Analysis of Voltage Profile Problems Due to the Penetration of Distributed Generation in Low-Voltage Secondary Distribution Networks

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***Abstract*—This paper presents a comprehensive analysis of the possible impacts of different penetration levels of distributed generation (DG) on voltage profiles in low-voltage secondary distribution networks. Detailed models of all system components are utilized in a study that performs hundreds of time-domain simulations of large networked distribution systems using the Electromagnetic Transients Program (EMTP). DGs are allocated in a probabilistic fashion to account for the uncertainties of future installations. The main contribution of this paper is the determination of the maximum amount of DG that secondary distribution networks can withstand without exhibiting undervoltage and overvoltage problems or unexpected load disconnections. This information is important for network planning engineers to facilitate the extension of the maximum penetration limit. The results show that depending on the location, type, and size of the installed DGs, small amounts of DG may cause overvoltage problems. However, large amounts of DG may not cause any voltage problems when properly selected.**

***Index Terms*—Distributed generation (DG), low-voltage secondary networks, maximum penetration of DG, voltage quality.**

I. Introduction

DISTRIBUTED generation (DG) is becoming an increasingly viable option for the future of power systems. Despite its higher price, the installation of DGs in distribution systems offers advantages over the traditional unidirectional flow of power from a distant generator. For example, DG reduces the load that needs to be supplied from the substation. Although not generalized today, DG could be used to control voltage [1]–[5] or dampen power oscillations [6]. There are, however, several challenges that DG pose to the safe and reliable operation of a distribution system [7]–[15].

A comprehensive literature review revealed that there are no systematic studies reporting the effects of DG penetration in meshed low-voltage (LV) secondary networks. There are, however, a considerable number of research papers reporting the advantages and disadvantages of DG for radial distribution systems; see, for example, [16]–[23]. The operation strategies of radial and networked systems are quite different from each other. For example, radial systems allow for bidirectional load flow, but may require a different coordination of protection. However, in secondary networks, reverse power from the LV network to the medium-voltage (MV) feeders is not possible. For safety reasons, all network transformers include network protectors that trip when reverse power is sensed. The requirement for the unidirectional active power flow in the secondary networks imposes additional constraints that are not present in radial systems and vice-versa.

This paper presents the first attempt to quantify the possible negative impacts on the voltage profile of different penetration levels of DG in secondary networked distribution systems. The study is intended to elucidate what will happen if customers are allowed to freely install DGs on their premises and DGs become widespread. In our analysis, we focus on the situation when the maximum DG output coincides with light (minimum) load conditions. This case is recognized in the literature as the worst-case scenario [24], [25]. To simulate possible future scenarios, we have probabilistically allocated DG in increments of 10% of the light load. The study is carried out using a very detailed representation of the system components. Hundreds of time-domain simulations with the Electromagnetic Transients Program (EMTP) are performed to determine if a given allocation of DGs would produce voltage profile problems.

We have found (Section IV) that even with very small DG penetration, there may be unacceptably low or high voltages at certain loads when DG units are installed at the wrong location. However, very large amounts of DG power (up to 100% of light load) installed with the adequate strategy allow acceptable operating conditions. Under the present operating strategy of secondary networks, no power can be exported from the network to the system.

Intermittent DG technologies, such as solar photovoltaic (PV) or wind conversion systems, could also affect secondary voltage

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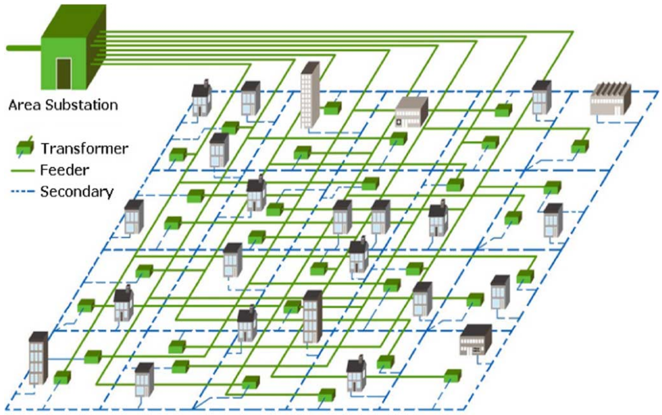

Fig. 1. Schematic of an LV secondary network. Loads and network transformers are tied together in a highly meshed network.

