Wound sector	L_{leakage} [μH]	Regulation (%)	Inrush [A]
360°-360°	303	2.2	310.0
300°-360°	442	2.3	298.8
240°-360°	1510	4.4	274.7
180°-360°	2020	5.7	235.0
180°–180°	8750	23.0	235.0

inductance. This method increases the stray fluxes for transformers with larger unwound angles. This issue may affect the electromagnetic emissions compliance for an underload transformer. Caution must be used when the EMI emissions are critical. The same technique has been implemented in [21] to control the leakage inductance. The increase in leakage inductance can be an advantage for power electronic applications where a specified (or enlarged) leakage inductance is required to substitute a series inductor for filtering or tuning purposes [21], [25]–[31]. In these applications, the converters of the device can be fitted with soft starting features to eliminate inrush current issues.

There exist other devices such as uninterruptable power supplies, audio amplifiers, isolation transformers for electronic medical devices, computers, closed circuit surveillance cameras, digital printers, test and measurement pieces of equipment, audio–video power conditioners, and airborne and marine applications that can use inrush mitigation methods. These products operate for 50/60/400 Hz and do not use switching electronic components to serve as soft starter. Hence, mitigation techniques are essential for cases where inrush currents need to be controlled. Ideally, voltage regulation (change of voltage in secondary when load varies from zero to nominal) would be zero for these transformers. As discussed, the sector winding method, when both primary and secondary windings are wound partially, increases the leakage substantially that may produce large voltage drops [21].

Frequently, it is desirable to reduce leakage inductance of transformers while increasing saturation inductance. Laboratory experiments demonstrate that leakage inductance can be reduced if the secondary winding is wound around 360°. Therefore, the primary winding, which is subject to energization, can be wound in a sector to mitigate inrush currents while the secondary winding may cover the 360° of core to avoid large increases of the leakage inductance. This technique may increase the size of the final toroidal transformer, but it is effective to simultaneously decrease the inrush currents and keep the leakage inductance at a desirable value.

The aforementioned method was tested on three 1-kVA transformers; 180°-360°, 240°-360°, and 300°-360°. Design parameters are similar to the 180°–180° and 360°-360° transformers presented in Table III. The leakage inductance (L_{leakage}) and the voltage regulation for all the transformers are presented in Table V. Measurement results show that the method can maintain the leakage inductance within an acceptable range. Comparing the 360°-360°, 180°-360°, and 180°–180° transformers, one can see that if the primary winding is sectored (180°), inrush current is reduced. In the case of a sectored secondary (180°), the regulation increases to 23%. However, if the secondary is not sectored, the regulation is only 5.7%, which is acceptable for most applications.

VI. CONCLUSION

The significance of sector windings to increase saturation inductance of toroidal transformers, and, consequently, mitigate inrush currents has been demonstrated. It can be concluded that for larger transformers with lower terminal resistance, the peak of inrush currents is extremely dependent on saturation inductance. Therefore, sector windings are more effective on larger transformers. This method does not negatively affect the reliability of the system since there is no requirement for using any auxiliary devices.

In a well-designed transformer, the volume of copper used for primary and secondary windings is usually balanced. Hence, to prevent an increase in size and simultaneously reduce inrush currents, it is recommended to use 180° for both primary and secondary windings when voltage regulation is not crucial. For applications where low-voltage regulation is needed, the secondary winding can be wound in 360° to control both leakage inductance and inrush currents.

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