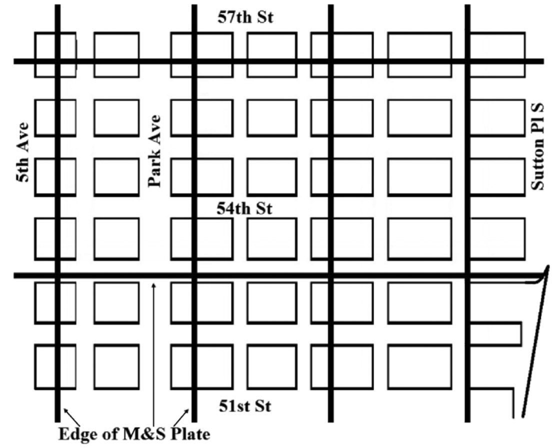

Fig. 2. Street map of the location of the Sutton. The edges of M&S plates are included as a reference.

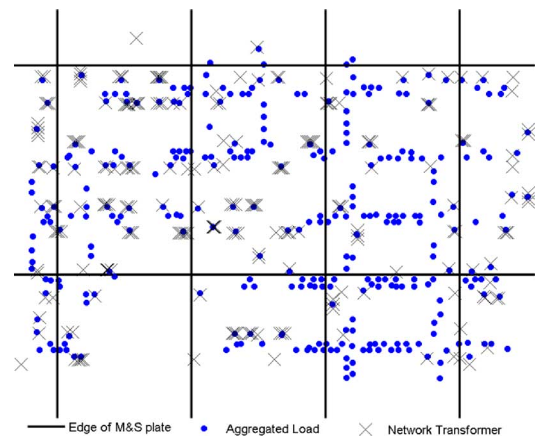

Fig. 3. Location of loads and transformers in the Sutton network using as reference the edges of the M&S plates.

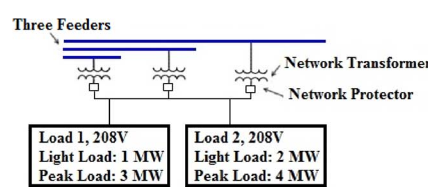

Fig. 4. Typical configuration of an isolated spot network.

profiles due to flicker. In addition, intermittent DG technologies frequently require optimal-management strategies to maximize power delivery since the maximum power output may fluctuate [26]. However, this topic is beyond the scope of a first study on the effects of DG penetration in secondary networks. The issues related to intermittent DG are to be studied in future research.

II. Secondary Network Under Study

A. Description of LV Secondary Networks

An LV secondary network is a distribution system configuration typical of the downtown cores of most cities in North America. An area substation commonly supplies power to two (or more) independent underground networks through a number of MV radial feeders. Each feeder delivers power through several tens of network transformers that reduce the voltage to the utilization level (say 208/120 V). All transformer secondaries and loads are tied together in a highly meshed LV network as shown in Fig. 1. This arrangement offers the highest levels of reliability of any standard configuration in use today [22].

Important components of an LV secondary network are the network protectors [27], [28]. These protective devices are installed on the secondary side of the network transformers and automatically disconnect them from the secondary grid when the power starts to flow in the reverse direction (i.e., from the LV network to the feeders). The network protectors automatically reclose when the conditions for the direct power flow in the system are restored.

B. Description of the Sutton Network

The selected network for the study is one of the 34 networks that supplies the Manhattan service area. It has 12 primary feeders, 1041 feeder cable sections, 1375 secondary cable sections, 27 substation breakers, 224 transformers, 224 network protections, 1375 secondary grid sections, and 311 aggregated loads. These loads correspond to 284 independent customers, 17 spot networks mostly at 460 V, and one isolated spot network connected at 208 V. The overall geographical map is shown in Fig. 2.

The Sutton network is limited by Fifth Avenue, Sutton Place South, 57th street, and 51st street. For identification purposes, this network is divided by Consolidated Edison into smaller rectangular areas called mains and services (M&S) plates. A simplified map showing locations of the network loads and transformers in the Sutton network is shown in Fig. 3. In this figure, the lines corresponding to the edges of the M&S plates of Fig. 2 are used as reference. The typical structure of an isolated spot network is presented in Fig. 4.

The network operates at 13.8 kV from the area substation through the primary feeder sections where the distribution transformers are connected to step-down the voltage to 208 V for regular customers and 460 V for larger power consumption customers.

C. Network Model Validation

The analysis presented in this paper is based on the detailed three-phase EMTP model developed in [29]. This model includes a very accurate representation of all main network elements including relay protection devices. The electrical loads were represented as constant impedances using a built-in EMTP load model. The results of our time-domain simulations with the EMTP were verified against the field-validated load-flow and short-circuit program of Con Edison [poly voltage load (PVL)]. The comparison included peak and light load conditions, a set of first and second contingencies, and three-phase short circuits at various locations in the network. In addition, the EMTP simulations were able to reproduce several transient events recorded with a power-quality (PQ) node [29].

III. DG Allocation Methods

In this paper, DG units will be referred to as deterministic or nondeterministic, depending on whether their location, type, and size are known parameters or not. Deterministic DGs are those already installed in the distribution network. The nondeterministic DGs are probabilistically placed at the customer sites to perform analysis of different "*what if*" (hypothetical) scenarios (i.e., to study the influence of type, size, and location of the distributed generators on the voltage profile). In this analysis, each scenario corresponds to a different distribution of DGs.

A. Gibbs Sampler and Monte Carlo Method

In this paper, the Gibbs sampler algorithm [30], [31] is used to generate three key parameters for the allocation of the nondeterministic DGs: type, size, and location. The Gibbs sampler algorithm is one of the Markov Chain Monte Carlo methods; it is commonly used for the generation of random variables from a marginal distribution directly without having to calculate the probability density via integration. An important advantage of the Gibbs sampler as a tool for the statistical study of DG penetration is that it enables low-dimensional conditional distributions (avoiding the use of a complicated multivariate distribution). In addition, more parameters for DGs can be added easily, for example, the cost of DG.

A brief description of the Gibbs sampler algorithm is given next. (See [30] for more details.) Suppose that a target distribution $\pi(x)$ corresponds to a joint distribution of several variables $\pi(x_1, x_2, \ldots, x_k)$. This joint distribution is assumed to exist and be proper. Each of the x_i terms could represent a block of several random variables grouped together. Let $\pi(x_j)$ represent the marginal distribution of the jth block of variables, x_j, and let $\pi(x_j|x_1, \ldots, x_{(j-1)}, x_{(j+1)}, \ldots, x_k)$ represent the full conditional distribution of the jth block of variables. The Gibbs sampler utilizes a set of full conditional distributions associated with the target distribution of interest in order to define a Markov chain. The Gibbs sampler can be implemented with the following iterative sampling scheme:

1) Select initial value $x^{(0)} = (x_1^{(0)}, x_2^{(0)}, \ldots, x_k^{(0)})$.
2) Set the counter index $i = 0$.
3) Simulate the sequence of random draws:

$$x_1^{(i+1)} \sim \pi\left(x_1|x_2^{(i)}, x_3^{(i)}, \ldots, x_k^{(i)}\right),$$
$$x_2^{(i+1)} \sim \pi\left(x_2|x_1^{(i+1)}, x_3^{(i)}, \ldots, x_k^{(i)}\right),$$
$$\vdots$$
$$x_k^{(i+1)} \sim \pi\left(x_k|x_1^{(i+1)}, x_2^{(i+1)}, \ldots, x_{k-1}^{(i+1)}\right)$$

and form

$$x^{(i+1)} = \left(x_1^{(i+1)}, x_2^{(i+1)}, \ldots, x_k^{(i+1)}\right).$$

4) Set $i \leftarrow i+1$ and return to Step 3) until i is sufficiently large so that each component $x_j^{(i)}$ is very nearly a random draw from the marginal distribution $\pi(x_j)$. See [30] for details.

Notice that in Step 3) of the Gibbs sampling algorithm, it is required to sample random draws once from each of the full conditional distributions and that the values of the conditioning variables are sequentially updated one by one. This sampling algorithm defines a valid Markov Chain Monte Carlo method as described in [30].

TABLE I
Load Group Classification per Size

Group Location	Power Demand Range
Very Large Load	Larger than 1 MW
Large Load	200 kW - 1 MW
Medium Load	50 kW - 200 kW
Small Load	10 kW - 50 kW
Very Small Load	0 kW - 10 kW

B. Overview of DG Characteristics (Interconnection)

The types of distributed generators considered for this study are inverter based, induction, and synchronous. According to local utility regulations, the selection of DG for a particular location has a direct relationship to the output-power range of that generator. The output range of inverter-based, induction, and synchronous generators is 0.3 kW to 2 MW, 40 kW to 2 MW, and 225 kW to 2 MW, respectively.

The probability of choosing a particular DG type and its output power for installation at any customer location depend on the power demand at this location. It is more likely that customers will choose the size of the DGs in accordance with the power consumption. Thus, for a DG unit having a particular large rating, the probability of being installed at locations with large demand is higher. For allocation purposes only, the 311 loads in the network are classified into groups based on the power demand (Table I). The simulations and analyses are based on explicit representation of each one of the 311 loads.

C. Probabilistic Approach for Allocating Nondeterministic DG Units

The selected conditional probability functions to allocate nondeterministic DGs were designed in accordance with the IEEE standard for interconnecting distributed resources with electric power systems [32]; local utility requirements [33],

TABLE II
CONDITIONAL PROBABILITY FUNCTIONS FOR CHOOSING A SIZE OF THE SYNCHRONOUS DG GIVEN THE SIZE OF THE CUSTOMER LOAD

Given Size of Customer Load	Probability of Choosing a Size of DG		
	1 to 2 MW	500 kW to 1 MW	200 to 500 kW
Larger than 1 MW	3/6	2/6	1/6
200 kW to 1 MW	1/6	3/6	2/6
50 kW to 200 kW	1/6	2/6	3/6
10 kW to 50 kW	1/6	1/6	4/6
0 kW to 10 kW	1/8	1/8	6/8

TABLE III
CONDITIONAL PROBABILITY FUNCTIONS FOR CHOOSING A CUSTOMER IN THE LOAD GROUP GIVEN THE SIZE OF THE SYNCHRONOUS DG

Given Size of DG	Probability of Choosing a Customer in the Load Group				
	Larger than 1 MW	200 kW to 1 MW	50 kW to 200 kW	10 kW to 50 kW	0 kW to 10 kW
1 to 2 MW	3/8	2/8	1/8	1/8	1/8
500 kW to 1 MW	2/9	3/9	2/9	1/9	1/9
200 kW to 500 kW	1/9	2/9	3/9	2/9	1/9

TABLE IV
INDIVIDUAL PROBABILITY FUNCTIONS OF SYNCHRONOUS DG SIZE

1 MW – 2 MW	500 kW – 1 MW	200 kW – 500 kW
0.2312	0.3267	0.4421

TABLE V
INDIVIDUAL PROBABILITY FUNCTIONS OF SYNCHRONOUS DG LOCATION

Larger than 1 MW	200 kW to 1 MW	50 kW to 200 kW	10 kW to 50 kW	0 kW to 50 kW
0.2084	0.2649	0.2489	0.1634	0.1144

[34]; and physical conditions of the selected distribution network. This implies a basic assumption of this study that large customers are allowed to install larger DG units. On the other hand, larger size DGs will have a higher probability to be installed at customer locations with larger load than customers with smaller loads.

Table II provides the specified conditional probability functions for choosing the size of a synchronous DG given the customer location. For example, for a given customer in the group of "Larger than 1 MW," the probability that a DG between 1 and 2 MW exists will be chosen is 3/6.

Table III corresponds to the specified conditional probability functions for choosing a customer location given the size of a synchronous DG. For example, if the power of the selected DG is between 1 and 2 MW, then the probability that a customer from the load group "Larger than 1 MW" will be chosen is 3/8.

Individual probabilities of DG locations and sizes are obtained by applying the Gibbs sampling algorithm with the conditional probability functions described before. These probabilities are shown in Tables IV and V, respectively. For example, the individual probability of selecting a synchronous-type DG of size 1 to 2 MW is 0.231. It should be noted that Tables II–V are the examples for the synchronous-type DGs; similar tables exist for other types of DGs and are not presented due to space constraints.

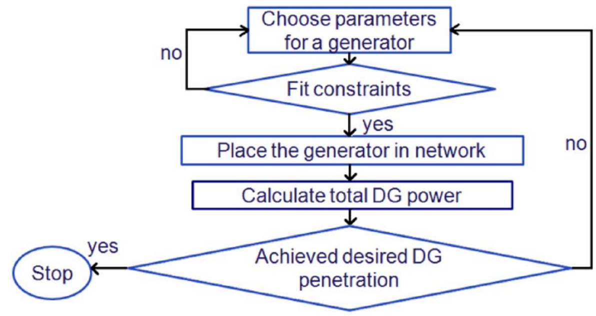

Fig. 5. Allocation algorithm for nondeterministic DG units.

The following constraints are considered for the allocation of the DGs: 1) a DG can only be installed at customer locations; 2) only "small" DG units are permitted in LV networks. It means that a DG unit cannot exceed 2-MW output power [33], [34]; 3) only one type of the DG is allowed per location; 4) each location may have multiple DG units of the same type; 5) the DG units supplying 460-V loads cannot exceed their light load demand for that particular location; 6) the DG units supplying 208-V loads cannot exceed their maximum of the light load demand and 85% of the peak load demand for that particular location; and 7) the total DG power in a spot network cannot exceed the total light load demand.

The aforementioned constraints are based on regulatory requirements and physical limitations of the distribution networks in the metropolitan areas. The operation of these networks under completely deregulated power markets raises concerns of reliability, stability, and PQ. Massive reinforcements of the existing urban networks to accommodate new generation units in the LV secondary grid are very time-consuming and require tremendous investments. At the same time, a persistently growing requirement to allow for the large-scale penetration of the distributed generation exists. As a result, an approach based on "connect and forget" principle becomes more attractive. According to this approach, newly connected DG units should not drive the network beyond its current operation and physical constraints. The constraints must ensure service continuity and reliability. For example, if constraint (6) is applied to the spot network in Fig. 4 without constraint (7), then the total maximum DG power that can be allocated would be greater than the total light load demand. Therefore, the spot network would be disconnected by the network protectors when sensing backfeed power.

The procedure of allocating nondeterministic DG units is presented in Fig. 5. The nondeterministic DG units are generated from the conditional and individual probability functions and can be placed at the particular location only when none of the allocation constraints is violated.

Fig. 6 shows a few examples of the nondeterministic DG allocations obtained by applying the algorithm of Fig. 5. Fig. 6(a) and (b) corresponds to two different cases of 20% DG penetration by applying the same distribution. Similarly, Fig. 6(c) and (d) corresponds to two different cases of 60% DG penetration. The results of nondeterministic DG allocations are not unique even when applying the same distributions for the same penetration level. Each one of the cases corresponds

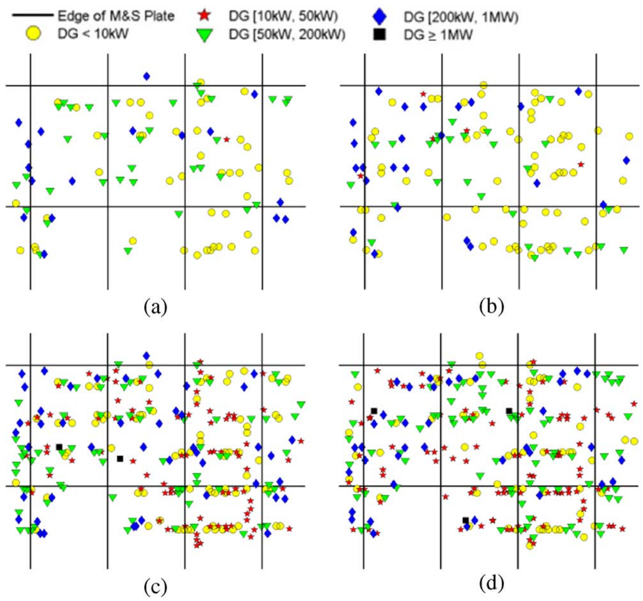

Fig. 6. Examples of DG allocation for different DG penetrations. (a) Case 1 at 20%. (b) Case 2 at 20%. (c) Case 1 at 60%. (d) Case 2 at 60%.

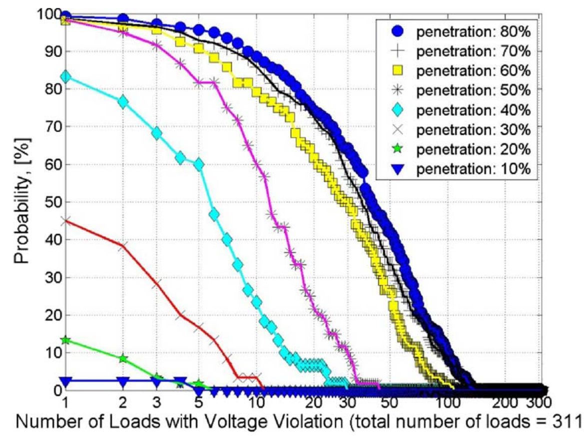

Fig. 7. Probability of having voltage violations of more than $\pm 5\%$ versus the number of loads with violation.

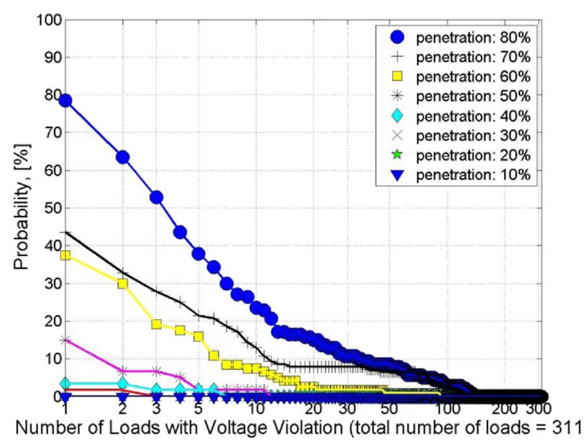

Fig. 8. Probability of having voltage violations of more than $\pm 10\%$ versus the number of loads with violation.

to a different "*what if*" scenario. The legend in Fig. 6 shows the sizes of the distributed generators of all types allocated at various locations in the network. It should be noted that they are not related to the size groups given in Tables II and III for the synchronous generators.

The importance of applying the proposed distributions (Tables II–V) is to avoid installing large-size DGs at small customer locations. As will be illustrated, this can cause voltage profile problems even at low penetration levels.

IV. DG Penetration Study

The study was performed with time-domain simulations using the EMTP for the following reasons: 1) commercial load-flow programs do not have an adequate model for the network protectors; 2) they also lack the model of all the protective and switching devices needed, in particular, undervoltage and overvoltage protection of DGs; 3) the models of DG are not as sophisticated as those in the EMTP, where all kinds of DG (synchronous, induction, and inverter based) can be represented in great detail.

A sufficiently long simulation time was used to reach steady state. For most cases, 800 ms was enough with an integration step of 80 μs (10 000 integration steps in total). However, some cases with high DG penetration took a longer simulation time to reach steady state. Each run takes about 40 to 90 min using a computer with a processor Intel Core i7 CPU 975 operating at 3.33 GHz and installed random-access memory (RAM) memory of 24 GB.

A. Simulation Results of the Proposed Distribution

The most important information to be extracted from the experiments is the voltage profile at the loads and transformer primaries, together with the status of the protection devices. The present study does not provide any strategy to prevent voltage violations in the distribution network. The strategies for active network management are reported elsewhere [35].

Prior to the installation of DGs, a review of the light load network voltage profile was performed, showing that the selected network is stable and suitable for the study. The obtained results confirmed that there are no network protectors open in the base case and that all per-unit voltages on the primary side of the transformers as well as at the load structures remained within 1% deviation from the nominal operating voltage. The network transformers' voltage profile for all primary feeders has been examined. For all feeders, the slope of lines, containing the voltage profiles at the primary side of the transformers along each feeder, is very close to zero; showing that the system has very good voltage regulation under the light load condition.

One can observe that in all simulations, the input voltage for the transformers is always within the acceptable range. Therefore, the voltage profile analysis in this paper is based on the number of loads having voltage violations for different DG power penetration levels. A voltage violation is defined as a load having a voltage deviation larger than $\pm 5\%$ from its rated voltage for normal operating conditions and $\pm 10\%$ for emergency conditions per standards [32] and [36].

Figs. 7 and 8 summarize the results of the hundreds of simulations aimed to find potential voltage profile problems. We plot the probability of having voltage violation versus the number of loads with problems for penetration levels varying from 10%

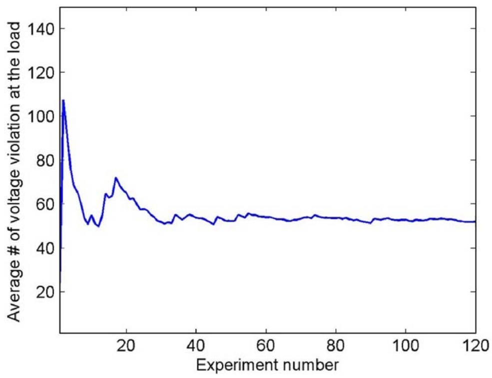

Fig. 9. Cumulative average of the number of loads with voltage violation (5%) as the number of experiments increases.

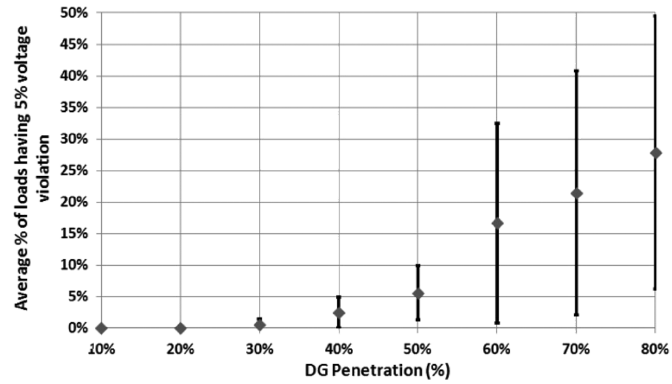

Fig. 10. Average percent of loads having voltage violations versus the DG penetration level.

TABLE VI
REQUIRED CASES FOR DIFFERENT DG PENETRATION LEVELS

DG Penetration [%]	Number of Stable cases	Number of Unstable Cases
10	40	0
20	60	0
30	60	0
40	60	0
50	60	0
60	120	1
70	140	2
80	140	4

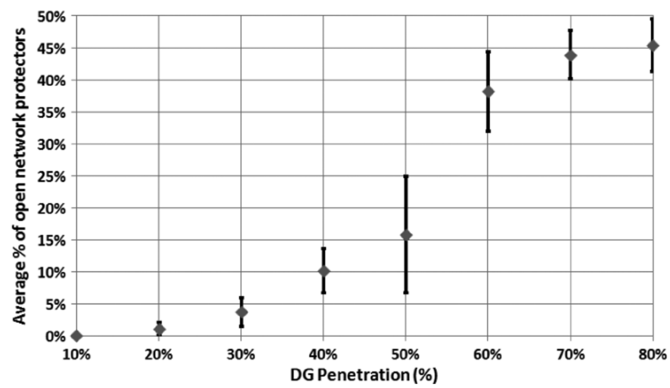

Fig. 11. Average percent of open network protectors versus the DG penetration level.

to 80% of the light load. One can see that as DG penetration increases, the probability of having voltage violations also increases. For example, looking at Fig. 7, one can see that the probability of having at least 10 loads with a voltage violation is 90% for a penetration level of 80%. The probability of having at least 10 loads with voltage violations is about 25% if the DG penetration is reduced to 40%.

To determine the number of probabilistic simulations that need to be performed to draw definitive conclusions, we note that loads with voltage violations are independent random variables for each case. Therefore, the Weak Law of Large Numbers applies, which implies that an average convergence to expected values can be reached given a sufficiently large number of experiments [37]. Thus, the number of necessary experiments is determined when the cumulative average number of loads with voltage violation converges as the number of experiments increases.

Fig. 9 illustrates such convergence properties for the average number of loads having voltage violation versus the number of cases at 60% DG penetration. One can observe that performing more than 120 simulations for this case will not change the conclusions in average. The total number of cases required for the study at different penetration levels is given in Table VI. Note that for larger penetrations, not only are a larger number of experiments required, but there are some unstable cases where most of the DGs were disconnected by their overvoltage protection; those unstable cases are excluded from the statistical analysis and will be discussed.

Other important information that can be extracted from each power penetration scenario includes: 1) the average percent of loads with voltage violations as given in Fig. 10 and 2) the average percent of open network protectors as shown in Fig. 11. The obtained standard deviations are given in each plot. A small standard deviation indicates that the data points tend to be close to the average value; in contrast, a large standard deviation indicates that data are spread over a wide range of values. One can see that standard deviations in Fig. 10 increase with DG penetration. This means that there are cases that have very few (or no) loads with voltage violations at higher DG penetrations. Therefore, it is important to understand how to distribute DGs so large penetration levels can be achieved without unacceptable voltage profiles. Fig. 11 shows that the average percent of open network protectors increases as the DG penetration increases.

It is important to compute the number of open network protectors because owners of DG would like to operate their units at a unity power factor to maximize economic benefits (maximizing generation of active power). Consequently, reactive power needs to be supplied by the utility and a sufficiently large number of network protectors spread out over the network need to be closed for that purpose. As the network protectors trip, there are fewer paths for the reactive power, increasing the possibility of voltage violations.

In this study, DG allocation using uniform distributions has been tested as well. The results are not shown here due to space limitations. However, the output power of the DG connected at some particular node is not related to the electrical load at this node. As a result, it is possible to allocate small DGs to

TABLE VII
UNSTABLE CASE DETAIL

Initial DG Penetration [%]	80
Final DG Penetration [%]	38
Violation over ±5% [%]	4
Violation over ±10% [%]	1
Max. Voltage [p.u.]	1.16
Min. Voltage [p.u.]	1.00
Open Network Protectors [%]	13
Reclosed Network Protectors [%]	56

large customers and large DGs to small customers. Therefore, the number of nodes with voltage violations and the amount of open network protectors are larger than when the load matching approach described in this paper is used. Moreover, it was determined that the attainable average power of the DG penetration is smaller using uniform distribution functions.

B. Example of an Unstable Case

During the process of finding the voltage profile, we have found a few unstable cases where the automatic protections reconfigure the network. The typical dynamic process of an unstable case is as follows: 1) most of network protectors open on sensing reverse power; 2) many of DG protections open on overvoltage; 3) most of the open network protectors reclose after DGs trip; and 4) the voltages at loads return to the acceptable range. The example shown in Table VII initially had 80% DG penetration and after shedding DG, it came to a steady-state condition with only 38% DG penetration.

C. Effects of DG Size

To study the effects of the size of the installed DG on the voltage profile, a set of deterministic tests was performed. The results from these scenarios show that DG penetration could reach 100% of the load when having proper allocation. Conversely, other results show that a small percent of DG power may cause voltage profile problems depending on the location.

1) Maximum DG Power Penetration (Best Case Scenario): To illustrate how large penetrations are possible with no voltage violations, a number of deterministic DG tests were performed. A DG is installed at each customer with penetrations ranging from 90% to 102% of their light load demand. Simulation results on the voltage profile and number of open network protectors are shown in Table VIII. For instance, the 90% case means that each load has DG power in the amount of 90% of its light load power. One can see that there are no voltage violations with DG penetrations of up to 95% of the network light load. One can also note that the network structure is not modified (no network protectors have opened). When the DG penetration exceeds 100%, the network structure is not reliable anymore (several network protectors open) and voltage violations start occurring.

2) Minimum DG Power Penetration (Worst Case Scenario): With deterministic simulations installing DG at certain locations, we are able to produce unacceptably low or high voltages at certain loads. A case with only one DG of 1.2 MW installed at load BC2858 (original demand of 450 kW) serves to illustrate the problem as shown in Table IX. The table gives the details of the case including information on DG and load. Note that there are two loads with voltage violations and four network protectors of the area have opened.

TABLE VIII
LOAD VOLTAGE VIOLATIONS FOR DETERMINISTIC LARGE PENETRATIONS

DG Penetration [%]	Violations over ±5% [%]	Max. Voltage [p.u.]	Min. Voltage [p.u.]	Open Network Protectors [%]
90	0	1.048	1.038	0
95	0	1.050	1.040	0
100	1	1.052	1.040	4
101	3	1.052	1.041	21
102	49	1.095	1.047	85

TABLE IX
WORST SCENARIO DETAIL

DG Penetration [%]	2.5
Number of Voltage Violations over ±5%	2
Max. Voltage [p.u.]	1.07
Min. Voltage [p.u.]	1.00
Number of Open Network Protectors	4
Load Voltage [p.u.]	1.07
Light Load Power [MW]	0.45
Peak Load Power [MW]	1.68
Size of DG [MW]	1.20

As shown in Table IX, the size of this DG is almost three times the light load of load BC2858. This causes the load to have an overvoltage violation. Moreover, a neighboring load also exceeds the permissible 5% overvoltage.

V. CONCLUSIONS

This paper has presented a method to quantify the negative impacts on voltage profile that a range of DG penetration levels may have in highly meshed LV secondary networks.

The simulations have been performed under the worst possible network conditions [i.e. when the minimum demand (light load) coincides with the maximum DG output power]. The DGs have been allocated using the Gibbs sampler algorithm and Monte Carlo methods. This allowed for the realistic simulation of the future DG expansions under physical and regulatory constraints. The study has been carried out using a very detailed representation of the system with hundreds of time-domain simulations using the EMTP. It was shown that the worst-case analysis has special significance in the case of urban distribution networks since any violation of the unidirectional power flow may result in the tripping of network protectors followed by serious voltage problems in the LV grid. For example, it was determined that small amounts of DG power (under 2.5%), installed at the wrong location, can produce unacceptable voltage profiles. At the same time, very large amounts of DG power (close to 100% of the network light load) installed with the adequate strategy enable acceptable operating conditions.

It was found that the primary factors that led to overvoltages and undervoltages are a surplus of DG power in localized areas of the secondary network that cause the tripping of the network protectors.

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